[70r]

Ockham
Tuesday. 22^{nd} Dec ^{r}

Dear M^{r} De Morgan   I now see exactly my mistake. I had overlooked that the Series in question is not one in successive Powers of x [‘like that in page 185’ inserted], but only in successive even powers of x.

I used once to regret these sort of errors, & to speak of time lost over them. But I have materially altered my mind on this subject. I often gain more from the discovery of a mistake of this sort, than from 10 acquisitions made at [70v] once & without any kind of difficulty. __
There is still one little thing in your Demonstration not perfectly clear to me. __ At the end you remark that “our result “gave the 503^{d} Term instead of “the 502^{nd}, which arose from “taking the whole number next “above $\sqrt{x+\frac{1}{4}}$ instead of an “intermediate fraction.” __
In examining the equation
$n =$ next whole number above
$\frac{5}{4}+\frac{1}{2}\sqrt{1000,000+\frac{1}{4}}$
I see clearly that $\frac{5}{4}+\frac{1}{2}(1001)$ is greater than $\frac{5}{4}+\frac{1}{2}\sqrt{1000,000+\frac{1}{4}}$; that the true answer would be $\frac{5}{4}+\frac{1}{2}\left(1000+\frac{1}{a}\right)$, $\frac{1}{a}$ being some [71r] fraction. We should then have had,
$n =$ nearest whole number above

$\frac{5}{4} + 500 + \frac{1}{2a}$, instead of =
$= \frac{5}{4} + 500 + \frac{1}{2}$
But, since $\frac{5}{4}$ is greater than 1,
the result must exceed 501 even
if we neglected the $\frac{1}{4}$ altogether;
and therefore at any rate n
(the next whole number above
$\frac{5}{4} + \frac{1}{2}\sqrt{Z + \frac{1}{4}}$), must be 502, &
$n + 1$ consequently = 503 . __
I do not therefore see that the
fact of taking 1001 instead of
the real square part of $Z + \frac{1}{4}$
does account for the discrepancy
in question . ____
[71v] I have now some question to
put respecting certain operations
with Incommensurables. Thanks to
your Treatise I think I understand
['this subject' inserted] pretty tolerably now. But there
are still one or two points of
Practical Application which I
am [something crossed out] busy in working up
previous to leaving the subject
altogether as a direct study, &
which I find not quite plain
sailing. __
I have been writing out in
the Mathematical Scrap-Book,
a full explanation of the
operations with Incommensurables
analogous to those of Multiplication,
Division, Raising of Powers &c,
and a day or two ago I was
[72r] about completing it with that
analogous to the extraction of
Roots, when I found I did
not fully understand the
process, _ that is beyond the
consideration of one Mean
Proportional. I have written
out & enclose my explanation
for one Mean Proportional,

& my difficulty in the case
of two or more Mean
Proportionals. ____
Also, I wished now to return
to the passage, page 29, lines 8
and 9 from the top, (Trigonometry)
which first suggested to me the
necessity of studying the subject
of Incommensurables; in order
that I might see if I could
[72v] now demonstrate the Proposition
of (46), for θ and $\sin\theta$ Incom=
=mensurable quantities. _ But I do
not find that I can. I believe
I understand the example referred
to in (4), the long & short
of which I understand to be
that if in the Right-Angled
Triangle [diagram in original] A, B, C are
Incommensurables, and V be any
given linear unit, then the
Ratio compounded of $A:V$ and $A:V$
added to the Ratio compounded of
$B:V$ & $B:V$, is equal to the
Ratio compounded of $C:V$ and
$C:V$. _
With respect to the Ratio of an
Angle with it's [*sic*] Sine, I began to
[73r] write it out as follows, after the
manner of pages 68 & 69 of the
Number & Magnitude: __
θ or $\theta:1$ is the Ratio of $\frac{AB}{AO}$
$\sin\theta$ or $\sin\theta:1$ is the Ratio of $\frac{BM}{AO}$,
$AB:AO$, $BM:AO$ being
Incommensurable Ratios, what
then does $\frac{\theta}{\sin\theta}$ really mean? _
In the first place we may
consider it to mean

$$\theta\,\frac{1}{\sin\theta}:1, \text{ or a Ratio}$$

compounded of the Ratio $\theta:1$ and
$\frac{1}{\sin\theta}$, or compounded of the
Ratios $AB:AO$ and $AO:BM$. _

But further than this I cannot get, nor see my way at all. _

I conclude that in Incommensurable language, a Ratio equal to 1 or a Ratio approximately to 1 can [73v] only mean a Ratio in which the Magnitude constituting the Antecedent is equal to the Magnitude constituting the Consequent, or is constantly approaching an equality to it, and therefore that if we take the above Ratio compounded of $AB : AO$ and $AO : BM$, or the Ratio $AB : BM$, & prove that AB constantly approaches in equality to BM, that is the desired Demonstration. __

I can only end by repeating what I have often said before, that I am very troublesome, & only wish I could do you any such service as you are doing me. Yours most truly

A. A. L