About Entropy in Large Classical Particle Systems
25-29 September 2023

Abstracts

Giada Basile (Sapienza University of Rome)

Title: Energy non-conserving paths for the Kac's walk

Abstract: I will present some recent results on large deviations for the Kac's walk which is described, in the kinetic limit, by the homogeneous Boltzmann equation. In particular, I will discuss the asymptotic probability of some atypical paths that violate energy conservation and I will introduce a large deviation rate function which takes into account these trajectories. Finally, I will propose a gradient flow formulation of the homogeneous Boltzmann equation related to the large deviation rate function. Joint work with L. Bertini, D. Benedetto, E. Caglioti and C. Orrieri.

Lorenzo Bertini (Sapienza University of Rome)

Title: Thermodynamic potentials for stationary non-equilibrium states

Abstract: The paradigmatic example of a non-equilibrium macroscopic system is a gas in contact with thermal reservoirs at different temperatures. In this situation, characterized by a flow of energy within the system, the Boltzmann-Gibbs prescription for the stationary state cannot be applied. By analyzing the large deviations of an underlying miscroscopic dynamics, the so-called macroscopic fluctuation theory provides a variational definition of suitable functionals that play the role of the standard equilibrium thermodynamic potentials. In specific models, these non-equilibrium potentials can be analyzed in detail: they show peculiar differences with respect to equilibrium, and exhibit novel types of phase transitions.

Freddy Bouchet (CNRS and ENS Paris)

Title: Relations between various entropy concepts and the quasi-potentials of classical kinetic theories

Abstract: I will review different concepts of entropy, their mathematical definitions and physical meaning. I will stress why they strictly differ for a finite number of particles, and how they might sometimes coincide, in the thermodynamical limit, at equilibrium. As far as dynamics is concerned, I will explain the relation between the microstate entropy and the relaxation dynamics of physical systems, following the classical understanding since the work of Boltzmann. I will emphasize the specific enlightening cases of physical systems which can be described by classical kinetic theories.

This classical framework and understanding, although perfectly valid and clear, does not cover a set of important questions related to entropy and its relation to the dynamics of physical systems. For instance, it does not quantify the probability of dynamical paths for physical systems and their relations with entropy.

I will describe recent results where we derived explicitly the functionals which describe the path (dynamical) large deviations for the empirical measure of dilute gases, plasma, systems of particles with long range interactions, and waves with weak turbulent interactions. The associated kinetic equations (the average evolution) are then either the Boltzmann, the Landau, the Balescu--Lenard—Guernsey, or the weak turbulence kinetic equations. After making the classic assumptions in theoretical physics textbooks for deriving the kinetic equation, our derivations of the path large deviation functionals are exact. For each of these systems, those results prove that entropy can be understood as the quasipotential of reversible stochastic processes, which describe the mesoscopic dynamics of the physical systems. This gives a full dynamical understanding of the concept of entropy, complementing the classical interpretation, for the specific cases of classical deterministic particle systems.

Jérôme Buzzi (University Paris-Saclay)

Title: Entropy, classifications, and structure theorems in dynamics

Abstract: Inspired by Shannon's information theory (1948), Kolmogorov and Sinai (1958) introduced entropy as a new invariant in ergodic theory. Ornstein (1971) crowned this classical period by showing that Bernoulli

schemes are isomorphic if and only if they have equal entropy. We will see some of the wonderful properties of entropy (Sinai, Jewett-Krieger, Hochman among others). We will look at generalizations of entropy beyond the setting of ergodic theory, towards some structural and classification results. If time permits, I will present some recent advances for surface diffeomorphisms (joint with Crovisier and Sarig, 2022).

Jean-René Chazottes (CNRS and Ecole Polytechnique)

Title: Entropy in dynamical systems and ergodic theory

Abstract: From its origins as a foundational, albeit somewhat nebulous, concept in statistical mechanics, the notion of entropy evolved into a central pillar of the mathematical theory of information, pioneered by Claude Shannon in his groundbreaking 1948 paper. Shortly thereafter, Kolmogorov had a brilliant insight, recognizing its role as a fundamental invariant within measure-preserving dynamical systems, which in turn sparked a revolution in ergodic theory. In this presentation, our aim is to blend a concise historical overview with explanations of specific concepts and ideas. We'll maintain a minimal emphasis on technical details, while highlighting the driving motivations and ramifications. It's important to acknowledge that offering a comprehensive treatment of either the historical or conceptual facets of this subject is an insurmountable task.

