[123r]

Ashley Combe
Thurs^{dy} Mor^{g}
9th Sep^{r} ['1841' added by later reader]

Dear M^{r} De Morgan. I have rather a large batch now for you altogether :
1^{stly}: I am in the middle of the article on Negative & Impossible Quantities ; & I have a question to put on page 134, (Second Column, lines 1, 2, 3, 4, 5 from the bottom)

$(a+bk)^{m+nk} = \varepsilon^A \cos B + k\,\varepsilon^A . \sin B$ &c

I have tried a little to demonstrate this Formula ; but before I proceed further in spending more time upon it, I think I may as well ask if it is <u>intended</u> to be <u>demonstrable by the Student</u>. For you know I sometimes try to do more than anyone means me to attempt. I have as yet only got thus far [something crossed] with the above formula : If in $(a+bk)^{m+nk}$,
r is given $= \sqrt{a^2+b^2}$, ['and' inserted] $\tan\theta = \frac{b}{a}$; then $\sin\theta = b$
$\cos\theta = a$
and $(a+bk)^{m+nk} = (\cos\theta + k.\sin\theta)^{m+nk} =$

$$= \left(\varepsilon^{k\theta}\right)^{m+nk} = \varepsilon^{k(m\theta)} \times \left\{\varepsilon^{k(n\theta)}\right\}^k$$

or $= (\cos .m\theta + k.\sin .m\theta) \times (\cos .n\theta + k.\sin .m\theta)^k$, and
[123v] I dare say that from some of these transformations, the <u>Second Side</u> of the given equation, with the determination of A and B, may be deduced. But it appears to me ['it must be' inserted] a very <u>complicated process</u> ; & therefore I should like to know before I undertook it, that I was not <u>wasting time</u> ['in' deleted] doing so. __
2^{dly}: I am plagued over page 135 of the Calculus. It is not that there is <u>any one thing</u> in it which I do not clearly see. But it is the <u>depth of the whole argument</u> which I cannot manage to discover. I should say that whole argument from "We now know &c" page 134, to "We can therefore take a function, "which, for a particular value of x, &c, &c" page 135.

It seems to me all to be <u>much ado about nothing</u>; and I do not see <u>what is arrived at</u> by means of it [something crossed out]. A very complicated process appears to be used in the 1st Paragraph of page 135, to prove that when <u>h is small</u> then the Increment in φx is <u>very nearly</u> represented by $\varphi' a + h$, which was already shown in page 134. And then suddenly in the Second Paragraph the Formula $\varphi a + \varphi' a(x-a)+$ $+\varphi'' a \frac{(x-a)^2}{2}$ is introduced, & I do not understand à quoi bon the closing conclusion drawn from it.

3dly: I am not sure that I agree to what you say in preference (for ascertaining Maxima & Minima) of the direct ascertainment of the value of $\varphi' x$, over [124r] the ordinary method. Because it seems to me in many cases impossible after you have determined 0 or α values of $\varphi' x$, to determine further that the sign does change at them & how it changes, unless by means of the ordinary rule. I have written out and enclose an example from Peacock, in which unless I had used the ordinary rule, after I had determined 0 values for $\varphi' x$, I should have been at my wits' end how to bring out the conclusion.

4thly: I send you a little Maxima & Minima Theorem of my own, which occurred to me by accident ; It is for $\varphi x = x^2 - mx$. After proving it by the Differential Calculus, I have given a direct proof of another sort. I merely wrote this ['direct proof' inserted], because it [something crossed out] occurred to me ; but it gave me a great deal of trouble, & I think was rather a work of supererogation; but I believe it is quite correct. You will find enclosed in the same sheet the demonstration of "What is the number whose excess above it's [*sic*] Square "Root is the least possible?" (see page 133 of the Calculus) ; and on the reverse side of this latter [something crossed out] is the "verification round the 4 Right Angles" for the continual increase together of x and it's [*sic*] tangent (See page 132). But here I have something further to add. In this Chapter VIII, we hear of Differential Co-efficient which become $= 0$, or $= \alpha$. In this very [124v] instance, $1 + \tan^2 x$ is alternately $= 1$, and $= \alpha$. __ Now according to my previous ideas, the terms Differential Co-efficient was only applied to some finite quantity ; and referring to pages 47, 48, where one acquired one's first ideas of a Differential Co-efficient, I think it is there clearly explained that the term is only used with reference to a finite limit. But in this Chapter VIII, there seems to be a considerable extension of meaning on

the subject. __
5[thly]. In page 132, it is very clearly deduced that the Ratio of a [something crossed out] Logarithm to it's [*sic*] number is increasing as long as x is $< \varepsilon$, and afterwards decreases. __

The proof is most obvious. But, unluckily, the conclusion seems to me to be contrary to the fact ; at least the first half of the conclusion, not the latter half.

On this principle : from the very nature of a Logarithm, it is obvious that (x being $> \varepsilon$), for equal increments to the log, x, there will be larger & larger Increments to x. The one being in arithmetical, the other in geometrical progression. Therefore clearly the Ratio of the Logarithm to the number, is a diminishing one. But then the same thing seems to me to apply [something crossed out] when $x < \varepsilon$. Surely there is then just the same [125r] arithmetical & geometrical progression for equal Increments of the Logarithms. I suppose there is some link that I have over-looked.

I send you two Integrations worked out. They are from Peacock. I in vain spent hours over the one marked 2, of which I could make nothing by any method that I devised ; until in despair, I looked thro' your Chapter XIII to see if I could there find any hints ; & accordingly at page 277, I found a general formula which included this case. But I do not believe I should ever have hit upon it by myself. The Integral marked 1, might of course be proved also in the same way ; tho' ['my' crossed out] the method ['I have used' inserted] is sufficient in this instance. __

I have written out no more papers on Forces. In fact there is only one more that is left for me, viz: $f = v\frac{dv}{ds}$. And for this I see no occasion ; for I am sure that I must thoro'ly understand it, after all I have written. ____
I quite see ['the truth' inserted] your remarks on my having treated Acceleration of Velocity as being identical with Force ; whereas, (as I now understand it), it is simply the measure of Force, & our only way of getting at expressions for this latter. On the subject [125v] of $v^2 = 2\int f ds + C$; I have considered it a great deal ; and any direct demonstration of it, after the

manner of my other papers, seems to me to be
quite impracticable. Neither $\int v.dv$, nor $\int f.ds$
[something crossed out] now appear to me to have any actual proto-types in the
real motion. Here then suggests itself to me the
question : "then are there certain truths & conclusions
"which can be arrived at by pure analysis, & in
"no other way?" And also, how far abstract
analytical expressions must express & mean
something real, or not. In short, it has
suggested to me a good deal of enquiry, which
I am desirous of being put in the way of
satisfying. __
By the bye, I fear that one little paper of mine
dropped out of the last packet. It was a little
pencil memorandum on ['the meaning of' inserted] $\int f.ds$; & there were
remarks upon it, (if you remember) in my accompanying
letter. It bears upon the above question.
I could write it out again, if it has been lost.__

Is not this a budget indeed? __

Yours most truly

A. A. Lovelace