

[174r] [(mostly) in AAL's hand]

Theorem. Page 199.

If N be a function of x and y , giving $\frac{dN}{dx} = p + q \frac{dy}{dx}$
then the equation $\frac{du}{dx \cdot dy} = V \cdot \frac{dN}{dx \cdot dy}$ is incongruous &
self-contradictory, except upon the assumption
that u is, as to x and y , a function of N ;
or contains x and y only thro' N .

Let $N = \psi(x, y)$ give $y = \chi(N, x)$, and
suppose, if possible, that the substitution of
this value of y in u gives $u = \beta(N, x)$, x
not disappearing with y . Then x and y
varying

$$\frac{du}{dx \cdot dy} = \frac{d\beta}{dN} \cdot \frac{dN}{dx} + \frac{d\beta}{dN} \cdot \frac{dN}{dy} + \frac{d\beta}{dx}$$

[in above line, $\frac{du}{dx \cdot dy}$ is crossed through in pencil, and '1' written above;

'= $\frac{du}{dx} + \frac{du}{dy}$ ' added in pencil at end of line — in ADM's hand?]

$$= \frac{d\beta}{dN} \cdot \left(\frac{dN}{dx} + \frac{dN}{dy} \right) + \frac{d\beta}{dx} = \frac{d\beta}{dN} \cdot \frac{dN}{dx \cdot dy} + \frac{d\beta}{dx} =$$
$$= V \cdot \frac{dN}{dx \cdot dy}, \text{ which equation being}$$

universal, is true on the supposition that x
does not vary, or that $\frac{d\beta}{dx} = 0$. This gives $\frac{d\beta}{dN} = V$;

$$\text{or } \frac{du}{dx \cdot dy} = V \frac{dN}{dx \cdot dy} + \frac{d\beta}{dx} = V \frac{dN}{dx \cdot dy}$$

because $\frac{d\beta}{dN}$ and V being independent of the variations
&c, &c. Hence $\frac{d\beta}{dx} = 0$ always; or β does not
contain x directly, &c.

I think the above is correct. I cannot see
[174v] the use (page 200) of introducing t in
the proof there given. Is it possible that
I have committed an error in my original
understanding of the enunciation [*sic*] of the Theorem;
& that the du ['of the equation' crossed out] and the dN
of the equation $du = V \cdot dN$, do not mean
the du and dN derived from differentiating
with respect to the quantities x and y ,
already introduced; but with respect
to ['some' crossed out] other given quantity?__

I suspect so.

[the following appears underneath in pencil — still in Ada's hand]

$$u = \beta(N, x)$$
$$\frac{du}{dx} = \frac{d\beta}{dN} \frac{dN}{dx} + \frac{d\beta}{dx}$$
$$\frac{d^2 u}{dx \cdot dy} =$$