New Perspectives in the Analytic Theory of Automorphic Forms
25-29 September 2023

Abstracts

Edgar Assing (University of Bonn)

Title: On (automorphic) density theorems for principal congruence subgroups in *SL_n*

Abstract: An automorphic density theorem is a quantitative statement about the number of non-tempered cusp forms, which can serve as a convenient replacement for the generalized Ramanujan Conjecture in several applications. In this talk we will discuss how the Kuznetsov formula can be used to prove such results. In particular, we will focus on the case of the principal congruence subgroup and highlight some interesting features that appear in this situation.

Raphaël Beuzart-Plessis (University Aix-Marseille)

Title: The Gan-Gross-Prasad conjecture for Bessel periods of unitary groups II

Abstract: In the second talk we will explain how to deduce the Gan-Gross-Prasad and Ichino-Ikeda conjectures for general Bessel periods on unitary groups from the results on periods of cuspidal Eisenstein series in codimension one. Time permitting, we will also give some glimpse of the proof of the spectral expansions.

Farrell Brumley (University Sorbonne Paris Nord)

Title: Concentration properties of theta lifts on orthogonal groups

Abstract: A well-known principle in quantum chaos suggests that, on a closed compact negatively curved Riemannian manifold, eigenfunctions should not manifest extreme localisation properties. Nevertheless, it is an emerging theme in arithmetic analysis that *intermediate* localisation properties can be realized in the setting of congruence manifolds, often for reasons related to functoriality.

I shall discuss joint work with Simon Marshall, in which we prove the existence of Maass forms with large sup norms on a wide class of anisotropic orthogonal groups. The argument combines a counting argument with a new period relation showing that a certain orthogonal period on O(n,m) distinguishes theta lifts from Sp2m. This generalizes a method of Rudnick and Sarnak in the rank one case, when m = 1. Our lower bound is naturally expressed as a ratio of the Plancherel measures for the groups O(n,m) and Sp2m(R), up to logarithmic factors, and strengthens the results of our previous work for such groups. In the case of odd-dimensional hyperbolic spaces, the growth exponent we obtain improves on a result of Donnelly, and is optimal under the purity conjecture of Sarnak.

Pierre-Henri Chaudouard (Jussieu)

Title: The Gan-Gross-Prasad conjecture for Bessel periods of unitary groups I

Abstract: In the first talk, we will explain a variant of the Gan-Gross-Prasad and Ichino-Ikeda conjectures that applies to the periods of cuspidal Eisenstein series on U(n)xU(n+1) along the diagonal subgroup U(n): this is an extension of previous results in the cuspidal case mainly due to Beuzart-Plessis, Liu, Zhang, and Zhu and Beuzart-Plessis, Chaudouard, and Zydor. We will give some ingredients of the proof which is based on a comparison of relative trace formula. The main novelty is the explicit computation of some spectral contributions.

Shai Evra (Hebrew University of Jerusalem)

Title: Cohomological Sarnak-Xue Density Hypothesis for SO_5

Abstract: The Generalized Ramanujan Conjecture (GRC) predicts that cuspidal automorphic representations of PGL_n are tempered locally everywhere. The naive generalization of this conjecture for other algebraic groups was shown to be false by Howe and Piatetski-Shapiro in the 70's. In their work from the 90's, Sarnak and Xue introduced a density version of the (GRC), called the Sarnak-Xue Density Hypothesis (SXDH), proved it for

SL_2, conjectured that it should hold in general and suggested that it should serve as a replacement for the (GRC) in applications. The (SXDH) was recently proved for SL_n by Assing, Blomer, Jana and Kamber, using techniques from analytic number theory.

In this talk I will describe a different approach to prove the (SXDH), in the restricted case of cohomological automorphic representations, using recent results coming from the Langlands program, and prove the conjecture for the split classical group SO_5, as well as for certain inner forms of it. Our main tool is the Arthur's endoscopic classification of automorphic representations of split classical groups, its extension by Taibi to inner forms, and the explicit description of the local A-packets by Schmidt in the special case of SO_5. If time permits we will show some of the following applications: Bounds on the growth of the Betti numbers of congruence manifolds of type SO(3,2) or SO(1,4), and the construction of density-Ramanujan complexes covered by the Bruhat-Tits building of SO_5 which satisfy the cutoff phenomena for the mixing time of their non-backtracking random walk.

This is based on a joint work with Mathilde Gerbelli-Gauthier and Henrik Gustafsson.

