[134r]

Ashley-Combe
Monday. 8th Novr ['1841' added by later reader]

Dear Mr De Morgan. I hope you intend to christen the "large boy" by the name of Podge, with which I am particularly pleased.
I am much obliged by your letter. I send a corrected version (now I believe quite right) of $\frac{d^2u}{dx^2} - u = X$; on my assumed supposition $\frac{dK}{dx}\varepsilon^x + \frac{dK'}{dx}\varepsilon^{-x} = 0$. As for my other assumption $K + K' = x^3$, it is so complicated a one that I have not thought it worth while to pursue it's [*sic*] development.
I cannot think how I could be so negligent as to forget that ε^{-x} is a function of $(-x)$ which is itself a function of x. A complete oversight ; as indeed most of the enquiries in my last letter seem to have been.
I should perhaps mention that lately [something crossed out] I have had my mind a good deal distracted by some circumstances of considerable annoyance & anxiety to me ; & I have certainly studied much less well & more negligently in consequence. Indeed the last few weeks I have not at all got on as I wished and intended ; & I find that to force myself, (when [134v] disinclined & distraite), beyond a certain point is very disadvantageous. So on these occasions I just keep gently going, without however attempting very much. I am hoping now to get a good lift again before long ; as I think I am returning to a more settled & concentrated state of mind. I mention all this as an excuse for some errors & over-sights which I conceive are more likely just at present to creep into my performances than would usually be the case. Now to business : Chapter VIII :
1. I send you two Problems on hypotheses of my own, intended as being worked out on the model of those in page 150. There are three different Hypotheses. In the one where I obtain $t = \frac{1}{\sqrt{2}}\int\frac{ds}{\sqrt{s^{-1}-a^{-1}}}$ I have not attempted to develop this Integral further. Perhaps I ought to have done so ; but it was only my object to get quite a general expression.
2. Page 141 ; (lines 9, 10, from the bottom) : Series in page 116 (of Chapter VI), it was shown that $\int\frac{ds}{\sqrt{2kx-x^2}} = \sin^{-1}\frac{x-k}{x} =$

$= \left(v \sin^{-1} \frac{x}{k}\right) + \left(\frac{w}{2}\right)$, I do not see how it can be said (page 141) that the Constant may have <u>any value</u> P.
3. I have never succeeded in <u>properly understanding</u> the Paragraph beginning page 134, ending page 135, on which I before applied to you ; & the paragraph of page 148 – (marked 2) – has only added to my mistiness on [135r] the subject. There is something or other which I cannot <u>get at</u> in the argument & it's [*sic*] objects. That of page 135 seems very like another way of arriving at Taylor's Theorem. The expression taken in line 25 from the top, I conclude to be arrived at as follows :
Having obtained $\varphi a + \varphi' a.(x - a)$; a function agreeing in value and diff-co with φx when $x = a$, let us now find a function agreeing not only in <u>these two points</u> but also in <u>second</u> diff-co with φx, when $x = a$; (the same <u>conditions being continued</u> of $\varphi' a$, $\varphi'' a$) ;
We see therefore that $\varphi' a$ must be of the form

$$\varphi'' a.x + m \quad \text{where} \quad m = \varphi' a - \varphi'' a.a$$

Substituting this in $\varphi a + \varphi' a.(x - a)$ we have

$$\begin{aligned}\varphi a + (\varphi'' a.x + m)(x - a) &= \varphi a + (\varphi'' a.\overline{x - a} + \varphi' a)(x - a) \\ &= \varphi a + \varphi' a.(x - a) + \varphi'' a(x - a)^2\end{aligned}$$

Similarly we may obtain $\varphi a + \varphi' a.(x - a) + \varphi'' a.(x - a)^2 + \varphi''' a.(x - a)^3$ (By the bye I don't see how you get $\frac{(x-a)^2}{2}$ and $\frac{(x-a)^3}{2.3}$, instead of $(x - a)^2$ and $(x - a)^3$ as I make it).
But I cannot perceive w<u>hat</u> all this is <u>for</u> ; & (as I mentioned below), paragraph 2 of page 148 has <u>added</u> to my blindness. _ I am sorry to plague you again about it. On receiving your former reply, I felt none the wiser ; but determined to <u>wait</u>, thinking I might see it as I went on, which is often the case [135v] with difficulties.
I now proceed to some miscellaneous matters.
1. I make nothing of the I<u>rreducible Case</u> in the Penny Cyclopedia. Is it perhaps for want of having read <u>Involution & Evolution</u>? I am puzzled <u>quite in the beginning</u> <u>of the Article</u>.
2. Article "Negative & Impossible Quantities" P. Cyclopedia – page 137
"If the logarithm of two Units inclined at angles θ and θ' be added, (the bases being inclined at angles φ and φ') ; the result is the logarithm of a Unit inclined &c, &c"
I cannot <u>develop this</u> ; but I enclose some remarks upon it.
3. In the treatise you sent me on the "Foundation of Algebra",

I cannot make out ['in' inserted] the least [something crossed out] (page 5), about the general
solution of $\varphi^2 x = ax$. I suspect I do not understand
the notation $f^{-1}x$. I quite understand $f^2 x$ or $\varphi^2 x$,
$f^n x$ or $\varphi^n x$. Judging by analogy, from page 82 of the
Differential & Integral Calculus, (where $\Delta^{-1}x$ is explained),
I conceive $f^{-1}x$ or $\varphi^{-1}x$ may mean "the quantity
"which having had an operation f or φ performed
"with & upon it, is $= x$." But I have considered
much over the last half of this page 5, & I can't
understand it. __
I have one or two other matters still to write about ;
but they do not press ; & this is plenty I think for
today. __ Pray congratulate Mrs De Morgan on the
arrival & prosperity of Podge. __

Yours most truly

A. A. L.