[7r] My dear Lady Lovelace

The Theorem in page 16 can be easily proved when the following is proved

[$\frac{a+b}{}$ crossed out] $\frac{a+a'}{b+b'}$ lies between $\frac{a}{b}$ and $\frac{a'}{b'}$

$\frac{a+a'}{b+b'} = \frac{a(1+\frac{a'}{a})}{b(1+\frac{b'}{b})} = \frac{a}{b} \times \frac{1+\frac{a'}{a}}{1+\frac{b'}{b}}$

Now if $\frac{a}{b}$ be greater than $\frac{a'}{b'}$

$ab' \cdots\cdots\cdots\cdots\cdots a'b$

$\frac{b'}{b} \cdots\cdots\cdots\cdots\cdots \frac{a'}{a}$ whence $\frac{1+\frac{a'}{a}}{1+\frac{b'}{b}}$ is less than 1

or $\frac{a}{b} \times \frac{1+\frac{a'}{a}}{1+\frac{b'}{b}}$ is less than $\frac{a}{b}$

or $\frac{a+a'}{b+b'}$ is less than $\frac{a}{b}$

Similarly, it may be shown that if $\frac{a}{b}$ be less than $\frac{a'}{b'}$ $\frac{a+a'}{b+b'}$ is greater than $\frac{a}{b}$. You will now I think, not have much difficulty in proving the whole. Page 48 [or 28?] contains the general view of this theorem

Page 29. Our conclusions are really the same. To say that [diagram in original] is a r$^{\text{t}}$ angled triangle, is to say that OP is straight and not curved. The following however will explain

[7v] [diagram in original] By the tangent of $\angle POM$ is meant the fraction $\frac{PM}{OM}$, which is, by similar triangles, the same thing for every point of OP.

If then $PM = \frac{2}{3}OM$, always, we have $\frac{PM}{OM} = \frac{2}{3}$ always, or the direction OP is always such as to make the angle POM the same, namely that angle which has $\frac{2}{3}$ for its tangent. To see all this fully something of Trigonometry and the application of algebra to geometry is required.

The Differential and Integral Calculus deal in the same elements, but the former separates one element from the mass and examines it, the latter puts together the different elements to make the whole mass.

The examination of $PQMN$ (p. 29) with a view to the relation between OM and MP is a case of the first: the <u>summation</u> of the rectangles in page 30, of the second.

Page 32. The reference is unnecessary.

The first series $1 + 4 + \&c$ is finite, the second infinite. It is not easy to see <u>à priori</u> why one problem should be attainable with given means and another not

so. It is stated here with a view to the following
common misapprehension.
[8r] It is thought that Newton and Leibnitz had some
remarkable new conception of principles, which is not
true. Archimedes and others [‘and others’ inserted] had a differential and integral
calculus, but not an algebraical system of sufficient
power to express very general truths.
Many persons before Newton knew, for instance that if
$\frac{(x+h)^n-x^n}{h}$ could be developed for any value of n,
the tangents of a great many curves could be drawn
and they knew this upon principles precisely the same
as Newton and Leibnitz knew it. But Newton
did develope $\frac{(x+h)^n-x^n}{h}$ and did that which they
could not do.

It was the additions made to the powers of algebra
in the seventeenth century, and not any new
conceptions of quantity, which made it worth while
to attempt that organization which has been called
the Differential Calculus

I should recommend your decidedly continuing the
Differential Calculus, warning you that you will
have long digressions to make in Algebra and
Trigonometry. I should recommend you to get my
Trigonometry, but not to attempt anything till I
send you a sketch of what to read in it. The
Algebra you must go through at some time or
[8v] other, adding to it the article

“Negative and impossible quantities
in the Penny Cyclopaedia.
I have no doubt of being able to talk this
matter over with you in town when you
arrive
In the mean while, as mechanical expertness
in differentiation is of the utmost consequence,
and as it is the most valuable exercise in
algebraical manipulation which you can
possibly have, I should recommend your
thoroughly acquiring and keeping up the
Chapter you are now upon.

Yours very truly
ADeMorgan

3 Grotes’ Place

Monday Augst 17/40

M^{r} Frend is rather better. I will add Lord Lovelace's name to my list of members.