**K-stability and Birational Geometry**
**25-29 September 2023**
**L4**

# Abstracts

**Hamid Abban** (University of Nottingham)

Title: Limits of quartic 3-folds

Abstract: What singular degenerations of varieties we allow in the moduli study is determined by the rules of the stability condition in hand. For Fano varieties K-stability provides an optimal scenario, but detecting such limits becomes difficult in practice. For hypersurfaces Geometric Invariant Theory gives another stability condition, and the comparison of the two moduli is often fruitful. One could extend the GIT picture to Kollár stability. I will talk about these stability conditions for quartic 3-folds. The K-stability study is a joint work with Cheltsov, Kasprzyk, Liu, and Petracci, while the Kollár stability study is an ongoing joint work with Fedorchuk and Krylov.

**Carolina Araujo** (IMPA)

Title: Gale duality, blowups and moduli spaces

Abstract: In this talk, we discuss the birational geometry of blowups of projective spaces at points in general position. For that, we explore Gale duality, a correspondence between sets of $n=r+s+2$ points in projective spaces $\mathbb{P}^s$ and $\mathbb{P}^r$. For small values of $s$, this duality has a remarkable geometric manifestation: the blowup of $\mathbb{P}^r$ at $n$ points can be realized as a moduli space of vector bundles on the blowup of $\mathbb{P}^s$ at the Gale dual points.

**Paolo Cascini** (Imperial College London)

Title: Foliation Adjunction

Abstract: We present an adjunction formula for foliations on varieties and we consider applications of the adjunction formula to the cone theorem for rank one foliations and the study of foliation singularities. Joint work with C. Spicer.

**Ana-Maria Castravet** (University Paris-Saclay)

Title: Higher Fano manifolds

Abstracts: Fano manifolds are complex projective manifolds having positive first Chern class. The positivity condition on the first Chern class has far reaching geometric and arithmetic implications. For instance, Fano manifolds are covered by rational curves, and families of Fano manifolds over one dimensional bases always admit holomorphic sections. In recent years, there has been some effort towards defining suitable higher analogues of the Fano condition. Higher Fano manifolds are expected to enjoy stronger versions of several of the nice properties of Fano manifolds.

In this talk, I will discuss higher Fano manifolds which are defined in terms of positivity of higher Chern characters. After a brief survey of what is currently known, I will present recent joint work with Carolina Araujo, Roya Beheshti, Kelly Jabbusch, Svetlana Makarova, Enrica Mazzon and Nivedita Viswanathan, regarding toric higher Fano manifolds. I will explain a strategy towards proving that projective spaces are the only higher Fano manifolds among smooth projective toric varieties.

**Giulio Codogni** (University Roma Tor Vergata)

Title: A lower bound on the volume of KSB moduli spaces

Abstract: Given a fibration f: (X,D) -> T, where (X,D) is a mildly singular pair and T a curve, the pushforward of the m-th power of the relative log canonical bundle is called m-th Hodge bundle. I will explain that, under suitable assumptions, if f has maximal variation then such bundles are ample for m greater than two. This

generalises to the singular setting classical results of Esnault-Viehweg and gives effective versions of results of Kollár and Kovacs-Patakfalvi. I will then propose two applications of this result.

The first is already mentioned in the title. KSB moduli spaces parametrizes log-canonically polarized pairs endowed with a Kaehler-Einstein metric. Fixed the dimension, the volume, and the coefficients of the boundary, coarse KSB moduli spaces are known to be projective varieties. I will focus on the irreducible components which parametrise at least one klt pair. Combining the above mentioned effective positivity result with a generalization of the classical Cornalba-Harris-Xiao slope inequality, I will give a lower bound on the volume of these moduli spaces. The lower bound depends only on the dimension, the coefficient of the boundary, and the size of the automorphism groups.

The second application is a lower on the top self-intersection of the relative log canonical bundle, and an upper bound on the cardinality of the relative automorphism group for fibrations in KSB-stable pairs such that at least one fiber is klt, in the same fashion of classical boundness results for KSB pairs. The talk is based on a joint work with L. Tasin and F. Viviani, and on a work in progress with Zs. Patakfalvi and L. Tasin.

**Kristin De Vleming** (University of Massachusetts)

Title: Moduli of boundary polarized log Calabi Yau pairs

Abstract: I will discuss joint work with Kenny Ascher, Dori Bejleri, Harold Blum, Giovanni Inchiostro, Yuchen Liu, and Xiaowei Wang on construction of moduli stacks and moduli spaces of boundary polarized log Calabi Yau pairs. Unlike moduli of canonically polarized varieties (respectively, Fano varieties) in which the moduli stack of KSB stable (respectively, K semistable) objects is bounded for fixed volume, dimension, the objects here form unbounded families. Despite this unbounded behavior, we define the notion of asymptotically good moduli space, and, in the case of plane curve pairs (P2, C), we construct a projective good moduli space parameterizing S-equivalence classes of such pairs.

