[77r]
Ockham
Sunday. 10^{th} Jan ^{y}

Dear M^{r} De Morgan. I send
you the [‘Series’ crossed out] Analysis of the
new Series I received in your
letter yesterday morning. I
believe I have made it out
quite correctly. In fact, the
Verification at the end proves
this. But, owing to a
carelessness in my [‘first’ inserted] inspection
of it, I have had the
trouble & advantage of analysing
two Series. I glanced too
hastily at it, & did not
observe that the factors of the
[77v] Denominators (of the Co-efficients),
are not powers of 2, but
simply multiples of 2. __
If you will open my Sheet,
you will find on the inside
my analysis of the Series I at
first mistook your’s [*sic*] for;
and I am not sorry this
has happened. I believe
both are correctly made out. __
You kindly request me
if I do not understand the
erasure in the former [‘small’ inserted] paper,
again to return it &c. Now
I do not agree to it; & [‘I’ inserted] still
fancy that we are in fact
meaning exactly the same
thing, only that you are
[78r] speaking of the n^{th} Term, & I
of the $n+1^{th}$. __ For
convenience of reference I again
return the former large paper
(& at any rate H. M’s
Post-Office will benefit). ____
I quite understand that

$\frac{1}{4} + \frac{1}{2}\sqrt{10^6 + \frac{1}{4} + a}$ is less than
501. Therefore as n is
the next whole number above
this fractional expression, $x =$
$= 501$. But n is not the
Term sought; the unknown
term to be determined being
by the conditions of the
Hypothesis & Demonstrations,
$n + 1$, & therefore $= 502$.
And if you will examine
[78v] your own ['former' inserted] Verification, you
will see that you there
determine the Term at which
Convergence begins, to be
A_{502}, or the 502nd Term,
which agrees with my
result $n + 1 = 502$. __
I think it is quite clear
that we are both agreed,
but that you were not aware
at the moment you made
the erasure that I was
not speaking of the next
whole number above $\frac{1}{4} + \frac{1}{2}\sqrt{10^6 + \frac{1}{4} + a}$
but of the next but one above
it. __
So much for the three Series: _
Now I must go to other
[79r] matters. I am indeed sending
you a Budget. __
I have been working hard
at the Differential Calculus,
& am putting together some
remarks upon Differential
Co-efficients (which in due
time will travel up to
Town for your approbation),
but in the progress of which
I am interrupted by a
slight objection to an old

matter of Demonstration,
which did not occur to me
at the time I was studying
it before, & sent you a
paper upon it ['from Ashley' inserted]. In the
course of the observations I
[79v] am now writing, I have ['had' inserted]
occasion to refer to the old
['general' inserted] Demonstration, (pages 46 & 47
of your Differential Calculus),
as to the finite existence of
a Differential Co-efficient
for all Functions of x; &
a slight flaw, or rather what
appears to me a flaw, in the
conclusions drawn, has occurred
to me. It is most clearly
proved that, θ being supposed
to diminish without limit,
the Fractions Q_1, Q_2 &c
must have finite limit, for
some value or other at all
events of $n\theta$ or h. But the
fractions in question do not
[80r] appear to me to be strictly
speaking analogous to $\frac{\Delta u}{\Delta x}$,
except the first of them $\frac{\varphi(a+\theta)-\varphi a}{\theta}$
and the last of them $\frac{\varphi(a+n\theta)-\varphi(a+\overline{n-1}\theta)}{\theta}$,
and for this reason.
In the expansion $\frac{\Delta u}{\Delta x}$ or
$\frac{\varphi(x+\theta)-\varphi x}{\theta}$, as θ alters
x does not alter, but remains
the same. In these fractions
on the contrary, which all
have the form $\frac{\varphi(a+k\theta)-\varphi(a+\overline{k-1}\theta)}{\theta}$
and in which $a+\overline{k-1}\theta$ [bar over $k-1$ should have little downward-pointing hooks at the ends]
stands for the x of the
expression $\frac{\Delta u}{\Delta x}$ or $\frac{\varphi(x+\theta)-\varphi x}{\theta}$,
not only does θ alter, but
from the conditions of the

Hypothesis & Demonstrations, $\overline{k-1}\theta$
[80v] & consequently $a + \overline{k-1}\,\theta$ must
likewise alter along with θ.
There is therefore a double
alteration in value going on
simultaneously, which appears
to me to make the Case quite
a different one from that
of $\frac{\Delta u}{\Delta x}$, & consequently to
invalidate all conclusions
deduced from the former with
respect to the latter. __
The validity of the Conclusions
with respect to the fractions
Q_1, Q_2 &c, you understand
I do not question. What I
question is the analogy between
these Fractions & the Fraction
$\frac{\Delta u}{\Delta x}$ or $\frac{\varphi(x+\theta)-\varphi x}{\theta}$ ['of' inserted] which
[81r] latter it is required to
investigate the Limits. __
I also have another slight
objection to make, not to the
extent of Conclusions established
respecting the Fractions Q_1, Q_2
&c having finite limits,
but to the Conclusions on that
point not going far enough, _
not going as far as they
might :__ "either these are
"finite limits, or some increase
"without limit and the rest
"diminish without limit ; if the
"latter, we shall have two
"contiguous fractions, one of which
"is as small as we please, and
"the other as great as we please,
"&c, &c, _ a phenomenon which
"which [*sic*] can only be true when
[81v] "Q_k is the fraction which is
"near to some singular value
"of the Fraction, & cannot be

“true of ordinary & calculable
“values of it &c.” _ Now it
appears to me that in no
possible case could such a
phenomenon as this be true,
when we consider how the
fractions are successively
formed one out of the others
by the substitution of $a + \theta$ for
a, θ too being as small as
we please. I therefore think
it might have been concluded
at once that there must
always be finite limits to
the Fractions Q_1, Q_2 &c,
[82r] and this whatever k or $n\theta$
may be. I suppose it is not
so, but I cannot conceive
the Case in which it could
be otherwise. __
I do not know if in writing
upon my two difficulties in
these pages 46, 47, 48, I have
expressed my objections (especially
in the former case of the
fractions Q_1, Q_2 not being
similar to $\frac{\Delta u}{\Delta x}$) with the
clearness necessary to enable
you to answer them, or indeed
to apprehend the precise points
which I dispute. It is not
always easy to write upon
these things, & at best one
must be lengthy. _ I shall be
[82v] exceedingly obliged if you will
also tell [‘me’ inserted] whether a little
Demonstration I enclose as to
the Differential Co-efficient
of x^n, is valid. It appears
to me perfectly so; & if it is,
I think I prefer it to your’s [*sic*]
in page 55. It strikes me

as having the advantage in
simplicity, & in referring to
fewer ['requisite' inserted] previous Propositions.
I have another
enquiry to make, respecting
something that has lately
occurred to me as to the
Demonstration of the Logarithmic
& Exponential Series in
your Algebra, but the real
truth is I am quite ashamed
[83r] to send any more; so will
at least defer this. ____
I am afraid you will indeed
say that the office of my
mathematical Counsellor or
Prime-Minister, is no joke.
I am much pleased to
find how very well I stand
work, & how my powers of
attention & continued effort
increase. I am never so
happy as when I am
really engaged in good
earnest; & it makes me most
wonderfully cheerful & merry
at other times, which is
curious & very satisfactory. __
What will you say when
[83v] you open this packet?____
Pray do not be very angry,
& exclaim that it really is
too bad. __

Yours most truly

A. A. Lovelace