[132r]

Ashley-Combe
Porlock
Somerset
Thurs^{dy} 4th Nov^{r} ['1841' inserted by later reader]

Dear M^{r} De Morgan. As I find my journey to Town is extremely uncertain, & may possibly even not take place at all, I will trouble you without further delay on the more important of my present points of difficulty. _

I will begin with those relating to Chapter 9th of the Calculus, which I am now studying. I have arrived at page 156.

page 132 : (at the bottom). I make $u = \cos^{-1}\left(\frac{1-\varepsilon^{2(C-x)}}{\varepsilon^{2(C-x)}+1}\right)$

instead of $u = \cos^{-1}\left(\frac{\varepsilon^{2(C-x)}-1}{\varepsilon^{2(C-x)}+1}\right)$

I enclose a paper with my version of it. _

page 153 : "For instance, we should not recommend "the student to write the preceding thus, $d^2.du + d^2x.du = 0$, "tho' is it certainly true that upon the implicit "suppositions with regard to the successive Increments, "$\Delta^2u.\Delta x + \Delta^2x.\Delta u$ diminishes without limit as compared "with $(\Delta x)^3$." Why this comparison with $(\Delta x)^3$?

[132v] Had the expression been $\frac{\Delta^2u.\Delta x+\Delta^2x.\Delta u}{(\Delta x)^3}$ instead of $\Delta^2u.\Delta x + \Delta^2x.\Delta u$, it would then be clear that if the Numerator diminished without limit with respect to the Denominator, the fraction itself would approach without limit to 0. But as it is, I see no purpose answered by a comparison with $(\Delta x)^3$.

Also, I not only do not see the object of this comparison, but I do not perceive the fact itself either.

Where is the proof that $\Delta^2u.\Delta x + \Delta^2x.\Delta u$ does diminish without limit with respect to $(\Delta x)^3$? ____

Page 135 : (at the top) : There is a slight misprint $C = K^2 + K^{12}$ instead of $C = K^2 + K'^2$

Page 156 : (line 9 from the top) : $u = C.\sin\theta + C'.\cos\theta + \frac{1}{2}\theta.\sin\theta$
(Explain this step?)

Now I cannot "explain this step".

In the previous line, we have :

(1)... $u = C\sin\theta + C'\cos\theta + \frac{1}{2}\theta.\sin\theta + \frac{1}{4}\cos\theta$ (quite clear)

(2)... And $u = \cos\theta - \frac{d^2u}{d\theta^2}$ (by hypothesis)

$$= \frac{1}{4}\cos\theta + \left(\frac{3}{4}\cos\theta - \frac{d^2u}{d\theta^2}\right)$$

whence one may conclude that

$$C.\sin\theta + C'\cos\theta + \tfrac{1}{2}\theta.\sin\theta = \tfrac{3}{4}\cos\theta - \frac{d^u}{d\theta^2}$$

But how $u = C\sin\theta + C'\cos\theta + \sin\theta.\frac{1}{2}\theta$ is to be deduced [133r] I do not discover : By subtracting $\frac{1}{4}\cos\theta$ from both sides of (1), we get

$$u - \tfrac{1}{4}\cos\theta = C\sin\theta + C'.\cos\theta + \tfrac{1}{2}\theta.\sin\theta$$

But unless $\frac{1}{4}\cos\theta = 0$, (which would only be the case I conceive if $\theta = \frac{w}{2}$), I do not see how to derive the equation in line 9 of the book.

Page 156 : Show that $\frac{d^2u}{dx^2} - u = X$ (a function of x)

gives $u = C\varepsilon^x + C'\varepsilon^{-x} + \frac{1}{2}\varepsilon^x\int\varepsilon^{-x}X.du - \frac{1}{2}\varepsilon^{-x}\int\varepsilon^x X.dx$

I have, $\frac{d^2u}{dx^2} - u = X \qquad u = K\varepsilon^x + K'\varepsilon^{-x}$

$\frac{du}{dx} = K\varepsilon^x + K'\varepsilon^{-x} + \frac{dK}{dx}\varepsilon^x + \frac{dK'}{dx}\varepsilon^{-x}$

Assume $\frac{dK}{dx}\varepsilon^x + \frac{dK'}{dx}\varepsilon^{-x} = 0$

Then $\frac{du}{dx} = K\varepsilon^x + K'\varepsilon^{-x}$, and $\frac{d^2u}{du^2} = K\varepsilon^x + K'\varepsilon^{-x} + {}$
$+\frac{dK}{dx}\varepsilon^x + \frac{dK'}{dx}\varepsilon^{-x}$

$$\left.\begin{aligned} X &= \tfrac{dK}{dx}\varepsilon^x + \tfrac{dK'}{dx}\varepsilon^{-x} \\ 0 &= \tfrac{dK}{dx}\varepsilon^x + \tfrac{dK'}{dx}\varepsilon^{-x}\end{aligned}\right\}$$ which tell nothing at all as to the values of $\frac{dK}{dx}$, $\frac{dK'}{dx}$ of K, & of K'

If we had $$\left.\begin{aligned} X &= \tfrac{dK}{dx}\varepsilon^x - \tfrac{dK'}{dx}\varepsilon^{-x} \\ 0 &= \tfrac{dK}{dx}\varepsilon^x + \tfrac{dK'}{dx}\varepsilon^{-x}\end{aligned}\right\}$$ the expression in the book will be then at once deduced.

[133v] But I do not see how to get these two latter equations co-existent. __

I enclose an attempt of mine, making the assumed to be $\frac{dK}{dx}\varepsilon^x + \frac{dK'}{dx}\varepsilon^{-x} = x^2$ instead of $= 0$; and also ['one' inserted] making this relation to be $K + K' = x^3$, but which latter I found led to such very complicated results that I proceeded but a little way, thinking it a probable loss of time to go on.

With the relationship $\frac{dK}{dx}\varepsilon^x + \frac{dK'}{dx}\varepsilon^{-x} = x^2$, I am as unsuccessful as with $= 0$. _

I defer to another letter some other difficulties of mine not relating to this Chapter, but partly to some remaining points in the 8th Chapter, & partly to miscellaneous matters. __

I hope M^{rs} De Morgan & the "large boy" continue to flourish. So M^{rs} De M has beat the Queen in the race, out & out! ____

Yours most truly

A. A. L.