[20r] My dear Lady Lovelace

With regard to the error in Peacock you will see that you have omitted a sign. It is very common to suppose that if φx differentiated gives ψx, then $\varphi(-x)$ gives $\psi(-x)$, but this should be $\psi(-x) \times$ diff.co.$(-x)$ or $\psi(-x) \times -1$. Thus

$$y = \varepsilon^x \qquad \frac{dy}{dx} = \varepsilon^x$$
$$y = \varepsilon^{-x} \qquad \frac{dy}{dx} = \varepsilon^{-x} \times (-1) = -\varepsilon^{-x}$$

As to the note, my copy of Peacock wants a few pages at the beginning by reason of certain thumbing of my own and others in 1825. I remember however that there is a note which I did not attend to, nor need you. But if curiosity prompts, pray sent it to me in writing.

As to $du = \varphi(x).dx$, you should not have written it $du = \varphi(x)$ as you proposed but

$$\frac{du}{dx} = \varphi x$$

The differential coeff$^{\text{t}}$ is the limit of $\frac{\Delta u}{\Delta x}$, and is a Total symbol. Those whose [*sic*] write

$$y = x^2 \quad \therefore \quad dy = 2xdx \text{ make an error}$$

but if $dy = 2xdx + \alpha$ be the truth, α diminishes without limit as compared with dx, when dx diminishes. Consequently α is of no use in finding [20v] any limit, and those who use differentials, as they are called, do not differ at the end of their process from those who make limiting ratios as they go along. You can however for the present transform Peacock's formula $du = A\,dx$ into $\frac{du}{dx} = A$.

There is the erratum you mention in Alg. p. 225

As to p. 226

$$\frac{1+b}{1-b} = \frac{1+x}{x}$$
$$(1+b)x = (1-b)(1+x)$$
$$x + bx = 1 + x - b - bx$$
$$bx = 1 - b - bx$$
$$2bx + b = 1$$
$$(2x+1)b = 1 \qquad b = \frac{1}{2x+1}$$

Verification $$\frac{1+\frac{1}{2x+1}}{1-\frac{1}{2x+1}} = \frac{2x+1+1}{2x+1-1} = \frac{2x+2}{2x}$$
$$= \frac{2(x+1)}{2x} = \frac{x+1}{x}$$

[21r] p. 212. To shew that for instance

$$n\,\frac{n-1}{2}\,\frac{n-2}{3}\,\frac{n-3}{4}\,\frac{n-4}{5} + m\,n\,\frac{n-1}{2}\,\frac{n-2}{3}\,\frac{n-3}{4} + m\,\frac{m-1}{2}\,n\,\frac{n-1}{2}\,\frac{n-2}{3}$$
$$+m\,\frac{m-1}{2}\,\frac{m-2}{3}\,n\,\frac{n-1}{2} + m\,\frac{m-1}{2}\,\frac{m-2}{3}\,\frac{m-3}{4}\,n + m\,\frac{m-1}{2}\,\frac{m-2}{3}\,\frac{m-3}{4}\,\frac{m-4}{5}$$

$= \overline{m+n}\,\frac{m+n-1}{2}\,\overline{\frac{m+n-2}{3}}\,\frac{m+n-3}{4}\,\frac{m+n-4}{5}$ without actual multiplication
m and n being whole numbers

1. $m\,\frac{m-1}{2}\,\frac{m-2}{3}\,\ldots\ldots\frac{m-(r-1)}{r}$ is the number of ways in which r can be taken out of m (see chapter on combinations in the Arithmetic)

If then we denote by (a, b) the number of ways in which a can be taken out of b, we have to prove that

$$(5,n) + (1,m)\times(4,n) + (2,m)\times(3,n) + (3,m)\times(2,n)$$
$$(4,m)\times(1,n) + (5,m) = (5, m+n)$$

Suppose ['the' crossed out?] $m+n$ counters to be divided into two parcels, one containing m and the other n counters
He who would take 5 out of them must either take

	0	out of the	m	and 5	out of the	n
or	1	$\cdots\cdots$	m	$\cdots$ 4	$\cdots\cdots$	n
or	2	$\cdots\cdots$	m	$\cdots$ 3	$\cdots\cdots$	n
	3	$\cdots\cdots$	m	$\cdots$ 2	$\cdots\cdots$	n
	4	$\cdots\cdots$	m	$\cdots$ 1	$\cdots\cdots$	n
	5	$\cdots\cdots$	m	$\cdots$ 0	$\cdots\cdots$	n

Now if to take say the third of these cases, we can take two of m in a ways and 3 out of n in b ways [21v] we can do both together in $a\times b$ ways. For if for instance there are 12 things in one lot and 7 in another, we can take one out of each lot in 12×7 ways, since any one of the twelve may come out with any one of the seven
Hence the number of distinct ['distinct' inserted] ways of bringing 2 out of m and 3 out of n together is

$$(2,m)\times(3,n)$$

I think you will now be able to make out that the preceding theorem is true when m and n are whole, whence, by the reasoning in the book it must be true when they are fractional.

This reasoning you do not see. It is an appeal to the nature of the method by which algebraical operations are performed. There is no difference of operation in the fundamental rules (addition subt$^{\text{n}}$ mult$^{\text{n}}$ & div$^{\text{n}}$) whether the symbols be whole nos or fractions. Hence if a theorem be true when the letters are <u>any</u> wh. nos, it remains true when they are fractions
For example, suppose it proved that for all whole nos

$$(a+b)\times(a+b) = a\times a + 2a\times b + b\times b$$

we should then, if we performed the operation $(a+b)\times(a+b)$ remembering that a and b are whole numbers find $a\times a+$ &c

$$\begin{array}{l} a+b \\ \underline{a+b} \\ aa+ab \\ \quad\;\; +ab+bb \\ \hline aa+2ab+bb \end{array}$$

[22r] Now in no part of this operation are you required to stop and do ['or omit' inserted] anything because the letters are whole numbers which you would not do or not omit if they were fractions. Consequently, the reservation that the letters are whole numbers cannot affect the result which if true with it is true without. This principle requires some algebraical practice to see the necessity of its truth.

The notation of functions is very abstract. Can you put your finger upon the part of Chapt. X at which there is difficulty

The equation

$$\varphi(x)\times\varphi(y)=\varphi(x+y)$$

is supposed to be universally true for all values of x and y. You have hitherto had to deal with equations in which value was the thing sought: now it is not value, but form. Perhaps you are thinking of the latter when it ought to be of the former.

With our remembrances to L.' Lovelace I am

Yours very truly

ADeMorgan

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