**Thibaut Delcroix** (University of Montpellier)

Title: Fano spherical varieties of small dimension and rank

Abstract: A spherical variety (X,G) is a normal complex algebraic variety X equipped with the action of a connected complex reductive group G such that a Borel subgroup B of G acts with an open dense orbit. The rank of (X,G) is the rank of the lattice of B-eigenvalues in the B-module of rational functions on X. I will present the classification of the 260 locally factorial Fano spherical varieties (X,G) of dimension four and of rank two or less, obtained in a joint work with Pierre-Louis Montagard. Those spherical varieties are described via combinatorial data, from which it is easy to read off geometric properties of the underlying variety X, such as the Picard rank, anticanonical degree, K-stability, etc.

**Ruadhai Dervan** (University of Glasgow)

Title: Divisorial stability of projective varieties

Abstract: The theme of this talk will be to extend some foundational ideas in the theory of K-stability of Fano varieties to more general projective varieties. I will first explain an approach to K-stability of general projective varieties (along with a choice of ample line bundle) using valuations (joint work with E. Legendre) and a strengthening of this involving ideas from non-Archimedean geometry, due to Boucksom-Jonsson, which goes by the name of "divisorial stability". I will then focus on explicit results around the behaviour of divisorial stability under finite covers, along with some surrounding ideas, which are joint work with T. Papazachariou.

**Kento Fujita** (Osaka University)

Title: K-stability of Casagrande--Druel varieties

Abstract: We focus on a certain subclass of Fano varieties named Casagrande—Druel varieties. Especially, we see the K-polystability of several Casagrande—Druel threefolds whose general members are in Mori--Mukai's list Nos. 3.9 and 4.2, and see the K-moduli spaces parametrizing those varieties. This is a joint work with Ivan Cheltsov, Tiago Duarte Guerreiro, Igor Krylov and Jesus Martinez Garcia.

**Lena Ji** (University of Michigan)

Title: The K-moduli of a family of conic bundle threefolds

Abstract: K-stability is a central tool in the construction of compact moduli spaces of Fano varieties, and more generally, log Fano pairs (X, cD). When D is a rational multiple of -K_X, Ascher–DeVleming–Liu developed a framework for the wall-crossing behavior. In this talk, I'll discuss an example of the non-proportional case, i.e. when D is not a multiple of -K_X. We study the K-moduli space for X=P^1 x P^2 and D a (2,2) divisor; a general member of this family gives rise to a conic bundle. We show that there is a wall-crossing at an irrational number c, thus exhibiting the first example of an irrational wall when the divisor D is irreducible. This is joint work in progress with Kristin DeVleming, Patrick Kennedy-Hunt, and Ming Hao Quek.

**Anne-Sophie Kaloghiros** (Brunel University)

Title: On one-dimensional K-moduli spaces of Fano 3-folds

Abstract: Relatively few examples of K-moduli spaces of smoothable K-polystable Fano varieities are known explicitly. In this talk, I will describe one-dimensional components of K-moduli spaces of smoothable K-polystable Fano 3-folds. In particular, I will discuss K-polystable limits of smooth Fano 3-folds in families 2.22, 3.12, 3.13 and 4.13.

**Eveline Legendre** (University Claude Bernard Lyon 1)

Title: The Einstein-Hilbert functional and the Donaldson-Futaki invariant

Abstract: I will present a correspondence between the constant scalar curvature Kähler problem on a polarized manifold and a family of CR-Yamabe problems on the associated circle bundle. Then I will explain the use of a CR-version of the Einstein-Hilbert functional in this context and show how it determines the K-stability of the polarized manifold. This is a joint work with Abdellah Lahdili and Carlo Scarpa.

**Jesus Martinez Garcia** (University of Essex)

Title: Computational Geometric Invariant Theory

Abstract: The study of quotients of varieties by reductive groups goes back to Hilbert's $14^{th}$ problem. Hilbert's approach, consisting on finding invariants, is computationally-heavy and it is difficult to approach even in simple cases. The introduction of the Hilbert-Mumford criterion by D. Mumford in the 60s moved the problem from studying the quotient as a whole to considering points in the quotient, removing some points (known as unstable points) to obtain a quotient which is an algebraic variety. This subject is now known as Geometric Invariant Theory (GIT) and it has the advantage of being computationally lighter than Hilbert's approach. For us, this is a natural problem in geometry (in particular to describe moduli spaces), but it also has applications to physics, computer vision or dynamical systems. In this project we develop some theory building on Mumford's GIT to describe the unstable, non-stable and strictly polystable locus of a projective algebraic variety with respect to a reductive group in a finite number of steps, including an implementation of our algorithms in SageMath. Furthermore, we also explain how to describe the quotient "X/G" and cover some applications, including one in K-stability. This is joint work with P. Gallardo, H-B. Moon and D. Swinarski.