Isabelle Gallagher (ENS)

Title: Large Deviations for hard spheres in the low density limit

Abstract: We shall study large deviations for a system of hard spheres in the low density limit, away from equilibrium. We shall see that large deviations are exponentially small in the average number of particles, for short times. The large deviation functional turns out to be the same as the one previously obtained by Fraydoun Rezakhanlou for dynamics with stochastic collisions, under some regularity assumptions. This is a joint work with Thierry Bodineau, Laure Saint-Raymond and Sergio Simonella.

François Golse (Ecole Polytechnique)

Title: The regularity problem for the Landau equation

Abstract: It is well known that the dynamics of particles interacting through the Coulomb potential cannot be described by the Boltzmann equation. In this case, the Boltzmann collision integral must be replaced with the Landau operator. In the late 1990's, Villani defined a notion of global, space-homogeneous solutions to the Landau equation, called H-solutions (in view of the importance of Boltzmann's H-Theorem in the definition of such solutions). This talk will review some recent progress on the regularity of Villani solutions of the Landau equation. (Based in particular on joint work with M.P. Gualdani, C. Imbert and A. Vasseur.)

Arnaud Guillin (Clermont-Auvergne University)

Title: E ntropy and mean field systems

Abstract: Entropy appears naturally as pseudo-distance between probability measures and as such may be used to quantify convergence to equilibrium, or propagation of chaos. We will mainly consider here functional inequalities such as logarithmic Sobolev inequalities and will show how it can be used for these goals in the case of mean field systems, singular or not. Joint works with Wei Liu, Liming Wu, Chaoen Zhang, Pierre Monmarché, and Pierre Le Bris.

Daniel Heydecker (MPIM)

Title: Dynamic Large Deviations: In search of matching bounds

Abstract: We discuss the dynamical large deviations of three models: Kac's model of the Boltzmann Equation and a rescaled zero-range process converging to the porous medium equation. For these models, we discuss the challenges and successes of finding matching bounds, and the connection to the equality case of classical inequalities.

François Huveneers (Kings College London)

Title: Spreading of a wave packet in a classical disordered anharmonic chain

Abstract: I consider the breakdown of Anderson localization in a one-dimensional anharmonic chain. A series of numerical works indicate that an initially localized wave packet spreads polynomially in time, while analytical studies rather suggest a much slower spreading. I will present new numerical and analytical results aimed to settle this conundrum. From work in collaboration with Wojciech De Roeck and Oskar Prosniak (KU Leuven).

Milton Jara (IMPA)

Title: Entropy methods in particle systems

Abstract: The aim of quantitative hydrodynamics is to provide quantitative estimates on the speed of convergence of scaling limits of interacting particle systems. In particular, the aim is to provide a quantitative version of what is known in the literature as the hydrodynamic limit of interacting particle systems. The key novelty is to use relative entropy as a measure of distance between stochastic processes. One of the main benefits of quantitative estimates is that one can use them to very precise scaling limits for fluctuations and/or large-time behavior of interacting particle systems, features that were not accessible by more established methodology.

Based on joint works with Enzo Aljovin (Rio de Janeiro), Patrícia Gonçalves (Lisbon), Rodrigo Marinho (Cachoeira do Sul), Otávio Menezes (Salvador) and Yangrui Xiang (Rio de Janeiro).

Daniel Lacker (Columbia University)

Title: Projected Langevin dynamics and entropic optimal transport

Abstract: The classical (overdamped) Langevin dynamics provide a natural algorithm for sampling from its invariant measure, which uniquely minimizes an energy functional over the space of probability measures, and which concentrates around the minimizer(s) of the associated potential when the noise parameter is small. We introduce analogous diffusion dynamics that sample from an entropy-regularized optimal transport (a.k.a. Schrodinger bridge), which uniquely minimizes the same energy functional but constrained to the set of couplings of two given marginal probability measures, and which concentrates around the optimal transport coupling(s) for small regularization parameter. More specifically, our dynamics possess two key properties: First, the law at each time remains a coupling if initialized as such. Second, the long-time limit is the unique solution of the entropic optimal transport problem. In addition, we show by means of a new logarithmic Sobolev inequality that the convergence holds exponentially fast, for sufficiently large regularization parameter and for (asymptotically) log-concave marginals. The special structure of entropy-regularization and an entropy-dissipation formula are critical in our analysis. This SDE is related, in the spirit of Otto calculus, to a gradient flow on the space of couplings, viewed as a submanifold of Wasserstein space. Joint work with Giovanni Conforti and Soumik Pal.

