[24r] My dear Lady Lovelace

I send back your worked question.
The second is right the first wrong in two places

I should recommend you to get out of the habit of writing d thus ['d' with flourish at top of stem] or thus ['d' with flourish and double stem]. If you have much to do with the Diff. Calculus, it will make a good deal of difference in time. The best way is to form all the letters as like those in the book as you can.

Pages 13–15 of Elementary Illustrations bear closely on the distinction of $\frac{du}{dx} = a$ and $du = adx$

It is not because $\overline{1+x}^m \times \overline{1+x}^n = \overline{1+x}^{m+n}$ when m & n are whole numbers that the same is true for fractions but because a certain other property which is therefore true makes it necessary in the case of fractions.

For instance, the logic is as follows

A is true when m is a whole number
Whenever A is true, B is true
B is of that nature, that if true when m is a whole number, it is also true when m is a fraction.
When B is true C is true
$\therefore C$ is true if A be true when m is a whole number

Thus, when m and n are whole numbers,

$$\overline{1+x}^m = 1 + mx + m\,\tfrac{m-1}{2}\,x^2 + \&\text{c} \qquad \overline{1+x}\,|^n = 1 + nx + \&\text{c}$$

[24v] But $\overline{1+x}^m \times \overline{1+x}^n = \overline{1+x}^{m+n}$ always
Therefore when m and n are whole numbers
$(1 + mx + \&\text{c}) \times (1 + nx + \&\text{c}) = 1 + \overline{m+n}x + \&\text{c}$
but this last property (without any reference to its mode of derivation) is true of m and n fractional or negative if true of whole numbers.

Hence, if $1 + mx + \&\text{c}$ be called φm

$$\varphi m \times \varphi n = \varphi(m+n)$$

But this (again by an independent process) is shown to be never universally true unless

$$\varphi m = c^m,\ c \text{ being independent of } m$$

whence $c^m = 1 + mx + m\,\frac{m-1}{2}\,x^2 + \cdots$.
But since c is independent of m, what it is when $m = 1$, it is always. Therefore

$$c^1 = 1 + 1.x + 1.\tfrac{1-1}{2}\,x^2 + \cdots \qquad \text{or } c = 1 + x$$

Let $\quad \varphi(xy) = x \times \varphi y \quad$ be always true
required φx

Make $y = 1$, then $\varphi(x) = x \times \varphi(1)$

$\varphi(1)$ is not yet determined; let it be c. The equation is either true for all values of c, or for some (not all)

$\varphi x = cx$

If any particular value were needed for c, it would be found by making $c = 1$. Do this and we have

$\varphi(1) = c$, or $c = c$, which is true for all values of c.

Suppose

$$\varphi(x) = \varphi(1).x + 2\varphi(1) - \overline{\varphi(1)}|^2$$

to be true for all values of x. It is then true when $x = 1$, giving

$$\varphi(1) = \varphi(1) + 2\varphi(1) - \overline{\varphi(1)}|^2$$

or $\overline{\varphi(1)}|^2 - 2\varphi(1) = 0$

or $\varphi(1) = 2$ only

$$\varphi(x) = 2x + 2.2 - 2^2 = 2x$$
$$\varphi(1) \qquad\qquad = 2$$

which agree with each other.

You must remember than [*sic*] when a form is <u>universally</u> true, it is true in all particular cases. Then in $2x = x + x$, I have a perfect right to say this is true when $x = 1$ and $\therefore 2 = 1 + 1$, and true <u>when</u> $x = 20$, or $40 = 20 + 20$: though I do ['do' crossed out] not thereby say that x can be 10 and 1 both at once.

$$\frac{1}{m}\log z = \left(z^{\frac{1}{m}} - 1\right) - \frac{1}{2}\left(z^{\frac{1}{m}} - 1\right)^2 + \cdots.$$

when m increases $z^{\frac{1}{m}} - 1$ diminishes
It is not $\log z$ which $z^{\frac{1}{m}} - 1$ approaches to, but $\frac{1}{m}\log z$, which also diminishes as m increases
I see that in page 219 it is thus

$$\log z = \frac{1}{m}\left\{\overline{z^m - 1} - \frac{1}{2}\overline{z^m - 1}^n + \cdots\right\}$$

and m <u>diminishes</u> without limit. Now as m diminishes
[25v] $\frac{1}{m}$ increases, and the assertion is that as
m diminishes, and therefore $z^m - 1$, the

product $\frac{1}{m}(z^m - 1)$
has one factor continually increasing & the other diminishing, so that their product approaches without limit to $\log z$.

In page 187, it is shown that if x be made sufficiently small, any term of $a + bx + cx^2 + \cdots$ may be made to contain all the rest as often as we please, that is may be made as great as we please compared with the sum of all the rest. Consequently m being small
$z^m - 1$ may be made as great as we please compared with the sum of all the terms of the series which follow, even if all were positive, still more when they counterbalance each other by difference of sign.

If $\quad \varphi(x+y) = \varphi x + \varphi y \quad$ be always true (hypothesis)
It is true when $x = 0$
It is also true when $y = -x$
This equation being always true, is the representation of a collection of an infinite number of truths
I do not say that these truths coexist
Put it thus. Let φ be such a function that, if a, b, c, d, &c be any quantities whatever
$\varphi(a+b) = \varphi a \times \varphi b$
$\varphi(b+c) = \varphi b \times \varphi c$
$\varphi(d+e) = \varphi d \times \varphi e \qquad$ &c &c
That is let $\varphi(x+y) = \varphi(x) \times \varphi(y)$ for all values of x and y
1. Let $a = 0$, then $\varphi b = \varphi(0) \times \varphi(b)$
or $\varphi(0) = 1$
Let $b = -c$, then $\varphi(0) = \varphi(b) \times \varphi(c)$
or $1 = \varphi(b) \times \varphi(-b)$
and so on.
There is a want of distinction between an equation made true by choice of values and one which is true of itself, independently of all values
$x = 3 - x \qquad x = \frac{3}{2}$, and then only
$x = 3x + a - (2x + a)$ is true for all values of x, though it cannot have more than one at a time.

There is the erratum in the Trigonometry, as you say

Yours very truly
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