[27r] My dear Lady Lovelace

I can soon put you out of your misery about p. 206.

You have shown correctly that $\varphi(x+y) = \varphi(x) + \varphi(y)$ can have no other solution than $\varphi x = ax$, but the preceding question is not of the same kind; it is not show that there can be no other solution except $\frac{1}{2}(a^x + a^{-x})$ but show that $\frac{1}{2}(a^x + a^{-x})$ is a solution: that is, try this solution

$$\varphi(x+y) = \tfrac{1}{2}(a^{x+y} + a^{-x-y})$$
$$\varphi(x-y) = \tfrac{1}{2}(a^{x-y} + a^{-x+y})$$
$$\varphi(x+y) + \varphi(x-y) = \tfrac{1}{2}(a^{x+y} + a^{-x-y} + a^{x-y} + a^{-x+y})$$
$$\begin{aligned} 2\varphi x.\varphi y &= 2.\tfrac{1}{2}(a^x + a^{-x}).\tfrac{1}{2}(a^y + a^{-y}) \\ &= \tfrac{1}{2}(a^x + a^{-x})(a^y + a^{-y}) \\ &= \tfrac{1}{2}(a^{x+y} + a^{-x+y} + a^{x-y} + a^{-x-y}) \end{aligned}$$

the same as before.

[27v] To prove that this can be the only solution would be above you

I think you have got all you were meant to get from the chapter on functions. The functional equations which can be fully solved are few in number

Yours very truly

ADeMorgan

69 G.S.
Mon^y^ M^g^