Thomas Leblé (University Paris-Cite)

Title: Some aspects of the specific relative entropy in the study of log-gases

Abstract: I will use the so-called "specific" relative entropy as a guide (and a pretext) to review some recent results about one- and two-dimensional log-gases. I will recall why and illustrate how this quantity appears naturally in infinite-volume statements. I will also mention rigidity properties that are known or suspected to hold for those systems - both at positive and at zero temperature - and how they could be detected (or not) by specific entropy computations.

Joel Lebowitz (Rutgers University)

Title: Boltzmann’s entropy for macroscopic systems: the dilute gas

Abstract: Bolzmann’s entropy is defined for individual macroscopic systems in a specified macrostate; equilibrium or not. In the latter case it satisfies the second law of thermodynamics which characterizes the time evolution of a typical isolated macroscopic system in a non-equilibrium macrostate. The time asymmetry of this observed evolution can be understood as arising from: a) the great disparity between microscopic and macroscopic sizes, b) initial conditions, and c) the fact that what we observe are typical behaviors — not all

imaginable ones. This will be illustrated for a classical dilute gas. Much of this work was done jointly with S. Goldstein.

Clément Mouhot (University of Cambridge)

Title: Entropy in PDE analysis

Abstract: We will present some examples of the central role played by entropy in PDE analysis (with not attempt at being exhaustive). In particular we will illustrate its role as a priori estimate for the Cauchy problem, its use as a relative distance to limit regimes, and its importance as a source of specific open problems (rigorous derivation from microscopic evolutions, quantitative rate of entropy production...).

Stefano Olla (University Paris Dauphine)

Title: Heat equation from a deterministic dynamics

Abstract: We derive the heat equation for the thermal energy under diffusive space-time scaling from a purely deterministic microscopic dynamics satisfying Newton equations perturbed by an external chaotic force acting like a magnetic field. Joint work with Giovanni Canestrari and Carlangelo Liverani.

Matt Rosenzweig (MIT)

Title: Mean-field limits for Riesz systems

Abstract: I will discuss the interplay of entropy, energy, and functional inequalities in establishing the mean-field convergence/propagation of chaos for the dynamics of interacting particle systems, specifically of Coulomb or Riesz type. I will also discuss the role of the trend to equilibrium for the mean-field equation and how it allows to deduce uniform-in-time convergence results or even establish the generation of chaos.

Juan Soler (University of Grenada)

Title: Mean-field limit of non-exchangeable multi-agent systems

Abstract: This talk deals with the derivation of the mean-field limit for multi-agent systems on a large class of sparse graphs. More specifically, the case of non-exchangeable multi-agent systems consisting of non-identical agents is addressed. The analysis does not only involve PDEs and stochastic analysis but also graph theory through a new concept of limits of sparse graphs (extended graphons) that reflect the structure of the connectivities in the network and has critical effects on the collective dynamics. The associated observables completely entangle the limit of connectivities with the initial conditions and solve an independent hierarchy of equations. They naturally extend the notion of marginals, and hierarchy of marginals to non-exchangeable systems. Some of the main restrictive hypothesis in the previous literature on the connectivities between the agents (dense graphs) and the cooperation between them (symmetric interactions) are removed.

Herbert Spohn (TU Munich)

Title: Integrable many-particle systems: generalised entropy and particle scattering

Abstract: For integrable many-body quantum systems, because of the Bethe ansatz, there is a link between the generalised free energy and scattering theory. For classical models there is no Bethe ansatz. Still a few years ago the respective link could be established for the Toda lattice. We will report on a recent related result for the Ablowitz-Ladik discretisation of the defocussing nonlinear Schroedinger equation.