Multi-valued Harmonicity

Abstracts

Cristiana De Filippis (University of Parma)

Title: On the Born-Infeld model

Abstract: The Born-Infeld theory replaces the quadratic Maxwell Lagrangian with a nonlinear model designed to render the self-energy of a point charge finite. From the PDE viewpoint, this comes at the price of a singular and nonuniformly elliptic structure, making the regularity of weak solutions a very delicate question. In this talk, I will discuss novel advances on the regularity of solutions to the electrostatic Born-Infeld equation. From recent, joint work with Giuseppe Mingione (Parma).

Simon Donaldson (Imperial College London)

Title: Multivalued maximal sections

Abstract: Maximal submanifolds are defined in spaces of indefinite signature as solutions of the Euler-Lagrange equations of the volume functional. We will consider a version of the theory involving flat vector bundles on the complement of codimension 2 submanifolds. We will outline some of the motivation from G2 geometry in 7 dimensions and similarities and differences with the case of multivalued harmonic 1-forms. In the last part of the talk we will discuss analytic aspects, particularly of the deformation theory.

Federico Francheschini (Stanford University)

Title: Scalar approach to Z_2 eigensections: thin obstacles and spectral partitions

Abstract: It has been noted that finding critical Z_2 eigensections is akin to solving an overdetermined elliptic PDE, with the critical set playing the role of a free boundary. The goal of this talk is to explore this analogy in more detail and build a dictionary between the two settings. First, we will explain how two well-studied scalar free boundary problems in fact produce critical Z_2 eigensections (but not all of them).
Then, we will survey the main results of these scalar problems and highlight how state-of-the-art results in the scalar theory—such as generic regularity and fine regularity properties of the free boundary—point toward direct counterparts in the emerging theory of Z_2 eigensections.

Andriy Haydys (Université Libre de Bruxelles)

Title: Deformation rigidity for Z/2 eigensections

Abstract: Recently, a lot of attention has been drawn by so-called Z/2 harmonic functions, whose regularity is closely related to certain Z/2 eigensections. I will talk about a rigidity result for generic Z/2 eigensections. This is a joint project with Siqi He and Andries Salm.

Siqi He (Morningside Center, CAS)

Title: Some examples of Z/2-harmonic 1-forms in dimension three

Abstract: A central difficulty in the study of Z/2-harmonic 1-forms is understanding their singular sets. In this talk, we will discuss some new methods for constructing examples in dimension three with interesting singular sets by varying the background metric.

Yang Li (University of Cambridge)

Title: Z/2 harmonic 1-forms from Fueter sections to monopole moduli bundles

Abstract: In the Donaldson-Segal programme, one aims to relate counting invariants for Calabi-Yau monopoles to the counting invariants of special Lagrangians with extra data. I will explain how this is formally related to Fueter sections with target in monopole moduli spaces, and how Z/2 harmonic 1-forms arise from the compactness problem.

Greg Parker (University of California)

Title: Choices of Infinite-Dimensional Gauge in Deformations of Submanifolds

Abstract: This is a talk about a theme I have noticed across several problems more than it is a talk about any particular result. In both gauge theory and the study of minimal submanifolds with singularities, one encounters nonlinear PDE problems coupled to the deformations of submanifolds. While these problems appear at first to "see" only the infinitesimal deformations of the submanifold, how these deformations are extended globally turns out to be an infinite-dimensional gauge choice with profound consequences for the analytic setup of the problem. I will give some examples where the naive choice of this gauge leads to a failure of ellipticity, boundedness and even tameness of certain differential operators, and how more geometrically informed choices of gauge resolve this.

Alessandra Pluda (University of Pisa)

Title: Inverse mean curvature flow and applications

Abstract: A classical solution to the inverse mean curvature flow is a time-dependent family of smooth immersions whose normal velocity, at each point and time, equals the inverse of the mean curvature. Over the years, the inverse mean curvature flow has proven to be an extremely useful tool, ranging from the proof of the Riemannian Penrose inequality to the characterization of Ricci-pinched 3-dimensional Riemannian manifolds. In this talk, I will focus on the inverse mean curvature flow as an instrument to prove geometric inequalities, from the isoperimetric inequality to a family of Willmore/Minkowski-type inequalities.

