

[3r] My dear Lady Lovelace

The solution of the $t^3 + t^2 + t$ problem is correct and I have no doubt from it that you fully understand the problem of the stone.

The difficulty you meet with in the variable coefficient would be fatal to the process if the coefficient increased without limit as the value of k diminishes without limit.

But in this case t which you call a variable, is really a given constant, for it is not t which varies, but the time which is first t , then $t + k$, then $t + 2k$, &c.

There is a little incorrectness in the phraseology of variable quantities. A quantity varies, it is first x , and then $x + a$; here it is usual to say that x varies, whereas it is not x , but the magnitude which x represents, which changes and is no longer represented by x , but by $x + a$.

Thus if we pass in thought from 10 seconds to 11 eleven [*sic*] seconds, we say let 10 vary, and become 11. Now 10 is a fixed symbol, and so is 11; it is we ourselves who vary our supposition, and pass from one to the other.

But even if the coefficients of k were variable, it would not vitiate the result, as may be thus shown Let $a + vk$ be an expression in which a is a constant k diminishes without limit, and v at the same time varies, say increases. Let v always remain finite, that is, let it not increase without limit while k diminishes

[3v] Suppose for instance, that it never exceeds a certain number (no matter how great) say a million. Then vk never exceeds $1000,000.k$. Now if k may be made as small as we please, so may $1000,000k$, and still more vk , which is less, or at least not greater.

That is vk diminishes without limit, and $a + vk$ has the limit a .

But if v increased without limit as k diminished, the case might be altered (not necessarily would). For example

1. Let $v = [\frac{1}{k} \text{ crossed out}] \frac{1+k}{k}$
 $a + vk = a + 1 + k$ and the limit is $a + 1$

2. Let $v = \frac{1+k}{k^2}$
 $a + vk = a + \frac{1+k}{k}$, and increases without limit

3. Let $v = \frac{1+k}{\sqrt{k}}$
 $a + vk = a + (1+k)\sqrt{k}$, and the limit is a , as
at first.

And $\frac{1+k}{k}$, $\frac{1+k}{k^2}$, $\frac{1+k}{\sqrt{k}}$ all increase without
limit as k diminishes without limit.

[4r] Your correction of the press is right. On the cover
of no 12 you will see a list of errata.

[diagram here in original] When two lines at right angles
are made standards of position, and
when the position of a point is
determined by its perpendicular
distance from the lines, as PA , PB ,
then PA and PB are called co-ordinates of P . One of
them, as BP is usually found by its equal OA , and called
the abscissa of P , the other, AP , is called the ordinate.

I have written something on the paper which I return.

My wife desires kind remembrances, and with our
united remembrance to Lord Lovelace

I remain

Yours very truly

ADeMorgan

3 Grottes' Place Blackheath

August 1, 1840

Should you decide on supporting our old printing
Society, I shall be very happy to have Lord Lovelace's name
inserted. We are beginning with an Old Saxon treatise on
astronomy with a translation.