Zeta and *L*-functions
Abstracts

Louis-Pierre Arguin (University of Oxford)

Title: The Riemann zeta-function on the critical line and (branching) random walks

Abstract: Fyodorov, Hiary, and Keating proposed ten years ago that the large values of the Riemann zeta-function in a short interval of the critical line could be precisely estimated by a stochastic process with logarithmically decaying correlations. In this talk, we will explain this connection and sketch the proof of their conjecture up to fluctuations of order one based on joint work with P. Bourgade and M. Radziwill. The analysis is very close to the study of large values of branching random walks going back to the seminal work of Bramson in the early 1980s. The random walk point of view is robust and can also be used to get precise estimates for the moments of the zeta function in short intervals, but also in long ones. In particular, we will discuss the connection with the Moments Conjecture of Keating and Snaith. This talk is based on joint work with many people, including E. Bailey, C. Chang, N. Creighton, J. Hamdan, and A. Roberts.

Vorrapan Chandee (Kansas State University)

Title: Moments of one-level densities for a large orthogonal family of *L*-functions

Abstract: Katz and Sarnak conjectured that the statistics of low-lying zeros of families of *L*-functions match with the scaling limit of eigenvalues from the random matrix theory. In this talk, I will discuss recent joint work with Yoonbok Lee and Xiannan Li on the nth centered moments of one level densities of a large orthogonal family of *L*-functions associated with holomorphic Hecke newforms of level q, averaged over q~Q. The nth centered moments are closely related to the n-level densities of low-lying zeros of *L*-functions. We verify the Katz-Sarnak conjecture for these statistics, in the range where the sum of the supports of the Fourier transforms of test functions lies in (-4, 4). Key challenges include identifying off-diagonal main terms and resolving a combinatorial problem in matching number-theoretic results with predictions from random matrix theory.

Alexandra Florea (University of California, Irvine)

Title: Simultaneous non-vanishing of *L*-functions at the central point

Abstract: In this talk, I will focus on simultaneous non-vanishing results for Dirichlet L-functions at the central point 1/2. Specifically, I will describe non-vanishing results (conditional on GRH) for two *L*-functions associated to Dirichlet characters in the same Galois orbit, and simultaneous non-vanishing of four *L*-functions as we vary over characters \chi modulo q, conditional on GRH and the Ramanujan-Petterson conjecture. This is based on joint works with Bui-Ngo and Bui-Milinovich.

Adam Harper (University of Warwick)

Title: The square of the Riemann zeta function gives rise to critical multiplicative chaos

Abstract: Multiplicative chaos is a class of random measures, that have recently been found to have strong connections with number theoretic objects like *L*-functions and character sums, and the phenomenon of ``better than squareroot cancellation". Saksman and Webb have conjectured that integrating test functions against absolute powers of the Riemann zeta function should give rise to these measures. The square of the zeta function is particularly interesting, since this should correspond to the so-called critical chaos. I will report on joint work (in preparation) of myself, Saksman and Webb, which proves their conjecture for zeta squared. I will try to give a gentle introduction to these problems, and also indicate some of the main proof ideas, which may be of independent interest.

Kaisa Matomäki (University of Turku)

Title: On optimality of mollifiers

Abstract: Mollifiers are used in a variety of contexts, for instance to study zeroes of *L*-functions. I will discuss the general question of finding optimal mollifiers and provide criteria to identify them provided the corresponding mollified moments can be computed. As an application, I will discuss non-vanishing of central values of Dirichlet *L*-functions. The talk is based on joint work with Martin Čech.

James Maynard (University of Oxford)

Title: New Zero density estimates

Abstract: The Riemann Hypothesis would have many fantastic consequences for the distribution of primes, but unfortunately it appears to be out of reach. However, it turns out several of these consequences do not require the full strength of the Riemann Hypothesis - it would be sufficient to show that 'most' zeros are 'close' to the 1/2 line. I'll talk about some results (joint with Larry Guth) which improve on a key bottleneck for these 'zero density' questions related to the number of possible zeros on the 3/4 line, with corresponding applications to the distribution of primes.

Paul Nelson (Aarhus University)

Title: Equidistribution and moments of *L*-functions

Abstract: I will survey how integral representations and period formulas recast equidistribution problems (for instance, involving unipotent shears) as moment problems for automorphic L-functions, a mechanism originating in earlier work of many authors. I will recall rank-one prototypes in which the varying vectors are pure translates of fixed ones (e.g. the triple-product setting), as well as the microlocal framework from my joint work with Venkatesh, where recent progress in homogeneous dynamics now yields effective consequences. I will then indicate a higher-rank variant, joint with Subhajit Jana, expressing GL(n) moments appear as L^2-norms of shears of fixed automorphic forms; here, implications for moments and subconvexity remain conditional on dynamical input not yet available.

Dan Petersen (Stockholm University)

Title: Moments in families of *L*-functions over function fields via homotopy theory

Abstract: This is a report of joint work with Bergström-Diaconu-Westerland and Miller-Patzt-Randal-Williams. There is a "recipe" due to Conrey-Farmer-Keating-Rubinstein-Snaith which allows for precise predictions for the asymptotics of moments of many different families of *L*-functions. Our work concerns the CFKRS predictions in the case of the quadratic family over function fields, i.e. the family of all *L*-functions attached to hyperelliptic curves over some fixed finite field. One can relate this problem to understanding the homology of the braid group with certain symplectic coefficients. With Bergström-Diaconu-Westerland we compute the stable homology groups of the braid groups with these coefficients, together with their structure as Galois representations. We moreover show that the answer matches the number-theoretic predictions. With Miller-Patzt-Randal-Williams we prove a uniform range for homological stability with these coefficients. Together, these results imply the CFKRS predictions for all moments in the function field case, for all sufficiently large (but fixed) q.

Ian Petrow (University College London)

Title: The Petersson / Bruggeman / Kuznetsov formulas for specified local components and some new cases of Weyl subconvexity

Abstract: The classical Petersson / Bruggeman / Kuznetsov (PBK) formula relates a sum of Fourier coefficients of a family of cusp forms defined in terms of their spectral parameters (equivalently, their associated representation of GL_2(R)) to a sum of Kloosterman sums. I will present some new PBK formulas (joint w/ Y. Hu and M.P Young) that relate a sum of Fourier coefficients of a family of cusp forms defined in terms of their associated representation of GL_2(Q_p) to a sum of generalized Kloosterman sums. As an application of these formulas, I will present some new estimates for cubic moments of *L*-functions, which lead to some new Weyl-strength subconvexity estimates for central values of *L*-functions of GL(2) cusp forms with trivial central character.

Lillian Pierce (Duke University)

Title: Multiplicative character sums: bounds and counting problems

Abstract: One fundamental motivation for bounding multiplicative character sums is their relationship to subconvexity estimates for Dirichlet L-functions. Another motivation has arisen more recently: using character sums to count integral points in thin sets. In this talk we will survey both motivations, outline recent progress on using character sums to count points, and describe some open problems.

Brad Rodgers (Queens University)

Title: Approximating arithmetic functions and random matrix theory

Abstract: Random matrix integrals frequently appear in connection to problems related to the distribution of the Riemann zeta-function and other L-functions. In this talk I hope to review the connection between the GUE Hypothesis for zeta zeros on the one hand and short-interval sums / random matrix integrals on the other. I will also describe an interpretation for related random matrix integrals in terms of a best-approximation problem for permutations and discuss ongoing work related to best-approximation problems over the integers.