PDE and Fluids
Abstracts

Maria Colombo (EPFL)

Title: Non-uniqueness and instability in vanishing viscosity solutions of the Euler equations

Abstract: The talk will provide an overview of the developments on non-uniqueness and instability of solutions of the Euler and Navier-Stokes equations, focussing on results about the sharpness of Yudovich well-posedness theorem for the 2 dimensional Euler equations. It will then highlight new results showing that solutions obtained in the vanishing viscosity limit from the (well-posed) Navier-Stokes equations can be nonunique.

Diego Córdoba (ICMAT)

Title: Vortex layer cascades and finite time singularities for incompressible fluids.

Abstract: We present a mechanism for finite time blow-up in the two-dimensional incompressible Boussinesq equations, achieved through a multi-layer degenerate pendula dynamics with a uniform $C^{1, \alpha}$ forcing term. The construction gives rise to a vortex layer cascade in which vorticity and temperature gradients concentrate successively, leading to singularity formation in finite time. We will outline the key ideas behind the cascade mechanism and discuss its implications for singularity formation in incompressible fluid equations. These results are joint works with Luis Martínez-Zoroa, Andrés Laín Sanclemente and Fan Zheng.

Tarek Elgindi (Duke University)

Title: Aspects of the long-time behavior of ideal fluids

Abstract: We will discuss various results related to the long-time behavior of inviscid fluids. We will start with a discussion of steady and traveling wave solutions. We will then discuss results related to small scale creation, filamentation, and mixing. We will do this based on joint works with many co-authors.

Javier Gómez-Serrano (Brown University)

Title: Discovery of unstable singularities

Abstract: In this talk I will explain how to construct numerically several new unstable singularities to certain equations in fluids (CCF, IPM, Boussinesq) using machine learning methods. Our approach combines curated machine learning architectures and training schemes with a high-precision Gauss-Newton optimizer, achieving accuracies that significantly surpass previous work across all discovered solutions, reaching near double-float machine precision, attaining a level of accuracy constrained only by the round-off errors of the GPU hardware, potentially leading to rigorous mathematical validation via computer-assisted proofs.

Joint work with Tristan Buckmaster, Gonzalo Cao-Labora, Yao Lai, Yongji Wang and Google Deepmind.

Zaher Hani (University of Michigan)

Title: Longtime derivation of the Boltzmann and fluid equations

Abstract: We will discuss recent joint work with Yu Deng (Chicago) and Xiao Ma (Michigan) on the rigorous derivation of the Boltzmann and fluid equations from Newton's laws. This executed a program proposed by Hilbert in his sixth problem from 1900, which asked for the derivation of fluid equations (like Euler’s and Navier-Stokes’) using Boltzmann’s kinetic theory as an intermediate step. The result follows parallel progress (by Deng and myself) in the wave setting, where colliding particles are replaced by interacting waves.

Alexander Kiselev (Duke University)

Title: Singularity suppression by fluid flow

Abstract: Transport by fluid flow can provide one of the less understood regularization mechanisms in PDE. In this talk, I will focus on the 2D Keller-Segel equation for chemotaxis set on a general domain and coupled via buoyancy with the fluid obeying Darcy's law - a much studied model of the incompressible fluid flow in porous media. It is well known that solutions to the 2D Keller-Segel equation can form singularities in finite time if the mass of the initial data is larger than critical. It turns out that if the equation is coupled with fluid flow obeying Darcy's law via buoyancy, this completely regularizes the system, leading to globally regular solutions for arbitrarily large initial data. One of the key ingredients in the proof is a generalized Nash inequality, which employs anisotropic norm that is natural in the context of the incompressible porous media flow. This talk is based on works joint with Kevin Hu, Naji Sarsam, and Yao Yao.

Matthew Novack (Purdue University)

Title: Dissipative Euler solutions and helicity

Abstract: In this talk, I will present a series of recent and ongoing works, in part joint with H. Kwon, V. Giri, and V. Vicol, in which we construct intermittent weak solutions to the 3D Euler equations. The techniques we have developed allow us to (1) prove a ”strong” version of Onsager’s conjecture, and (2) construct solutions with well-defined helicity which is not conserved.

Steve Shkoller (University of California, Davis)

Title: The sad story of the steepening soundwave

Abstract: For open sets of smooth, generic, initial data, solutions to the Euler equations develop finite time singularities in the form of gradient blowup such as shock formation, as well as amplitude blowup such as implosion. In the analysis of such solutions, a fundamental question is: how can the Euler solution be estimated and bounded as it evolves past the first time of singularity, often through a succession of "infinities" (in which entire families of fast acoustic soundwaves experience one catastrophe after another). I will describe a new geometric framework for this analysis in multiple space dimensions, and a new type of energy estimate which allows for uniform control of Sobolev norms as the Euler solution evolves along a hypersurface of blowups. This is joint work with Vlad Vicol.

Klaus Widmeyer (University of Zurich)

Title: Global axisymmetric Euler flows with rotation

Abstract: We discuss the construction of a class of global, dynamical solutions to the 3d incompressible Euler equations near the stationary state given by uniform "rigid body" rotation. At the heart of this result is a dispersive effect due to rotation, which we discuss with some context. In our approach, it is captured in a "method of partial symmetries", which is adapted to maximally exploit the symmetries of this anisotropic problem, both for the linear and nonlinear analysis, and allows to globally propagate sharp decay estimates. This is based on joint work with Y. Guo and B. Pausader (Brown University).