Andries Salm (University of Oxford)

Title: Metric perturbations of degenerate Z/2-harmonic 1-forms

Abstract: Many deformation theories for Z/2-harmonic objects have a non-degeneracy condition. This degeneracy condition is often given as a decay behaviour near the branching set. In this talk we study the behaviour of degenerate Z/2-harmonic 1-forms. For a natural class of examples, we prove that a degenerate Z/2-harmonic 1-form can be deformed to a nearby non-degenerate one using suitable perturbation of the ambient Riemannian metric.

Anna Skorobogatova (ETH Zurich)

Title: mod(q) area-minimizing currents: structure and singularities

Abstract: One well-studied framework for the Plateau problem uses currents with multiplicities modulo q, for a fixed integer q. This setting allows for minimizing surfaces to exhibit codimension 1 singularities like triple junctions, which are seen in physical soap films, and has close connections to the known regularity theory for stable minimal submanifolds, while at the same time providing the framework of a minimization problem.

I will give an overview of the history of the problem and discuss some recent structural results, with a focus on the local behavior of two-dimensional mod(q) minimizing currents in arbitrary codimension near particular singularity models, in the spirit of the work of Jean Taylor which treats the special case of two-dimensional mod(3) minimizing currents in $\mathbb{R}^3$.

Ryosuke Takahashi (National Cheng Kung University)

Title: Adiabatic limit of APS-boundary conditions, trace maps and Z_2-harmonic spinors

Abstract: Abstract. In this talk, we consider a closed 4-manifold M containing an embedded surface Σ. We will first review the Atiyah-Patodi-Singer (APS) index defined on the complement of a small tubular neighborhood of Σ and formulate its adiabatic limit as the tubular radius approaches zero. It will induce a “degenerate” boundary condition on Σ, which we continue to refer to as the APS boundary condition (D/+, D^+_{APS}). Secondly, we will introduce an alternative degenerate boundary condition on Σ, denoted ($D/^+$, D^+_{ψ}), which is defined by Z_2-harmonic spinors ψ. Our main result demonstrates that these two distinct boundary conditions yield the same index. This is joint work with Rafe Mazzeo and Andry Haydys.

Thomas Walpuski (Humboldt University)

Title: Universal moduli spaces of Z/2Z harmonic spinors

Abstract: More than a decade ago, Taubes and his disciples began to observe that the failure of compactness for a wide variety of “new” low-dimensional gauge theories leaves behind evidence in the form of a Z/2Z harmonic spinor. The latter have since appeared in related questions in gauge theory and calibrated geometry, and might to play a role in the conjectural construction of invariants of manifolds with special holonomy.

In light of the above it seems pressing to understand what kind of a phenomenon of the existence of Z/2Z harmonic spinor is; in particular, whether it persists under (small) deformations of the background parameters. This question of the deformation theory of Z/2Z harmonic spinors has been approached before in the work of Donaldson, Parker, and Takahashi. This talk presents joint work with Siqi He and Greg Parker that develops such a theory in a unified framework building upon earlier joint work with Gorapada Bera and a suggestion by Donaldson.

Neshan Wickramasekera (University of Cambridge)

Title: Planar frequency, asymptotic normal forms, and local topology of area-minimizing currents

Abstract: A fundamental problem in geometric measure theory is to understand the local structure of n-dimensional area-minimizing rectifiable currents T of codimension at least 2. Almgren's 1983 theory (subsequently made more accessible by De Lellis–Spadaro) provides a powerful framework establishing the sharp Hausdorff dimension upper bound n − 2 for the singular set. The work of White, Chang, and Micallef–White gives a remarkably detailed structure theory when T is 2-dimensional, in which case the singularities are isolated. When n ≥ 3, however, the local structure of T and the nature of its singularities remain more subtle, particularly at branch points, where planar tangent cones occur.

