[144r]

Ashley-Combe
Porlock
Somerset
Sunday Mor^g. 28^th Aug^st
['1842' added by later reader]

Dear M^r De Morgan. I am going on well ; __ ['quite' inserted] as I could wish. I have done much since I saw you ; & you will have all the results of the last few days in good time. I enclose you now two papers ; one on $f = \frac{dv}{dt}$, the other on $\int_a^{a'} f.dt$. You will have next those on $v\frac{dv}{dt} = f$, and $v^2 = 2\int f.ds + C$. This latter I think I have succeeded in analysing to my mind.
I have ['now' inserted] two observations to make : [something crossed out]
1^stly: I think I have detected a slight error in one of my former papers, that on $t = \int \frac{ds}{v}$. I return it for reference. In order in the [something crossed out] Summation [something crossed out]
$\left\{\frac{1}{\varphi s} + \frac{1}{\varphi(s+ds)} + \cdots \frac{1}{\varphi(2s)}\right\} ds$, to end with $\frac{1}{\varphi(2s)}$,
I should have begun with $\frac{1}{\varphi(s+ds)}$ not with $\frac{1}{\varphi s}$.
If the time elapsed during the first fraction of Space [144v] (starting from s) were ['made' inserted] $= \frac{1}{\varphi s}$, then the time for the last of the Fractions necessary to complete up to $2s$, would be $\frac{1}{\varphi(2s-ds}$, and not $\frac{1}{\varphi(2s)}$ which it ought to be.
I don't know that this affects the correctness of the ultimate limit of the Summation. But here, where the Summation itself is made to represent a hypothetical movement, it is clearly wrong.
The error is avoided in the former paper I had written on $s = \int v.dt$, which I likewise return to refer to this Point. _
2^ndly: In considering a priori the Integral $\int f.ds$, I am inclined still to adhere to my original opinion (expressed in the pencil Memorandum I showed you & ['which I' inserted] now return). I should premise that I now mention this merely as a curious subject of investigation, not because it is concerned in the [something crossed out] papers I am making out upon $v^2 = 2\int f.ds + C$, in which I have avoided the direct consideration of $\int_a^{a'} f.ds$. ____

I am disposed to contend that tho' ds
does here represent Space, that still the ds fraction
of any one of the terms of the Summation, say $\varphi(a+n.ds)ds$
means the same fraction of $\varphi(a+n.ds)$ which ds is of
[145r] a Unit of Space ; & therefore that since $\varphi(a+n.ds)$
represents Force, (or ['uniform' inserted] Acceleration of Velocity for 1 Second
in operation during the performance of the length ds),
the ds fraction of this expression must represent the
['ds part of this Force or the' inserted] actual Acceleration for $\frac{1}{ds}$ of a Second. I treat ds as
an abstract quantity. And so I conceive [something crossed out] dt must
be treated in $s=\int v.dt$, ['ds' inserted] in $t=\int \frac{ds}{v}$, dt in $\int f.dt$,
&c, &c. ____
I should tell you that I am much pleased with
the observation you added to my inverse demonstration
of $\int fx.\frac{dx}{dt}dt=\int fx.dx$, and that I quite
understand ['why' inserted] my proof can only be admissible on
the Infinitesimal Leibnitzian Theory. But this
theory is to my mind the only intelligible or
satisfactory one. In fact, (notwithstanding it's [*sic*] error),
I should call it the only true one. __
By and bye, you will have some observations
of mine upon Differential Co-efficients & Integrals,
abstractly considered. I have been thinking much
upon them. _
I am going on with Chapter VIII.
By the bye, I believe you will receive somehow tomorrow
[145v] a book (the 1st Vol of Lamé's Cours de Physique)
in which there is a passage which I will write
to you about as soon as I find time. __
I forgot to mention it to you on Thursday ; &
so have ordered the Book to be sent to you, that
I might write about it sometime. __

Believe me
Yours very truly
A. A. Lovelace