Hodge Theory and Algebraic Cycles Abstracts

Ekaterina Amerik (HSE University)

Title: Parabolic automorphisms of hyperkahler manifolds and related questions

Abstract: A parabolic automorphism f of a hyperkahler manifold is, essentially, an automorphism of infinite order preserving a lagrangian fibration. It can be shown that up to taking a power, it acts by fiberwise translations (recall that a general fiber of a lagrangian fibration is a torus). A few years ago, together with Verbitsky, we proved that the orbits of f must be dense in a general fiber. Cantat asked if, nevertheless, for a dense set of fibers the orbits are finite.

Clearly this is true if the "Betti map", sending a general point of the base to the corresponding translation, is generically of maximal rank. The Betti map has recently been studied by Andre-Corvaja-Zannier, Voisin, Gao. One can deduce the maximality of the rank from their results; the purpose of this talk is to explain an alternative proof of dynamical nature, a joint work with Cantat.

Benjamin Bakker (University of Illinois, Chicago)

Title: Baily--Borel compactifications of period images and the b-semiampleness conjecture

Abstract: Building on previous work of Satake and Baily, Baily and Borel proved in 1966 that arithmetic locally symmetric varieties admit canonical projective compactifications whose graded rings of functions are given by automorphic forms. Such varieties include moduli spaces of abelian varieties and have rich algebraic and arithmetic geometry. Griffiths suggested in 1970 that the same might be true for the image of any period map, which would then provide canonical compactifications of many more moduli spaces, including for instance those of Calabi--Yau varieties. In joint work with S. Filipazzi, M. Mauri, and J. Tsimerman, we confirm Griffiths' suggestion, and prove that the image of any period map admits a canonical functorial projective compactification. We also show how the same techniques yield a resolution to an important conjecture in birational geometry, the b-semiampleness conjecture of Prokhorov--Shokurov. Both proofs crucially use o-minimal GAGA, and the latter application additionally uses results of Ambro and Kollar on the geometry of minimal lc centers.

Anna Cadoret (Jussieu)

Title: Tate locus - conjectures and results

Abstract: Let k be a field and X a geometrically connected variety over k. The Tate or degeneracy locus of a l-adic local system on X is the etale counterpart of the Hodge locus of a VHS. While in the last decade tremendous progresses have been made in understanding the latter thanks to, in particular, techniques from ominimality, much less is known about the former. I will review the main conjectures (and mention briefly some applications) about this locus when k is a number field, and explain what we can currently prove. If time remains, I will sketch some of the proofs. This should include joint works with Jakob Stix and Akio Tamagawa.

Daniel Huybrechts (University of Bonn)

Title: Universal Brauer—Severi varieties and applications

Abstract: This is joint work with Frank Gounelas. Using techniques from classical algebraic geometry, we construct universal Brauer-Severi varieties of fixed period and index and study their geometry. We determine their cohomology and their Brauer and Picard groups and show that they are almost always simply connected.

Bruno Klingler (Humboldt University of Berlin)

Title: Special loci for local systems

Abstract: Given a local system on a complex algebraic variety, what are the subvarieties on which the monodromy drops? The talk will discuss these monodromy special loci, a natural generalisation of (the positive period dimension components of) the Hodge loci.

Eyal Markman (University of Massachusetts)

Title: Cycles on abelian 2n-folds of Weil type from secant sheaves on abelian n-folds

Abstract: In 1977 Weil identified a 2-dimensional space of rational classes of Hodge type (n,n) in the middle cohomology of every 2n-dimensional abelian variety with a suitable complex multiplication by an imaginary quadratic number field. These abelian varieties are said to be of Weil type and these Hodge classes are known as Weil classes.

The connected components of the moduli space of polarized abelian varieties A of Weil type have three discrete invariants, dim(A), the imaginary quadratic number field K, and the discriminant. The latter is the coset in $Q^*/Nm(K^*)$ of the determinant of a natural Hermitian form.

