[33r] My dear Lady Lovelace

I return the papers about series which
are all right, the old one is as you suppose

With reference to your remarks on the diff[l] calculus

1. You observe that

$$\frac{\varphi(x+n\theta+\theta)-\varphi(x+n\theta)}{\theta}$$

differs from $\frac{\varphi(x+\theta)-\varphi(x)}{\theta}$
in that while θ diminishes, $x+n\theta$ varies. So it [second 'it' crossed out] is,
and if n be <u>finite</u> and <u>fixed</u>, it might be shown that the
limits of the two are the same. But if n increase while
θ diminishes, in such manner that $n\theta$ is either equal to
or approaches the limit a, then the first fraction has
the same limit as $\frac{\varphi(x+a+\theta)-\varphi(x+a)}{\theta}$

To illustrate this, let φx be the ordinate of a
curve, the abscissa being x. If x remains fixed, the triangle
[diagram in original] (blotted) diminishes without
limit with θ; but if while
θ diminishes, the point A moves
in toward B, so as continually to
approach B, and to come as near as
[33v] we please to it, and yet never absolutely to reach
B as long as θ has any value, it is obvious that
the small triangle would ride along the curve,
perpetually diminishing its dimensions, and
continually approaching in figure nearer and
nearer to the figure of as small a triangle <u>at</u> B.
All this necessarily follows from the notion
of continuity
[diagram in original]

2. You want to extend what I have said
about continuous functions to all possible cases,
not being able to imagine a function which changes
its values suddenly. But for this you must
wait till you come to the mathematics of disconti-
nuous quantity. It is perfectly <u>possible</u> though the
calculation would be laborious, to find an algebraical
function which from $x=1$ to $x=2$ increases like the
ordinate of a straight line, from $x=2$ to $x=3$ draws the
[diagram in original] likeness of a human profile in a
different place, from $x=3$ to $x=4$
draws a part of a circle, from
$x=4$ to $x=5$ is nothing, and from

[34r] $x = 5$ to $x = 6$ makes any odd combination of lines or curves, perfectly irregular. None of the notions incidental to continuity must be applied to such a function

3. Your proof of the diff.co. of x^n is correct, <u>but it assumes the</u> <u>binomial</u> <u>theorem</u>. Now I endeavor to establish the diff.calc. without any assumption of an infinite series, in order that the theory of series may be established upon the differential calculus

Besides, if you take the common proof of the binomial theorem, you are reasoning in a circle, for that proof requires that it should be shown that $\frac{v^n - w^n}{v - w}$ has the limit nv^{n-1} as w approaches v. This is precisely the proposition which you have deduced <u>from</u> the binomial theorem.

Pray send your point about the exponential theorem.

And thank Lord Lovelace for pheasants and hare duly received this morning

Yours very truly

<u>ADeMorgan</u>

69 G.S. Wed$^{\text{y}}$