

[beginning of letter seems to be missing]

[91r] (unless the limit for $\frac{v^n-w^n}{v-w}$ is dispensed with in the demonstration for the Binomial Theorem, which it is not in your Algebra, nor am I aware that it can be dispensed with in any of the elementary proofs of that Theorem). — It had not struck me that, calling $(x + \theta) = v$, the form $\frac{(x+\theta)^n-x^n}{\theta}$ becomes $\frac{v^n-x^n}{v-x}$.

And by the bye, I may here remark that the curious transformations many formulae can undergo, the unexpected & to a beginner apparently [91v] impossible identity of forms exceedingly dissimilar at first sight, is I think one of the chief difficulties in the early part of mathematical studies. I am often reminded of certain sprites & fairies one reads of, who are at one's elbow in one shape now, & the next minute in a form the most dissimilar, and uncommonly deceptive, troublesome & tantalizing are the mathematical sprites & fairies sometimes ; like the types I have found for them in the world of Fiction. —

[92r] I will now go to the question I delayed asking before :

In the development of the Exponential Series

$$a^x = 1 + (\log a)x + \frac{(\log a)^2 x^2}{2} + \&c,$$

and the Logarithmic Series

$$\log a = (a - 1) - \frac{1}{2}(a - 1)^2 + \&c$$

deduced from it ; I object
to the necessity involved of
supposing x to be diminished
without limit, _ a supposition
[‘obviously’ inserted] quite necessary to the completion
of the Demonstration. It has
struck me that though this
supposition leaves the Demonstration
& Conclusions perfect for the
cases in which x is supposed
to diminish without limit, yet
[92v] it makes it valueless for the
many in which x may be
anything else which does not
diminish. _ No _ by the bye,
I think I begin to see it now ;
I am sure I do. It is as
follows : _ the supposition of
 x diminishing without limit
is merely a parenthetical
one, by means of which a
limit for a certain expression
 $\frac{a^x-1}{x}$ is deduced under those
circumstances ; & then the
argument proceeds, that having
already obtained in another
place, a [‘different’ inserted] limit for this same
expression under the same
[93r] circumstances, we at once
deduce the equality of these two
limits, from whence follows
&c, &c. Thus this supposition
of x diminishing without limit,
is not a portion of the main
argument, but only a totally
independent & parenthetical
hypothesis introduced in order
to prove something else which
is a part of the main
argument. _ Yes _ this is
it, I am sure. I had
had the same objection to

the Demonstration in Bourdon,
to which I have had the
curiosity to refer. I am
[93v] sometimes very much interested
in seeing how the same
conclusions are arrived at
in different ways by different
people ; and I happen to
have been inclined to compare
you & Bourdon in this
case of developing Exponential
& Logarithmic Series ; and
very amusing has it been to
me to see him begin exactly
where you end, &c. Your
demonstration is much the
best for practical purposes.
His is exceedingly general, &
the vast number of substitutions
[94r] of one thing for another make
it lengthy, & by no means very
simple to follow. ___ But it
is very well occasionally to
make these comparisons. ___

We are going to
Town on Monday the 25th,
for two or three nights, &
I will ask M^{rs} De Morgan's
& your permission to spend
Monday Evening with you,
going towards 8 o'clock,
as I did before. It would
give me great pleasure, &
may perhaps be not only
agreeable to me, but of use
[94v] too, as there are one or two
points [~~something crossed out~~] relating to my future plans
which I rather think of
speaking to you upon. ___

By the bye, Lord
Lovelace & I are both of us
much vexed, at our own

negligence in letting the Xmas
Vacation go bye [*sic*], without
proposing to you & your
lady & children to visit us
here, which you might
perhaps have been able to do
during Holiday-time. I fear
you may now be unable to
think of it ; but pray consider
[95r] the question with her ; if not
for any immediate use, at
any rate for the next occasion.
The fact is, that we have
so much the habit of thinking
of you only in connexion
with Town & engagements there,
that it only suddenly occurred
to us whether you might not
be able to breathe country
air like other people. —
You would come by Railway,
& we would send the
carriage to the Station for
you.

Yours most truly

A. A. L