[84r]

Ockham

Sunday. 17th Jan y

Dear Mr De Morgan. Many thanks for your reply to my enquiries. I believe I now understand about the limit of $\frac{\varphi(x+n\theta+\theta)-\varphi(x+n\theta)}{\theta}$ not being affected by $n\theta$ being a gradually varying quantity. I think your explanation of it amounts to this : that provided [something crossed out] $(x+n\theta)$ varies only towards a fixed limit, either of increase or diminution ; then [84v] the result of the Subtraction of $\varphi(x+n\theta)$ from $\varphi(x+n\theta+\theta)$ remains just the same as if, (calling $(x+n\theta)=Z$), Z were a fixed quantity. ____ Now by the conditions of the Demonstration in question, (in your pages 46 & 47), when a decrease takes place in θ, a certain simultaneous increase takes place in n. That is to say, suppose θ has at any one moment a certain value corresponding to which n has the value k. If I alter θ to a lesser value χ, then say that the corresponding [85r] value of n, necessary to fulfil the constant condition $n\theta=h$, is not k, but $k+m=p$. What happens now? _ Why as follows, I believe : there were, before θ became χ, k fractions ; there are now $k+m$, or p fractions. _ In ['each of' inserted] the k former fractions, [something crossed out] Z will

have diminished, towards a
fixed limit ['of diminution' inserted] x ; in ['each of' inserted] the m
new fractions introduced, Z
will be greater than in the
old k fractions ; but there
is a fixed limit of increase,
h, which it can never pass,
[85v] up to the very last Term
of the Series of Fractions. _
Therefore tho' the quantity
$x + n\theta$ or Z varies necessarily
with a variation in the value
of θ, yet it varies within
fixed limits either of
diminution or increase, & thus
the result of the subtraction
$\varphi(Z + \theta) - \varphi(Z)$ is not
affected. _
I hope I have made
myself clear. I think it is
now distinct & consistent in
my head. _

I see that my proof of
the limit for the function x^n
is a piece of circular arguments,
[86r] containing the enquiry which
I was in fact aiming at
in the former paper, but
which required to be
separated from the confusion
attendant on my erroneous
statements on other points.
I merely return the old
paper with the present one,
because it might perhaps be
convenient to compare them.

On the other side
of the sheet containing the
remarks on $\frac{a^\theta - 1}{\theta}$, you
will find an enquiry
which struck me lately
quite by accident in

[86v] referring to some old
matters. ____
I ought to make many
apologies I am sure for
this most abundant
budget. _ I am very
anxious about the matter
of the successive Differential
Co-efficients, & their
finiteness & continuity. I
think it troubles my
mind more than any
obstacles generally do. I
have a sort of feeling
that I ought to have
understood it before, &
[87r] that it is not a legitimate
difficulty. __
With many thanks,
Yours most truly
A. A. Lovelace