Dear Mr. De Morgan. I have rather a large batch now for you altogether:

1stly: I am in the middle of the article on Negative & Impossible Quantities; & I have a question to put on page 134, (Second Column, lines 1, 2, 3, 4, 5 from the bottom)

$$(a + bk)^{m+nk} = \varepsilon^A \cos B + k \varepsilon^A \sin B \& c$$

I have tried a little to demonstrate this Formula; but before I proceed further in spending more time upon it, I think I may as well ask if it is intended to be demonstrable by the Student. For you know I sometimes try to do more than anyone means me to attempt. I have as yet only got thus far [something crossed] with the above formula: If in $(a + bk)^{m+nk}$, $r$ is given $= \sqrt{a^2 + b^2}$, ['and' inserted] $\tan \theta = \frac{b}{a}$; then $\sin \theta = b \cos \theta = a$

and $(a + bk)^{m+nk} = (\cos \theta + k. \sin \theta)^{m+nk} = (\varepsilon^{k\theta})^{m+nk} = \varepsilon^{k(m\theta)} \times \{\varepsilon^{k(n\theta)}\}^k$

or $= (\cos . m\theta + k. \sin . m\theta) \times (\cos . n\theta + k. \sin . m\theta)^k$, and

I dare say that from some of these transformations, the Second Side of the given equation, with the determination of $A$ and $B$, may be deduced. But it appears to me ['it must be' inserted] a very complicated process; & therefore I should like to know before I undertook it, that I was not wasting time ['in' deleted] doing so.

2ndly: I am plagued over page 135 of the Calculus. It is not that there is any one thing in it which I do not clearly see. But it is the depth of the whole argument which I cannot manage to discover. I should say that whole argument from “We now know &c” page 134, to “We can therefore take a function,” “which, for a particular value of $x$, &c, &c” page 135.

It seems to me all to be much ado about nothing; and I do not see what is arrived at by means of it [something crossed out]. A very complicated process appears to be used in the 1st Paragraph of page 135, to prove that when $h$ is small then the Increment in $\phi x$ is very nearly represented by $\phi' a + h$, which was
already shown in page 134. And then suddenly in
the Second Paragraph the Formula \( \varphi a + \varphi'(x - a) + \\
+ \frac{\varphi''a}{2}(x - a)^2 \) is introduced, & I do not understand
\( \text{à quoi bon} \) the closing conclusion drawn from it.

3\(^{\text{th}}\): I am not sure that I agree to what you
say in preference (for ascertaining Maxima & Minima)
of the direct ascertainment of the value of \( \varphi'x \), over
[124r] the ordinary method. Because it seems to me in
many cases impossible after you have determined 0 or
\( \alpha \) values of \( \varphi'x \), to determine further that the sign
does change at them & how it changes, unless by means
of the ordinary rule. I have written out and
enclose an example from Peacock, in which unless
I had used the ordinary rule, after I had
determined 0 values for \( \varphi'x \), I should have been
at my wits' end how to bring out the conclusion.

4\(^{\text{th}}\): I send you a little Maxima & Minima
Theorem of my own, which occurred to me by accident ;
It is for \( \varphi x = x^2 - mx \). After proving it by the
Differential Calculus, I have given a direct proof
of another sort. I merely wrote this ['direct proof' inserted], because it
[something crossed out] occurred to me ; but it gave me a great deal of
trouble, & I think was rather a work of supererogation;
but I believe it is quite correct. You will find
enclosed in the same sheet the demonstration of
“What is the number whose excess above its [sic] Square
“Root is the least possible?” (see page 133 of the Calculus) ;
and on the reverse side of this latter [something crossed out]
is the “verification round the 4 Right Angles” for the
continual increase together of \( x \) and its [sic] tangent (See
page 132). But here I have something further to
add. In this Chapter VIII, we hear of Differential
Co-efficient which become = 0, or = \( \alpha \). In this very
[124v] instance, \( 1 + \tan^2 x \) is alternately = 1, and = \( \alpha \)._ __
Now according to my previous ideas, the terms
Differential Co-efficient was only applied to some
finite quantity ; and referring to pages 47, 48,
where one acquired one's first ideas of a Differential
Co-efficient, I think it is there clearly explained
that the term is only used with reference to a
finite limit. But in this Chapter VIII, there
seems to be a considerable extension of meaning on
5thly. In page 132, it is very clearly deduced that the Ratio of a [something crossed out] Logarithm to it’s [sic] number is increasing as long as \( x < \varepsilon \), and afterwards decreases.

The proof is most obvious. But, unluckily, the conclusion seems to me to be contrary to the fact; at least the first half of the conclusion, not the latter half.

On this principle: from the very nature of a Logarithm, it is obvious that \( x > \varepsilon \), for equal increments to the log, \( x \), there will be [something crossed out] larger & larger Increments to \( x \). The one being in arithmetical, the other in geometrical progression. Therefore clearly the Ratio of the Logarithm to the number, is a diminishing one. But then the same thing seems to me to apply [something crossed out] when \( x < \varepsilon \). Surely there is then just the same [125r] arithmetical & geometrical progression for equal Increments of the Logarithms. I suppose there is some link that I have over-looked.

I send you two Integrations worked out. They are from Peacock. I in vain spent hours over the one marked 2, of which I could make nothing by any method that I devised; until in despair, I looked thro’ your Chapter XIII to see if I could there find any hints; & accordingly at page 277, I found a general formula which included this case. But I do not believe I should ever have hit upon it by myself. The Integral marked 1, might of course be proved also in the same way; tho’ [‘my’ crossed out] the method [‘I have used’ inserted] is sufficient in this instance.

I have written out no more papers on Forces. In fact there is only one more that is left for me, viz: \( f = \frac{dv}{ds} \). And for this I see no occasion; for I am sure that I must thor’ly understand it, after all I have written. I quite see [‘the truth’ inserted] your remarks on my having treated Acceleration of Velocity as being identical with Force; whereas, (as I now understand it), it is simply the measure of Force, & our only way of getting at expressions for this latter. On the subject [125v] of \( v^2 = 2 \int f ds + C \); I have considered it a great deal; and any direct demonstration of it, after the
manner of my other papers, seems to me to be quite impracticable. Neither $\int v \, dv$, nor $\int f \, ds$
[Something crossed out] now appear to me to have any actual proto-types in the real motion. Here then suggests itself to me the question: "then are there certain truths & conclusions "which can be arrived at by pure analysis, & in "no other way?" And also, how far abstract analytical expressions must express & mean something real, or not. In short, it has suggested to me a good deal of enquiry, which I am desirous of being put in the way of satisfying.

By the bye, I fear that one little paper of mine dropped out of the last packet. It was a little pencil memorandum on ['the meaning of' inserted] $\int f \, ds$; & there were remarks upon it, (if you remember) in my accompanying letter. It bears upon the above question.

I could write it out again, if it has been lost.

Is not this a budget indeed? __

Yours most truly

A. A. Lovelace