[152r]

Ockham Park
Surrey

Dear M[r] De Morgan. I am indeed extremely obliged to you for all your late communications. In two or three days more, I shall have several observations & to send you in reply to some of them. _

My object in writing today, is to make another enquiry concerning the substitution of $\varphi_1(a+h) - \varphi_1 a$ for $\varphi(a+h) - \varphi a$ in page 100, for I perceive on carefully examining the passage, that I do not quite understand it

$\varphi_1 x = \varphi x + C$, which ['last side' inserted] means the Primitive Function ['of $\varphi' x$' inserted]
and the Primitive Function means the Function which differentiated gives $\varphi' x$

Therefore $\varphi_1(a+h) = \varphi(a+h) + C$

And $\varphi_1 a = \varphi a + C$

Consequently $\varphi_1(a+h) - \varphi_1 a = \varphi(a+h) + C - (\varphi a + C)$

$= \varphi(a+h) - \varphi a$

[152v] This is my version of it. But you tell me,

$\varphi_1(a+h) = \varphi(a+h) + C$

$\varphi_1 a = \varphi a$, (which ought to be I say

$\varphi_1 a = \varphi a + C$)

From which we should have,

$\varphi_1(a+h) - \varphi_1 a = \varphi(a+h) + C - \varphi a$

Consequently $\varphi_1(a+h) - \varphi_1 a$ is not equal to $\varphi(a+h) - \varphi a$ as is required to be proved, but is $= \varphi(a+h) - \varphi a + C$. _

I cannot unravel this at all. __

Second[ly]: [something crossed out] I do not see why the Indefinite Integral only is $= \varphi x + C =$ Primitive Function.
of $\varphi' x$

The argument at the top of page 101 seems to [something crossed out] me to apply equally to the Definite Integral

As follows : It is proved that

$$\varphi_1(a+h) - \varphi_1 a = \int_a^{a+h} \varphi' x.dx$$

$\varphi_1 a$ is just as much here an arbitrary Constant
as it is in $\varphi_1 x - \varphi_1 a = \int_a^x \varphi' x.dx$
Therefore $\int_a^{a+h} \varphi' x.dx = \varphi_1(a+h) + C_1$
$= \varphi(a+h) + C + C_1$
$= \varphi(a+h) +$ an arbitrary Constant
[153r] just as with $\varphi_1 x - \varphi_1 a = \int_a^x \varphi' x.dx$

Thirdly : With respect ['to' inserted] the assumption that when a is arbitrary, then any function of a, say φa, is also arbitrary or may be anything we please, seems to me not always valid. __
For instance if $\varphi a = a^0$, it must be always = 1. We may assume a = anything we like, but φa will not in this case be arbitrary. _
It is curious how many little things I [something crossed out] discover in this Chapter, which in looking back upon them, I find I have only half-understood.

I shall be exceedingly obliged, if you can answer these points soon ; I think a word almost may explain them, & they rather annoy me. ____

Believe me, with many thanks

Yours very truly

A. A. Lovelace