We prove that the Weil classes are algebraic for all abelian sixfolds of Weil type of discriminant -1, for all imaginary quadratic number fields. The algebraicity of the Weil classes follows for all abelian fourfolds of Weil type (for all discriminants and all imaginary quadratic number fields), by a degeneration argument of C. Schoen. The Hodge conjecture for abelian fourfolds is known to follow from the above result.

Alexander Petrov (MIT)

Title: Coniveau filtration in p-adic cohomology

Abstract: Generalized Hodge conjecture in Grothendieck's formulation has the following purely algebraic consequence: if a smooth proper variety X over C has no non-zero global i-forms, then the restriction map on cohomology $H^i(X)-H^i(U)$ is zero for some non-empty Zariski open U in X. I will describe an approach to checking this special case of the Hodge conjecture via p-adic Hodge theory. We prove that for a smooth proper scheme X over Z_p with no non-zero global i-forms the restriction map from prismatic cohomology of X to prismatic cohomology of every affine open in X vanishes in the separated quotient of prismatic cohomology. This generalizes an observation of Katz that the analogous vanishing holds for mod P de Rham cohomology. I will discuss a proof of this vanishing result, its relation to Milnor-Bloch-Kato conjecture, and, perhaps more interestingly, what the obstruction is to deducing the above special case of the Hodge conjecture from this vanishing. This is joint work with Hélène Esnault and Mark Kisin.

Stefan Schreieder (Leibniz University Hannover)

Title: Matroids and the integral Hodge conjecture for abelian varieties.

Abstract: Hodge originally formulated his conjecture with integral coefficients. Atiyah and Hirzebruch showed in 1962 that this fails for torsion classes, and later Kollár gave non-torsion counterexamples. Since then, the integral Hodge conjecture has become a property of individual varieties rather than a universal statement. A central open case concerned abelian varieties. In this talk, I will explain how ideas from the theory of regular matroids allow us to prove that the integral Hodge conjecture fails for large classes of abelian varieties. As a consequence, building on work of Voisin, we obtain that very general cubic threefolds are not stably rational, strengthening the classical result of Clemens and Griffiths that smooth cubic threefolds are not rational. Joint work with Philip Engel and Olivier de Gaay Fortman.

Junliang Shen (Yale University)

Title: Cohomology of the universal Jacobian and compactifications

Abstract: The Jacobian variety is a fundamental object associated with a curve. The universal Jacobian over the moduli space of curves combines the geometric complexity of both abelian varieties and the moduli of curves. The purpose of this talk is to discuss two fundamental questions concerning the cohomology/Chow theory of universal Jacobians: (1) the dependence of the cup product (or intersection product) on the degree.

(2) the dependence on the choice of compactification. Our approach combines the Fourier transform techniques of Beauville and Deninger–Murre, introduced over 30 years ago, with recent developments in the study of perverse filtrations associated with abelian fibrations. Based on joint work with Younghan Bae, Davesh Maulik, and Qizheng Yin.

Salim Tayou (Dartmouth College)

Title: The non-abelian Hodge locus

Abstract: Classical finiteness results of Arakelov and Parshin state that a fixed quasi-projective curve can only carry finitely many non-isotrivial families of smooth projective curves of fixed genus g. These results have been generalized by Faltings and Deligne for polarized variations of Hodge structure of arbitrary weight. In this talk, I will explain a further generalization which only depends on the topology of the base and not the algebraic structure, giving thus a partial answer to a question asked by Deligne. I will then explain an application proving the algebraicity of the non-abelian Hodge locus, partially solving a conjecture of Simpson. The results in this talk are joint work with Philip Engel.

Claire Voisin (CNRS, Jussieu)

Title: Universal 0-cycles and the integral Hodge conjecture

Abstract: The universal 0-cycle of a smooth projective variety X, parameterized by the Albanese variety Alb(X), is a generalization of the Poincaré divisor for a curve. However it does not always exists, and this corresponds to counterexamples to the integral Hodge conjecture on Alb(X) x X. I will discuss an example where a universal 0-cycle does not exist, although X has a representable group of zero-cycles. This answers a question of Colliot-Thélène. The construction relies on another counterexample to the integral Hodge conjecture by Benoist and Ottem.