[121r]

Ockham.

Friday. [something crossed out] 21st Augst

Dear M^{r} De Morgan. You have received safely I hope my packet of yesterday, & my packet sent on Tuesday. _

I now re-enclose you the paper marked 1. There is another Integral added at the bottom. Also I have altered one or two little minutiae in the development of $\int \frac{dx}{\sqrt{2ax+x^2}}$ above, which you had omitted to correct. _

I quite understand your observations upon it, & see the mistake I had made ; & which related to the Differential dy, and $d(\varphi x.x)$

If $y = \varphi x.x$, then $dy = d(\varphi x.x) = \frac{d(\varphi x.x)}{dx} dx =$

$$= \left\{ x.\frac{d(\varphi x)}{dx} + \varphi x.\frac{dx}{dx} \right\} dx = x.\left(\frac{d(\varphi x)}{dx} dx\right) + \varphi x.dx =$$

$$= x.d\varphi x + \varphi x.dx$$

Or if $y^2 = \varphi x.x$, then $dy^2 = x.d.\varphi x + \varphi x.dx$

$$\text{or } \frac{d(y^2)}{dy} dy = x.d.\varphi x + \varphi x.dx$$

$$\text{or } 2ydy = x.d.\varphi x + \varphi x.dx, \text{ and } ydy = \tfrac{1}{2} x.d.\varphi x + \tfrac{1}{2}\varphi x.dx$$

[121v] This is all now right in my head.

In $\int \frac{dx}{\sqrt{2ax+x^2}}$ we arrive then in my corrected paper, at $\int \frac{dx}{\sqrt{2ax+x^2}} = \log(x + a + \sqrt{2ax+x^2})$

$$= \log\left(\frac{x}{2} + \frac{a}{2} + \frac{\sqrt{2ax+x^2}}{2}\right) + \log 2$$

which, as you observe "again with the book all but "the log .2, which being a Constant, matters nothing". Very true ; but why did you then insist the $\log 2$ in page 116?_ it seems as if put in on purpose to be effaced in the parenthesis (Omit the Constant). And it might just as well have been $\log 3$, $\log 4$, log (anything in the world). ____

As to my two papers marked 2 (& which I again return, merely for the convenience of reference), I see that in order to make them valid, as applying each to two separate & different velocities, they should be re-written (which is not worth while), & the terms of the enunications altered as follows :

"If two quantities V, V' be respectively equal to the "Ratios $\frac{S}{T}$, $\frac{S'}{T'}$, and if $V : V' = T : T'$, then the values "S, S' must be to each other as the squares of T, T'

"are to one another" &c, &c
[122r] At last I believe I have it quite correctly. __
As for $\frac{dy}{dx} = \frac{y}{x}$, I see my fallacy about $\frac{y}{x}$ being
a fixed quantity. _
About page 113, "The first form becomes impossible
"when x is greater than $\sqrt{c}$, for &c", I fancy I
[something crossed out] had a little misunderstood
the mathematical meaning of the words impossible
quantity. I have loosely interpreted it as being
equivalent to "an absurdity", or at least to
"an absurdity, unless an extension be made in the
"ordinary meaning of words". _ And in this
instance I perceived that if the Logarithm be
an odd number, there would be no absurdity
even without extension in the meaning of terms ;__
because that it would then merely imply a
negative Base ; which negative Base, would I think
be admitted theoretically (tho' inconveniently practically)
on the common beginner's instruction on the Theory of
Logarithms. __ Am I right? __
By the bye this subject reminds me that I think
I find a mistake in page 117, line 13 (from the top)
"(n an integer) $\int_{-a}^{+a} x^n dx = 0$ when n is odd, $= \frac{2a^{n+1}}{n+1}$ when n is even"
[122v] It seems to me just the reverse, thus :
$= 0$ when n is even, $= \frac{2a^{n+1}}{n+1}$ when n is odd
I have it as follows :

$$\int_{-a}^{+a} x^n dx = \frac{a^{n+1}}{n+1} - \frac{(-a)^{n+1}}{n+1} = \frac{a^{n+1}-(-a)^{n+1}}{n+1} =$$

$$= \frac{a^{n+1}-a^{n+1}}{n+1} \text{ or } 0 \text{ if } n+1 \text{ be even}$$

(I now see it while working ; for if
$n+1$ be even, n must be odd.)
and vice versa.

So I need not trouble you upon this ; as I have
solved my difficulty whilst stating it. I had only
looked at this Integral in ['a' inserted] great hurry, this morning.

I hope on Sunday to send you two
remaining papers I have to make out, on the
Accelerating Force subject ;
upon $f = \frac{dv}{dt}$, and $v = \int f dt$
I think I have been encouraged by your great
kindness, so as to give you really no Sinecure
just at this moment.

Yours most truly
A. A. Lovelace