Wee Teck Gan (National University of Singapore)

Title: Triality and the Spin L-function of PGSp(6)

Abstract: I will discuss a joint work with Gaetan Chenevier, in which we exploit the theory of triality to construct the Spin lifting from PGSp(6) to GL(8). This allows one to establish the expected analytic properties of the Spin L-function of Siegel modular forms of genus 3.

Mathilde Gerbelli-Gauthier (McGill University)

Title: Counting non-tempered automorphic forms using endoscopy

Abstract: In this talk, we consider the limit multiplicity question (and some variants): how many automorphic forms of fixed infinity-type and level N are there as N grows? The question is well-understood when the archimedean representation is a discrete series, and we focus on non-tempered cohomological representations on unitary groups. Using an inductive argument which relies on the stabilization of the trace formula and the endoscopic classification, we give asymptotic counts of multiplicities, and prove the Sarnak-Xue conjecture at split level for (almost!) all cohomological representations of unitary groups. Additionally, for some representations, we derive an average Sato-Tate result in which the measure is the one predicted by functoriality. This is joint work with Rahul Dalal.

Subhajit Jana (Queen Mary University of London)

Title: On the L2-bound of the Eisenstein series

Abstract: We will talk about the local L2 bounds of the Eisenstein series on the general reductive groups. First, we will discuss how the Maass—Selberg relations, when used to understand the L2 norm of an Eisenstein series, yields a complicated combinatorial problem. Second, we will discuss how the ideas from Finis--Lapid--Muller's fine spectral expansion of the Arthur--Selberg trace formula may be used to bypass the problem. This is joint work with Amitay Kamber.

Amitay Kamber (University of Cambridge)

Title: Optimal Lifting and Sarnak's Density Conjecture

Abstract: Consider the mod q map $SL_n(Z) \to SL_n(Z/qZ)$, which is onto by a simple case of the strong approximation theorem. This translates to a finite balls-to-bins problem if we restrict the map to elements $\gamma \in SL_n(Z)$ whose entries are bounded by T.

The optimal lifting theorem states that once T is large enough to allow for a bit more balls than the number of bins, almost-every element of $\SL_n(Z/qZ)$ is in the image. On the other hand, the big hole phenomenon says that T needs to be a lot larger to cover all the elements.

A version of optimal lifting is expected to hold very generally, for all congruence subgroups of arithmetic groups, and is closely related to Sarnak's density conjecture. We will explain all this and also the relation to certain bounds on Eisenstein series. based on joint works with K. Golubev, S. Jana, and P. Varju.

Erez Lapid (Weizmann Institute of Science)

Title: Generic automorphic representations of classical groups: old and new

Abstract: A simple consequence of the work of Jacquet-Shalika in the 1980s on the Rankin-Selberg L-functions is a relation between Whittaker coefficients of cusp forms on GL_n and the Petersson inner product. For classical groups, a conjectural analogue of this relation was formulated a decade ago and proved in special cases. It is based on the descent construction of Ginzburg-Rallis-Soudry. I will revisit this theme in connection with the recent solution of the Hiraga-Ichino-Ikeda conjecture (in the local setup) by Beuzart-Plessis. Joint work with Zhengyu Mao.

Jasmin Matz (University of Copenhagen)

Title: Towards a symplectic version of Duke's theorem

Abstract: In generalization of Duke's seminal work on the equidistribution of Heegner points and closed geodesics on the modular surface, one can ask about the distribution of closed torus orbits on more general locally symmetric spaces. The case of SL(n) over an arbitrary number field was studied by Einsiedler, Lindenstrauss, Michel, and Venkatesh. In joint ongoing work with Farrell Brumley we now study the case of Sp(4).

Simon Marshall (University of Wisconsin-Madison)

Title: Subconvexity for L-functions on U(n) x U(n+1) in the depth aspect

Abstract: We present a subconvex bound for L-functions on U(n) x U(n+1) in the depth aspect, for any n. We do this by first using the unitary Ichino-Ikeda formula to relate the central L-value to an automorphic period integral. There is a `trivial' bound for this period integral, which corresponds to the convexity bound for the L-value if the test vector is chosen correctly. By using the method of arithmetic amplification, we are able to improve over this trivial bound for the period, and hence obtain subconvexity.

