[127r]

Ashley-Combe
Sunday. 19th Sepr ['1841' added by later reader]

Dear Mr De Morgan. I have more to say to you than ever ; ( beginning with many thanks for your bountiful replies to my last packet). __
I will begin with the Article on Negative & Impossible Quantities, on which I have a good deal to remark. I have finished it ; & I think with on the whole great success. I need scarcely say that I like it parenthetically. __ I enclose you the demonstration of the formula $(a + bk)^{m+nk} = \varepsilon^A. \cos B + k.\varepsilon^A. \sin B$, which I found exceedingly easy, after your observations. ____

I should tell you that the allusions to the Irreducible Case of Cubic Equations in this Article, has so excited my curiosity on the subject, that I have attacked the chapter on Cubic Equations in ['page 47' inserted] of R. Murphy's treatise on the Theory of Equations (Library Useful Knowledge), hoping there to gain some light on the subject. For I know not to what exactly this alluded, (my Algebra wits, as you say, not having been quite proportionally stretched with some of my other wits). I have got thro' the first two pages ; and [127v] shall have to write you some remarks upon these, either in this letter, or in one as soon as possible. But as yet I meet with nothing about

$$\sqrt[3]{(a + b\sqrt{-1})} + \sqrt[3]{(a - b\sqrt{-1})}$$

I hope I shall be able to understand the rest of the Chapter. __
At the bottom of my demonstration of $(a + bk)^{m+nk}$, you will find a memorandum (simple as to the working out) of the formula $\cos.(a + bk)$, see page 137 of the Cyclopedia. You there say that such a formula may be interpreted by it's [*sic*] identical expression on the Second Side. That is to say I imagine that the meaning of $\cos(a + b\sqrt{-1})$, which (as before pointed out in the case of the line $h$) is a misapplication of symbols, may be got at thro' an examination of the results arrived at by ['the application of' inserted] symbolical rules to this unmeaning or mis-meaning expression. That if in a calculation, such an answer as $\cos(a + b\sqrt{-1})$ were worked out, the answer means in fact

[something crossed out]
the remaining side of a parallelogram in which $\cos\alpha\frac{\varepsilon^b+\varepsilon^{-b}}{2}$ is a diagonal, and $\sin\alpha.\frac{\varepsilon^b-\varepsilon^{-b}}{2}.k$ the other side : the diagonal itself being a $4^{\text{th}}$ Proportional to 1, $\cos\alpha$, $\frac{\varepsilon^b+\varepsilon^{-b}}{2}$, inclined to 1 ['(that is to the Unit-Line)' inserted] as the $\cos\alpha$ is ; & the remaining side being a $4^{\text{th}}$ Proportional to 1, $\sin\alpha$, $\frac{\varepsilon^b-\varepsilon^{-b}}{2}$ inclined to 1 at an angle equal to the sum of a [128r] Right-Angle and the angle made by $\sin.\alpha$ with the Unit-Line.

I enclose you an explanation I have written out (according to the Definition of this Geometrical Algebra), of the two formulae for the Sine and Cosine. I am at work now on the Trigonometrical Chapters of the Differential Calculus.

I do not agree to what is said in page 119 ['(of the Calculus)' inserted] that results would be the same whether we worked [something crossed out] algebraically with forms expressive of quantities or not. It is true that ['in' inserted] the form $a+\sqrt{m}-\sqrt{n}$, if $(-1$ be substituted for $m$ and $n$, the results come out the same as if we work with $a$ only. but were the form $a+\sqrt{m}$, $a-\sqrt{m}$, $a\times\sqrt{m}$, or fifty others one can thin of, surely the substitution of $(-1)$ for $m$ will not bring out results the same as if we worked with $a$ only; and in fact can only do so when the impossible expression is so introduced as to neutralize itself, if I may so speak. I think I have explained myself clearly. __

It cannot help striking me that this extension of Algebra ought to lead to a further extension similar in nature, to Geometry in Three-Dimensions ; & that again perhaps to a further extension into some unknown region, & so on ad-infinitum possibly. And that it is especially the consideration of an angle $=\sqrt{-1}$, which should lead to this ; a symbol, which when it appears, sees to me in no way more [128v] satisfactorily accounted for & explained than was formerly the appearances ['which' inserted] Bombelli in some degree cleared up by showing that at any rate they (tho' in themselves unintelligible) led to intelligible & true results. You do hint in parts of page 136 at the possibility of something of this sort. __

I enclose you also a paper I have written explaining a difficulty of mine in the Definitions of this Geometrical Algebra. __

It appears to me that there is no getting on at all without this Algebra. In the 3d Chapter of your Trigonometry (which I have just been going thro'), though there are no impossible quantities introduced ; yet how unintelligible are such formulae as $2ac. \cos B$, $a. \sin B$, or any in short where lines are multiplied into lines, if one only takes the common notion of a line into a line being a Rectangle. ____

I cannot send more today ; but I have many other matters to write on ; especially the Logarithmic Theory at the end of the Article. I am considering it very carefully ; & studying at the same time the Article on Logarithms in the Cyclopedia. And I believe I shall have much to say on it all.
The passage I wanted to ask you about in Lamé's [129r] 1st Vol, is pages 54, 55, 56, on the Resultant of the pressures of a liquid on a vase. I want to know if I ought to understand these three pages, or if they entail some knowledge of mathematical (especially of trigonometrical) application to Mechanics, which I do not yet possess. __

I hope you receive game regularly.

Yours most truly
A. A. Lovelace

P.S. Did you ever hear of a Science called Descriptive Geometry? I think Monge is the originator of it. ____