**Takuzo Okada** (Saga University)

Title: Birationally solid Fano 3-fold hypersurfaces

Abstract: Fano 3-folds that are embedded as (quasismooth) hypersurfaces in weighted projective 4-spaces are classified and they form 130 families. Among them 95 families consist of Fano 3-fold weighted hypersurfaces of Fano index 1, and Cheltsov-Park proved that they are all birationally rigid. Recently, Abban-Cheltsov-Park showed that none of Fano 3-fold weighted hypersurfaces of Fano index at least 2 is birationally rigid. The aim of this talk is to explain birational properties of these Fano 3-fold weighted hypersurfaces of Fano index at least 2, and explain the classification of those with the property of "birational solidity" which is a notion weaker than birational rigidity.

**Zsolt Patakfalvi** (EPFL)

Title: Varieties with nef anti-canonical has surjective Albanese

Abstract: I will present a joint work with Sho Ejiri showing that smooth projective varieties with nef anti-canonical divisor have surjective Albanese morphism. The statement was conjectured in the Kähler setting by Demailly-Peternell-Schneider in 1993, and it was shown in characteristic zero by Zhang in 1996. Our contribution is that it also holds in positive characteristic, and hence over any field. This is the first arbitrary dimensional positive characteristic result on varieties with nef anti-canonical divisor that is not sensitive to wild behavior: wild action of Frobenius on cohomology, wild singularities of the general fibers over the Albanese image, etc. I will also mention a few corollaries and generalizations.

**Arman Sarikyan** (University of Edinburgh)

Title: Varieties with an ample divisors of numerically trivial canonical class

Abstract: A three-dimensional non-Gorenstein Fano variety with at most canonical singularities is called a Fano-Enriques threefold if it contains an ample linear system, whose generic element is an Enriques surface with at most canonical singularities. Although there is no complete classification of Fano-Enriques threefolds yet, but there are some partial results. For instance, L. Bayle has classified Fano-Enriques threefolds with terminal cyclic quotient singularities in terms of their canonical covers, which are smooth Fano threefolds in this case. The rationality of Fano-Enriques threefolds is an open classical problem that goes back to the works of G. Fano and F. Enriques. In this talk we will discuss the rationality of Fano-Enriques threefolds with terminal cyclic quotient singularities.

**Cristiano Spotti** (Aarhus University)

Title: Some examples of bubbling for Kähler-Einstein metrics

Abstract: When Kähler-Einstein manifolds degenerate forming singularities, it is natural from a differential geometric perspective to study metric limits (in the pointed Gromov-Hausdorff sense) *at all possible scales*. In this talk, I will describe some concrete examples, focusing mostly on non-collapsing low dimensional cases and discussing how one can hope that algebraic geometry can detect the full multiscale metric bubble tree. The talk is based on joint work with Martín de Borbon.

**Hendrik Suess** (University of Jena)

Title: Boundedness of K-semistable hypersurface cone singularities via normalised volume

Abstract: It has been conjectured that the notions of normalised volume and K-stability can be used to bound klt cone singularities. There has been huge progress in this direction due to results by Ziquan Zhuang. Here, we present an elementary argument which shows that in the hypersurface case the class of K-semistable cone singularities with normalised volume bounded from below by a positive constant is indeed bounded. This is (part of) joint work with Yuchen Liu and Joaquín Moraga.

**Nivedita Viswanathan** (University of Nottingham)

Title: Log Canonical Thresholds of high multiplicity plane curves

Abstract: Given a reduced plane curve *C_d* of degree *d* in $\mathbb{C}^2$, a classical question is to understand the singularities of it. Over the years many different measures of singularities have been explored, such as Multiplicity, Milnor number, Tjurina number to name a few. In this talk, I will focus on another invariant called the log canonical threshold, which has a long standing relation with the notion of K-stability. Firstly, for all curves of degree $d \leq 5$, I will explicitly show the exhaustive list of all possible log canonical threshold values that the curve *C_d* can take at a singular point *p* on it. Then, we will see how imposing restrictions on the multiplicity of the curve *C_d* at the point *p* can help us in saying more about this invariant. This is joint work with Erik Paemurru.