Werner Müller (University of Bonn)

Title: On the growth of torsion in the cohomology of arithmetic groups

Abstract: There are deep connections between cohomology of arithmetic groups, the theory of automorphic forms and number theory. This concerns the cohomology with complex coefficients. Recent developments indicate that torsion classes may play a similar role. In the light of this a basic question is what kind of torsion one can expect in the cohomology of a given arithmetic group. Starting with a review of the work of Bergeron and Venkatesh, I will discuss various aspects of the growth of torsion in the cohomology of locally symmetric spaces of finite volume, associated to arithmetic groups. Here ``growth'' means that we consider either towers of coverings or special sequences of local systems of growing rank associated to algebraic representations of the underlying reductive group. The method is analytic and is based on the study of the Ray-Singer analytic torsion of the locally symmetric spaces. I will review some of the recent results and discuss some open problems.

Paul Nelson (Aarhus University)

Title: The orbit method, microlocal analysis and applications to L-functions

Abstract: L-functions are generalizations of the Riemann zeta function. Their analytic properties control the asymptotic behavior of prime numbers in various refined senses. Conjecturally, every L-function is a "standard L-function" arising from an automorphic form. A problem of recurring interest, with widespread applications, has been to establish nontrivial bounds for L-functions. I will survey some recent results addressing this problem. The proofs involve the analysis of integrals of automorphic forms, approached through the lens of representation theory. I will emphasize the role played by the orbit method, developed in a quantitative form along the lines of microlocal analysis. The results/methods to be surveyed are the subject of the following papers/preprints:

https://arxiv.org/abs/1805.07750
https://arxiv.org/abs/2012.02187
https://arxiv.org/abs/2109.15230

Ian Petrow (University College London)

Title: A Petersson/Kuznetsov formula with non-archimedean test functions, the spectral large sieve, and subconvexity

Abstract: The Petersson/Kuznetsov formula is a spectral summation device for classical automorphic forms that allows one to select forms according to the irreducible admissible unitary representation of $PGL_2(R)$ that they generate. We present a generalized Petersson/Kuznetsov formula where one may instead select automorphic forms according to the irreducible admissible unitary representation of $PGL_2(Q_p)$ that they generate. In the case where one selects a single supercuspidal representation of $PGL_2(Q_p)$ with even conductor exponent (resp. a pair of supercuspidal representations with odd conductor exponent) we present an elegant expression for corresponding Kloosterman sum. As applications, we present a spectral large sieve inequality for these families of automorphic forms, and also applications to cubic moments and subconvexity. Everything in this talk is joint work in progress with M.P. Young and Y. Hu.

Abhishek Saha (Queen Mary University of London)

Title: Mass equidistribution for Saito-Kurokawa lifts

Abstract: The Quantum Unique Ergodicity (QUE) conjecture was proved in the classical case of Maass forms on the upper-half plane by Lindenstrauss and Soundararajan. The analogous mass equidistribution statement for holomorphic cusp forms in the weight aspect is a theorem due to Holowinsky and Soundararajan. In this talk, I will discuss some joint work with Jesse Jaasaari and Steve Lester on the higher rank analogue of the result of Holowinsky and Soundararajan for the case of holomorphic Siegel cusp forms. Our main result establishes mass equidistribution for Saito-Kurokawa lifts (which are special types of Siegel cusp forms of degree 2) assuming the Generalized Riemann Hypothesis (GRH). We also show that this implies the equidistribution of zero divisors of Saito-Kurokawa lifts.

Sug Woo Shin (University of California, Berkeley)

Title: Endoscopic classification for unitary groups

Abstract: Arthur proved the endoscopic classification theorem for automorphic representations of quasi-split symplectic and orthogonal groups about a decade ago. It was extended to quasi-split unitary groups by Mok, and partly to non-quasi-split unitary groups by Kaletha, Minguez, White, and myself. In particular, this strengthens the cohomological base change for unitary groups due to Clozel and Labesse. All of these theorems, except Clozel-Labesse's base change, are conditional on certain expected results. I will discuss their current status as well as a related ongoing project with Atobe, Gan, Ichino, Kaletha, and Minguez.

Jesse Thorner (University of Illinois, Urbana-Champaign)

Title: A new zero-free region for Rankin-Selberg *L*-functions

Abstract: I will present a new zero-free region for all $\mathrm{GL}(1)$-twists of $\mathrm{GL}(m)\times\mathrm{GL}(n)$ Rankin-Selberg *L*-functions. The proof is inspired by Siegel's celebrated lower bound for Dirichlet *L*-functions at $s=1$. This is joint work with Gergely Harcos.