[18r] My dear Lady Lovelace

First as to your <u>non</u> mathematical question: I do not think an English jury would have found Mad. Laffarge guilty; but the presumptions of guilt and innocence can only be perfectly made by those who have the same national opinions and feelings as the accused. I think it very possible that a Frenchman may know a Frenchman to be guilty upon grounds which an Englishman would not understand; for instance, a particular act may be such as a Frenchman may know a Frenchman would not do, unless he had committed a murder before; and such acts may have been proved for aught I (who have not read much of the trial) can tell. It is the same thing in our courts of justice: judges and counsel can by experience make things which would appear to you or me almost indifferent, carry very positive conclusions to their own minds.

Now as to the part of your difficulty contained between X X in the remarks. It matters nothing (p. 210) whether $\frac{n-p}{p+1}x$ is negative or positive. If you perfectly understand why I neglect the sign in the fractional case, the same reason applies to the negative one. When n is negative then $n-p$ (p being essentially $+$) is necessarily negative. <u>Consequently</u>, (x being positive) the terms alternate in sign from the very beginning, whereas when n is positive and fractional, they do not begin to alternate until p passes n. But our matter is to determine convergency, and if a series of positive terms be convergent, so will be the series of similar terms alternating. It is on the <u>absolute</u> <u>magnitude</u> of $\frac{n-p}{p+1}x$, independent of sign, that it depends whether the terms shall ultimately diminish so as to create convergence, or not. Now the limit of this is x, whence follows as in the book
[18v] It is important to remember in results which <u>depend</u> <u>entirely</u> on limits that they have nothing to do with any vagaries which the quantity tending to a limit chooses to play, provided that, when it has sown its wild oats, it settles down into a steady approach to its limit. The sins of its youth are not to be remembered against it. Now $\frac{n-p}{p+1}x$ when n is positive, remains positive until p passes n and then settles into incurable negativeness. But when x is ['positive' crossed out] negative, it is negative from the beginning Now this matters nothing as to a result which depends only on the limit to which $\frac{n-p}{p+1}x$ approaches as p is increased without limit.

As to the point marked B, remember that placing this doubtful assumption, namely the expansibility of functions of x in whole powers of x, out of doubt by instances,

has been a prevailing vice of algebraical writers, and
one which is to be carefully avoided. It was once thought, by instance, that x^2+x+41 must be a prime number, whenever x is a whole number, for

$$\begin{array}{llrl}
x=0 & x^2+x+41 & =41 & \text{a prime n}^{\text{o}} \\
=1 & & =43 & \cdots\cdots\cdots \\
=2 & & =47 & \cdots\cdots\cdots \\
=3 & & =53 & \cdots\cdots\cdots \\
=4 & & =61 & \cdots\cdots\cdots \\
=5 & & =71 & \cdots\cdots\cdots \\
=6 & & =83 & \cdots\cdots\cdots \\
=7 & & =97 & \cdots\cdots\cdots
\end{array}$$

Now x^2+x+41, though it gives nothing but prime numbers up to $x=39$ inclusive, yet gives a composite number when $x=40$; for it then is

$$40\times 40+40+41 \quad \text{or} \quad 41\times 40+41 \text{ or } 41\times 41$$

and when $x=41$ it is

$$41\times 42+41 \quad \text{or} \quad 43\times 41$$

and for higher values it gives sometimes prime numbers sometimes not, like other functions. So much for instances.

C. The supposition as to the meaning of the non-arithmetical roots is right (Read from p. 109 "We shall now proceed" to p. 113 inclusive). When we use $\sqrt{-1}$, which we must do at present, if at all, without full explanation, it is to be remembered that we say two expressions are equal when they are algebraically the same, that is, when each side has all the algebraical properties of the other. ['M' crossed out] Numerical accordance must not be looked for when one or both sides are numerically unintelligible, and algebraical accordance merely means that everything which is true of one side is true of the other. It is then unnecessary to consider any restrictions which may be necessary when numerical accordance is that which is denoted by =.
But this is touching on even a higher algebra than the one before you
Suppose

$$\tfrac{1}{1-x}=1+x+x^2+x^3+\cdots\text{ ad inf.}$$

which is certainly true in the arithmetical sense when [19v] $x<1$. But if $x>1$, say $x=2$, we have

$$\tfrac{1}{1-2} \text{ or } -1=1+2+4+8+16+\,\&c$$

which, arithmetically considered is absurd. But nevertheless -1 and $1 + 2 + 4 + 8 +$ &c have the same properties

This point is treated in the chapter on the meaning of the sign =.

My wife desires to be kindly remembered

I remain

Yours very truly

ADeMorgan

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It is fair to tell you that the use of divergent series is condemned altogether by some modern names of very great note. For myself I am fully satisfied that they have an algebraical truth wholly independent of arithmetical considerations; but I am also satisfied that this is the most difficult question in mathematics.