[14r] My dear Lady Lovelace

Your inquiries were received just after I
had dispatched the receipt for Lord L's subscription to the Hist. Soc.

While I think of it (the Hist. Soc. reminds me) Nicolas Occam, or Ockham, or of Ockham, who flourished about 1350, took his name I rather think, from the same place as your little boy. He was a mathematician, and one of the most remarkable English metaphysicians before Locke. It is very likely that the late Ld King may on both accounts, Ockham and metaphysics, [', Ockham and metaphysics,' inserted] have collected something about him, or that Lord Lovelace may be in possession of something relating to him. If so, it can certainly be made useful. His <u>logic</u> was printed very early but is so scarce that I have never been able to get sight of a copy.

Now to your queries. <u>Festina lente</u>, and above all never estimate progress by the number of pages. You can hardly be a judge of the progress you make, and I should say that it is more likely you progress rapidly upon a point that makes you think for an hour, than upon an hour's quick reading, even when you feel satisfied. That which you say about the comparison of what you do with what you see can be done was equally said by Newton when he compared himself to a boy who had picked up a few pebbles from the shore; and the last words of Laplace were 'Ce que nous connaissons est peu de chose; ce que nous ignorons est immense' So that you have respectable authority for supposing that you will never get rid of that feeling; and it is no use trying to catch the horizon.
[14v] Peacocks examples will be of more use than any book.

As to the functional equation. You must distinguish in algebra questions of <u>quantity</u> from questions of <u>form</u>. For example "given $x + 8 = 10$, required x," is a question of quantity but "given x, an arbitrary variable, required a function of x in which if the function itself be substituted for x, x shall be the result" is a question of <u>form</u>, independent of <u>value</u>, for it is to be true for all values of x. One solution is

$$\frac{1-x}{1+x}$$

for x substitute the function itself, this gives $\frac{1+\frac{1-x}{1+x}}{1+\frac{1-x}{1+x}}$

or $\frac{1+x-(1-x)}{1+x+(1-x)}$ or $\frac{2x}{2}$ or x.

Another solution is $1 - x$, since $1 - (1 - x)$ is x;
a third is $-x$, since $-(-x)$ is x.

Now suppose $\varphi(x+y) = \varphi x + \varphi y$
x^2 does <u>not</u> satisfy this; $\overline{x+y\,}|^2$ is not x^2+y^2
ax does $\quad a(x+y)$ is $ax+ay$

Let $\varphi x = x^a \qquad \varphi(xy) = (xy)^a = x^a y^a = \varphi x + \varphi y$
$\varphi x = a^x \qquad \varphi x \times \varphi y = a^x \times a^y = a^{x+y} = \varphi(x+y)$
[15r] $\varphi x = ax + b \qquad \frac{\varphi x - \varphi y}{\varphi x - \varphi z} = \frac{ax+b-(ay+b)}{ax+b-(az+b)} = \frac{ax-ay}{ax-az} = \frac{x-y}{x-z}$
A functional equation is one which has for its [something crossed out]
<u>unknown</u> the form proper to satisfy a certain condition
Example. What function of x is that which is not
altered by changing x into $1-x$, let x be what it
may. Or, required φx so that

$$\varphi x = \varphi(1-x)$$

One solution is $\varphi x = 1 - 2x + 2x^2$
for $\varphi(1-x) = 1 - 2(1-x) + 2(1-x)^2$
$= 1 - 2 + 2x + 2 - 4x + 2x^2$
$= 1 - 2x + 2x^2$ as before.

The equation of a curve means that equation
which must necessarily be true of the coordinates of
every point in it, and obviously depends upon 1. The
point chosen from which to measure coordinates 2.
The direction chosen for the coordinates. 3. The nature
and position of the curve. For example let the curve
be a circle, the point chosen its center, and the axes
of coordinates two lines at right angles. Let the
[diagram in original] radius be a; then at every point x and
y must be the two sides of a right angled
triangle whose hypotenuse is a; or

$$x^2 + y^2 = a^2$$

which being an equation true at every point
[15v] of the circle, is called the equation of the
circle

My wife returns to day from Highgate.
M^r Frend continues very comfortable, and
neither mends nor grows worse. I hope Ld Lovelace
and the little people are well. The old Ockham
will be a poor example for the young one, though

he was a monk, as I suppose. I would have been nothing else had I lived in his day

Yours very truly

ADeMorgan

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Monday Sep^{t} 15/40