

# Symplectic Geometry, Low Dimensional Topology, and Quantum Fields

## 25-29 September 2023

### Abstracts

**Mohammed Abouzaid** (Columbia University)

Title: Flow categories and derived orbifolds

Abstract: In joint work with McLean and Smith, we showed that differential geometric methods can be used to provide concrete models for moduli spaces of stable pseudo-holomorphic maps as 0-loci of equivariant vector bundles on manifolds carrying an actions of the unitary group; we called such objects "global Kuranishi structures". This leads one to be tempted to use this formalism as a basis for a general theory of bordism that would be universal among those with relevance to Floer theory. Unfortunately, I do not know how to prove the Mayer-Vietoris property without appealing to Pardon's result on the existence of enough vector bundles. I will thus explain how this result appears in a key step of forthcoming joint work with Blumberg, which constructs a stable infinity category of flow categories enriched in derived orbifolds, which achieves the goal of tautologically assigning a homotopy type to each Floer flow category.

**Mina Aganagic** (University of California, Berkeley)

Title: Homological link invariants from Floer theory

Abstract: I will describe a theory which generalizes Heegard-Floer theory from  $\mathfrak{gl}(1|1)$  to arbitrary Lie (super) algebras. The theory categorifies quantum group link invariants. The corresponding category of A-branes has many special features which render it solvable explicitly. In this talk, I will describe how the theory is solved, and how homological link invariants arise from it. I will focus on the two simplest cases, the  $\mathfrak{gl}(1|1)$  theory itself, and the  $\mathfrak{su}(2)$  theory, categorifying respectively the Alexander and the Jones polynomials.

**Aleksander Doan** (University College London and University of Cambridge)

Title: An invitation to the Fueter equation

Abstract: This talk is motivated by the idea that there should be a category associated with a pair of complex Lagrangians in a hyperkahler manifold. The building blocks of this putative category are pseudo-holomorphic strips and solutions to the Fueter equation, a nonlinear generalization of the Dirac equation. I will discuss some questions and results in analysis inspired by this proposal, as well as the special case of cotangent bundles. This talk is based on joint work with Semon Rezchikov.

**Kenji Fukaya** (Stony Brook University)

Title: Gromov Hausdorff convergence of filtered A infinity category and its (potential) application to Lagrangian correspondence and Gauge theory

Abstract: I will explain A infinity category version of Gromov Hausdorff convergence and its expected application to the study of Lagrangian correspondence and filtered version of Atiyah-Floer conjecture

**Sergei Gukov** (Caltech)

Title:  $SL(2, \mathbb{C})$  Floer theory and quantum groups at generic  $q$

**Monica Jinwoo Kang** (Caltech)

Title: Finding isomorphic superconformal field theories

Abstract: When do two different looking quantum field theories describe the same physics? This is essentially asking when the quantum field theories are isomorphic. In the case of topological quantum field theories, there

are sometimes a way to determine them via topological invariants. For a superconformal field theory, what would be the minimal set of “invariants” to determine when they are isomorphic? I will discuss some approaches to this question in the context of superconformal field theories in four and six dimensions. Utilizing 4d class S theories that also admits 6d (1,0) SCFT origins, I will explain how a certain class of 4d N=2 SCFTs, which a priori look like distinct theories, can be shown to describe the same physics.

**Rinat Kashaev** (University of Geneva)

Title: Generalized TQFTs from local fields

Abstract: Based on a particular quantum dilogarithm associated to a local field  $F$  and by using the similar techniques as for the Teichmüller TQFTs, one can construct three generalised distribution valued 3d TQFTs on triangulated manifolds, two of which are of Turaev-Viro type. The associated 3-manifold invariants are positive real distribution valued invariants expected to be related with representations of  $\pi_1$  into  $PSL_2(F)$ . Joint work with Stavros Garoufalidis, arXiv:2306.01331.

**Maxim Kontsevich** (IHES)

Title: Singularity spectra of critical loci

Abstract: By mirror symmetry, the Fukaya category of a symplectic manifold is expected to be equivalent to a deformation of the category of matrix factorizations associated with a mirror Landau-Ginzburg potential. I'll report on an ongoing project with D.Auroux and L.Katzarkov concerning Steenbrink-Varchenko singularity spectra of such categories.

**Pietro Longhi** (Uppsala Univeristy)

Title: Quiver structures of knot invariants, open strings, and recursion

Abstract: Generating series of HOMFLYPT polynomials colored by symmetric representations have been found to coincide with partition functions of motivic Donaldson-Thomas invariants of symmetric quivers, after a suitable identification of variables. I will discuss an interpretation of this relation based on string theory, where quivers encode interactions of M2 branes mediated by an M5 brane. Invariance of this picture under deformations leads to a generalization of the knots-quivers correspondence corroborated by wall-crossing type phenomena associated with skein relations among M2 brane boundaries. If time permits, a generalization to multiple M5 branes will be discussed. Based on joint works with Ekholm and Kucharski and ongoing work with Ekholm.

