Theorem. Page 199.

If \( N \) be a function of \( x \) and \( y \), giving 
\[
\frac{dN}{dx} = p + q \frac{dy}{dx}
\]
then the equation 
\[
\frac{du}{dx} \frac{dy}{dx} = V \cdot \frac{dN}{dx} \frac{dy}{dx}
\]
is incongruous & self-contradictory, except upon the assumption that \( u \) is, as to \( x \) and \( y \), a function of \( N \); or contains \( x \) and \( y \) only thro’ \( N \).

Let \( N = \psi(x, y) \) give 
\[
y = \chi(N, x)
\]
and suppose, if possible, that the substitution of this value of \( y \) in \( u \) gives 
\[
u = \beta(N, x), x
\]
not disappearing with \( y \). Then \( x \) and \( y \) varying
\[
\frac{du}{dx} \frac{dy}{dx} = \frac{d\beta}{dN} \frac{dN}{dx} + \frac{d\beta}{dx} \frac{dN}{dy}
\]
in above line, \( \frac{du}{dx} \frac{dy}{dx} \) is crossed through in pencil, and ‘1’ written above;
\[
\frac{d\beta}{dx} = \frac{d\beta}{dN} \cdot \frac{dN}{dx} + \frac{d\beta}{dy} \cdot \frac{dN}{dy} + \frac{d\beta}{dx}
\]
which equation being universal, is true on the supposition that \( x \) does not vary, or that \( \frac{d\beta}{dx} = 0 \). This gives \( \frac{d\beta}{dN} = V \); or
\[
\frac{du}{dx} \frac{dy}{dx} = \frac{dN}{dx} \frac{dy}{dx} + \frac{d\beta}{dx} = \frac{dN}{dx} \frac{dy}{dx}
\]
because \( \frac{d\beta}{dN} \) and \( V \) being independent of the variations &c, &c. Hence \( \frac{d\beta}{dx} = 0 \) always; or \( \beta \) does not contain \( x \) directly, &c.

I think the above is correct. I cannot see the use (page 200) of introducing \( t \) in the proof there given. Is it possible that I have committed an error in my original understanding of the enunciation [sic] of the Theorem; & that the \( du \) [‘of the equation’ crossed out] and the \( dN \) of the equation \( du = V.dN \), do not mean the \( du \) and \( dN \) derived from differentiating with respect to the quantities \( x \) and \( y \), already introduced; but with respect to [‘some’ crossed out] other given quantity?

I suspect so.

[the following appears underneath in pencil — still in Ada’s hand] 
\[
u = \beta(N, x)
\]
\[
\frac{du}{dx} = \frac{d\beta}{dN} \frac{dN}{dx} + \frac{d\beta}{dx}
\]
\[
\frac{du}{dx} \frac{dy}{dx} = \frac{d\beta}{dN} \frac{dN}{dx} \frac{dy}{dx} = \frac{d\beta}{dx}
\]