In a series of papers with Brian Krummel, we develop a new framework for this problem in arbitrary dimension n. Uniform decay estimates at branch points are the primary organising principle in this framework, which connects these estimates with the dimension and structure of the singular set, as well as with the structure of T, in a unified approach. A central conceptual novelty is the introduction of an intrinsic frequency function, planar frequency, which satisfies a monotonicity property and takes values ≤ 1 on stationary cones. These properties provide control of the decay rate of T towards planes and lead to a quantitative decomposition of the singular set.
I will describe this framework and some of its main consequences, including a more direct proof of Almgren's n−2 bound and Hn−2-almost everywhere uniqueness of tangent cones. Most importantly, at Hn−2-almost every branch point Z, the current admits an asymptotic normal form: there is a unique tangent plane, an intrinsic branching order OT (Z) > 1 taking rational values, and a unique, nonzero, OT (Z)-homogeneous cylindrical multi-valued tangent function, together with quantitative remainder decay.

This normal form has strong consequences for both the structure of the singular set and the local topology of T. In particular, it yields a local decomposition of the singular set into finitely many disjoint, locally compact, locally (n−2)-rectifiable pieces. It also gives a "full-projection" criterion, in terms of branching order, under which certain branch points Z are classical: near Z, the support of T is homeomorphic to an n-disk and admits a C1,μ parametrization while the singular set is an (n − 2)-dimensional embedded C1,μ submanifold consisting only of branch points. This is a higher-dimensional analogue of the Chang–Micallef–White structural description in dimension 2. The full-projection condition is automatically true when n = 2, and explicit algebraic surfaces show that it is necessary for the conclusion in general dimensions.

A central theme of the talk will be how planar frequency reduces the role of the Almgren center manifolds in the branch-point analysis: center manifolds are needed only in a specific regime, where they enjoy additional simplifying properties and are shown to be canonical up to branching order. This reduction is key to obtaining the asymptotic normal form. I will also briefly contrast this approach with the contemporaneous work of De Lellis, Minter, and Skorobogatova, which establishes a.e. uniqueness of tangent cones and countable rectifiability of the singular set using a different approach based on iterative center manifolds at every branch point.

Therese Wu (University of Houston)

Title: Novel local models for homogeneous Z/2 harmonic forms and spinors on R^4

Abstract: Homogeneous Z/2-harmonic forms and spinors arise as rescaling limits of divergent sequences of solutions to certain four-dimensional generalizations of the Seiberg–Witten and (anti)-self-dual Yang–Mills equations. We describe novel local singularity models for Z/2 harmonic 1-forms, self-dual 2-forms and spinors in dimension 4. These models are homogeneous versions on R^4 whose singular sets are cones on the 1-skeleta of certain regular 4-dimensional polytopes.

Dashen Yan (Stanford University)

Title: Dynamical Aspects of Multivalued Harmonic 1-Forms

Abstract: This talk explores the dynamical properties of multivalued harmonic 1-forms and their application to new geometric constructions.

In the first part, I will discuss Calabi’s intrinsic characterization of harmonic 1-forms in the multivalued setting. This perspective leads to a better understanding of the dynamics of \mathbb{Z}_2-harmonic forms and allows us to establish a “soft” gluing theorem for various such forms without relying on analytic tools.

In the second part, I will introduce branched harmonic k-fold covers, higher-dimensional analogues of spectral covers on Riemann surfaces. I will focus on examples obtained by resolving intersections of harmonic linear graphs in T^*\mathbb{R}^3, and demonstrate the existence of gradient trivalent graphs in the linearized setting of the Donaldson–Scaduto picture.

This talk is based on joint work with Jiahuang Chen and Siqi He, as well as ongoing joint work with Gorapada Bera.