**Lenhard Ng** (Duke University)

Title: New algebraic invariants of Legendrian links

Abstract: I'll discuss a package of new holomorphic-curve invariants of Legendrian knots and links. These come from an L-infinity structure extending the Chekanov-Eliashberg dg algebra and derived from rational symplectic field theory. The L-infinity structure induces, among other things, a Poisson bracket on Legendrian contact homology and a symplectic structure on the augmentation variety of a Legendrian link. In the special case where the Legendrian link is the rainbow closure of an arbitrary positive braid, the Poisson bracket generalizes a previously-constructed bracket of Flaschka-Newell, and the symplectic structure agrees with previously-constructed symplectic structures from cluster theory. Parts of this are joint work in progress with Roger Casals, Honghao Gao, Linhui Shen, and Daping Weng.

**Sunghyuk Park** (Harvard University)

Title: Quantum UV-IR map and curve counts in skeins

Abstract: Quantum UV-IR map (a.k.a. q-nonabelianization map), introduced by Neitzke and Yan, is a map from UV line defects in a 4d N=2 theory of class S to those of the IR. Mathematically, it can be described as a map between skein modules and is a close cousin of quantum trace map of Bonahon and Wong. In this talk, I will discuss how quantum UV-IR map can be generalized to a map between HOMFLYPT skein modules, using skein-valued curve counts of Ekholm and Shende.

**Vivek Shende** (University of California, Berkeley)

Title: Quantum cluster transformations from skein valued curve counting

Abstract: It is well understood that cluster transformations appear in Fukaya categorical computations as a reflection of wall crossing phenomena. In the talk, I will explain how and why quantum cluster transformations make their appearance in a higher genus curve counting formalism.

**Ivan Smith** (University of Cambridge)

Title: Bordism of flow modules and exact Lagrangians

Abstract: We discuss constraints on exact Lagrangian embeddings in Stein manifolds that arise from Floer homotopical considerations in the spirit of ongoing work of Abouzaid and Blumberg. Novel aspects include an obstruction theory for bordism classes of flow modules with Baas-Sullivan singularities. This talk reports on joint work in progress with Noah Porcelli.

**Yan Soibelman** (Kansas State University)

Title: Holomorphic Floer Theory and resurgence

Abstract: I am going to explain how to apply the ideas of Holomorphic Floer Theory (the topic we have been developing with Maxim Kontsevich since 2014) to resurgence of formal series arising in different areas of mathematics and physics.

**Thomas Walpuski** (Humboldt University Berlin)

Title: The Gopakumar–Vafa finiteness conjecture

Abstract: The purpose of this talk is to illustrate an application of the powerful machinery of geometric measure theory to a conjecture in Gromov–Witten theory arising from physics. Very roughly speaking, the Gromov–Witten invariants of a symplectic manifold  $(X, \omega)$  equipped with a tamed almost complex structure  $J$  are obtained by counting pseudo-holomorphic maps from mildly singular Riemann surfaces into  $(X, J)$ . It turns out that Gromov–Witten invariants are quite complicated (or “have a rich internal structure”). This is true especially for if  $(X, \omega)$  is a symplectic Calabi–Yau 3–fold (that is:  $\dim X = 6$ ,  $c_1(X, \omega) = 0$ ).

In 1998, using arguments from M–theory, Gopakumar and Vafa argued that there are integer BPS invariants of symplectic Calabi–Yau 3–folds. Unfortunately, they did not give a direct mathematical definition of their BPS invariants, but they predicted that they are related to the Gromov–Witten invariants by a transformation of the generating series. The Gopakumar–Vafa conjecture asserts that if one defines the BPS invariants indirectly through this procedure, then they satisfy an integrality and a (genus) finiteness condition.

The integrality conjecture has been resolved by Ionel and Parker. A key innovation of their proof is the introduction of the cluster formalism: an ingenious device to side-step questions regarding multiple covers and super-rigidity. Their argument could not resolve the finiteness conjecture, however. The reason for this is that it relies on Gromov’s compactness theorem for pseudo-holomorphic maps which requires an a priori genus bound. It turns out, however, that Gromov’s compactness theorem can (and should!) be replaced with the work of Federer–Flemming, Allard, and De Lellis–Spadaro–Spolaor. This upgrade of Ionel and Parker’s cluster formalism proves both the integrality and finiteness conjecture.

This talk is based on joint work with Eleny Ionel and Aleksander Doan.

**Tian Yang** (Texas A&M University)

Title: Recent progress on the Volume Conjectures for various quantum invariants

Abstract: In this talk I will recall the volume conjectures of various 3-manifold invariants constructed from TQFTs, including the colored Jones polynomials (Kashaev, Murakami–Murakami), the Reshetikhin–Turaev and the Turaev–Viro invariants (Chen–Yang) and their relative versions, and introduce some of the recent progresses.