[146r]

Ashley-Combe
Wed[dy]. 16[th] Nov[r]

['1842' or '1847' added by later reader]

[I think this might date from earlier than 1842, as it contains material that seems to fit better with the earlier letters — I'll try to slot it in]

Dear M[r] De Morgan. I am very much obliged for your long letter. The Formula in Peacock comes out quite correct now that I have written diff. co of $(b-x)^4 = 4(b-x)^3 \times (-1) = -4(b-x)^3$. It is odd that notwithstanding the caution you gave me in Town on this very point, I should have fallen into the trap. There is nothing like one's own blunders after all for instruction. __ I do not however understand why example (19) page 4, has not come out wrong also in my working out. I enclose a copy of my solution, and it appears to me it ought to be wrong, because I surely should have had diff. co of $(1-x)^4 = 4(1-x)^3 \times (-1) = -4(1-x)^3$, whereas I have diff. co of $(1-x)^4 = 4(1-x)^3$. __

On looking over my development again very carefully, I am inclined to think that my solution [146v] $\frac{(1+x)^2}{(1-x)^5} \times (7+x)$, comes out right only because I have managed to make another blunder of a sign in the course of the proofs, which has corrected the first blunder. I therefore now write on the other side of the paper, what I think it should be. __ The note in page 2 I do not imagine to be of any consequence. It is on "rendering the Differentiation "of complicated Functions sometimes much easier" by means of three Theorems from Maclaurin's Fluxions.

Certainly had I thought a little more upon what I read some weeks ago, before I wrote my last letter to you, I should not have sent the question about $du = \varphi(x) \times dx$ [flourishes at tops of stems of 'd's, here and after].
I ['must have' inserted] forgot exactly what a Differential Co-efficient means, when I did so. __ But how is it then that in your 1[st] Chapter of the Differential Calculus" there is no mention of the multiplication by dx?"

I conclude that the real Differential Co-efficient
is $\frac{du}{dx} = \varphi(x)$, and that Peacock's solutions are"
not strictly speaking Differential Co-efficients. __"
I think pages 13 to 15 of your Elementary
[147r] Illustrations bear considerably upon the observations
in your letter, do they not? __
Your explanation of Euler's proof of the Binomial
Theorem is perfectly satisfactory to me. Unluckily
I have not any Book here which contains the
Theory of Combinations. I wanted to refer to
this when reading page 215, as I have forgotten
it in it's [*sic*] particulars. However this can very
well wait a short time, & I have only to take
the Formula for Combinations for granted meanwhile.
The necessity of the truth of $(1+x)^n \times (1+x)^m = (1+x)^{n+m}$
for all values of n and m, since it is true
when they are whole numbers, I shall probably
see more clearly at some further time. ____

I can explain exactly what my
difficulty is in Chapter X. _ "For instance, if we
"know that $\varphi(xy) = x \times \varphi y$, supposing this always
"true, it is true when $y = 1$, which gives $\varphi(x) =$
"$= x \times \varphi(1)$. But $\varphi(1)$ is an independent quantity,
"made by writing 1 instead of y in $\varphi(y)$. Let us
"call it c &c. _"
It is this substitution of 1 and of c, and
consequent ascertainment of the form which will
[147v] satisfy the equation, which is all dark to me.
It is ditto in lines 12, 13, & 14 from the top. __

I understand quite well I believe from
"We have seen that if $\varphi x = c^x$ &c", all through
the next page.
That I do not comprehend at all the means of
deducing from a Functional Equation the form
which will satisfy it, is I think clear from
my being quite unable to solve the example
at the end of the Chapter "Shew that the equation
"$\varphi(x+y) + \varphi(x-y) = 2\varphi x \times \varphi y$ is satisfied
"by $\varphi x = \frac{1}{2}(a^x + a^{-x})$". I have tried
several times, substituting first 1 for x, then
1 for y. but I can make nothing whatever
of it, and I think it is evident there is

something that has preceded, which I have
not understood. _ The 2nd example given for
practice "Shew that $\varphi(x+y) = \varphi x + \varphi y$ can
"have no other solution than $\varphi x = ax$", I
have not attempted. ____
I have a question to ask upon page 229.

"By extracting a sufficiently high root of z, we
[148r] "can bring z^m as near to 1 as we please, or
"make $z^m - 1$ as small as we please ; that is
"(page 187) $z^m - 1$ may be made as nearly equal
"to the sum of the whole series as we please". _

I cannot find what it is that is referred
to in page 187 ; and Secondly, it appears to me
somewhat of a contradiction that a quantity
$z^m - 1$ which can certainly be made as small
as we please by the diminution of m, should
become as near as we please to a fixed
limit or sum (the $\log z$ I conclude is the
sum of the series, referred to), since by continued
diminution the quantity $z^m - 1$ may become a
great deal less than the sum of the Series, &
keep receding from it. __
To return to Chapter X, there is one other
thing in it that I do not understand. Page
205, lines 5, 6, 7 from the bottom. It seems
to me fallacious to substitute first one value
0, for a letter ; & then another value, let $y = -x$,
[148v] in the same equation & in a manner at the
same time. How can the two suppositions
consist together at all. ____

I go on well with the Trigonometry, &
have nearly finished the Number & Magnitude.
I think there is another Erratum in page
34 of the Trigonometry, line 13 from the bottom

$$= \frac{OM}{ON} \cdot \frac{ON}{OP} - \frac{NR}{NP} \cdot \frac{NP}{NO} \text{ \&c}$$

should be $-\frac{NR}{NP} \cdot \frac{NP}{OP}$

I am really ashamed to send you such
troublesome letters. ____

Believe me

Yours most truly

A. A. Lovelace