[115r]

Ockham Park

Sunday. 15th Augst

['1841' added by later reader]

Dear Mr De Morgan. You must be beginning to think me lost. I have been however hard at work, with the exception of 10 days complete interruption from company. __ I have now many thing to enquire. __ First of all; can I spend an evening with Mrs De Morgan & yourself on Tuesday the 24th? On that day we go to Town to remain till Friday, when we move down to Ashley for 2 months at least. __ I would endeavour to be early in Gower St; before eight or not later than eight. And I feel as if I should have many mathematical things to discuss. ____

Now to my business : ____

1stly: I send you a paper marked 1, containing my development of two Integrals in page 116,

$\int \frac{dx}{\sqrt{2ax-x^2}} = \sin^{-1}\left(\frac{x-a}{a}\right)$

And $\int \frac{dx}{\sqrt{2ax+x^2}} = \log(x+a+\sqrt{2ax+x^2}) + \log 2$

The former one I think is plain enough, & I and the book are quite agreed upon it. Not so with [115v] the latter one, & I begin to suspect the book. I cannot make it anything but

$$\int \frac{dx}{\sqrt{2ax+x^2}} = \log(x+a+\sqrt{2ax+x^2})$$

or else $$= \log(\tfrac{x}{2}+a+\tfrac{\sqrt{2ax+x^2}}{2}) + \log 2$$

I have tried various methods ; ____ but the only one which I find hold good [*sic*] at all, is that applied in page 115 to $\int \frac{dx}{\sqrt{a^2+x^2}}$, & which seems clearly to bring out my result above. By the bye I have a remark to make on the Integration of $\int \frac{dx}{\sqrt{a^2+x^2}}$ as developped [*sic*] in page 115. __

Line 10 (from the bottom), you have $xdx = ydy$:_ This is obvious, & similarly I deduce in my paper No 1, $(2a+x)dx = ydy$. _ But I see no use in what follows, "and $ydx + xdx = ydx + ydy$". _ It is equally obvious with the former equation, but seems to me to have no purpose in bringing out the results, which I deduce as follows :

Since $xdx = ydy$, we have $\frac{dx}{y} = \frac{dy}{x}$
Therefore by the Theorem of page 48, or at least ['by' inserted] a Corollary of it, we have $\frac{dx+dy}{x+y} = \frac{dx}{y}$, whence &c, &c.
And this is the method also which I have used [116r] in developping [*sic*] $\int \frac{dx}{\sqrt{2ax+x^2}}$. ——

2ndly: Page 113, lines 16 ['&c 17' inserted] from the bottom, you say "The "first form becomes impossible when x is greater than "$\sqrt{c}$, for in that case the Integral becomes the "Logarithm of a Negative Quantity". Now there are surely certain cases in which negative quantities may be powers, & therefore may have Logarithms.
All the odd whole numbers may surely be the Logarithms of Negative Quantities. __

$(-a) \times (-a) = a^2$ But $(-a) \times (-a) \times (-a) = -a^3$
or $(+a) \times (+a) = a^2$ $(+a) \times (+a) \times (+a) = a^3$

3 is here surely the Logarithm of a Negative Quantity.
Similarly a negative quantity multiplied into itself any odd number of times will give a negative result.

3dly: In the Paper marked 3, which I return again ['for reference' inserted] ; I perfectly understand the proof by means of the Logarithms (added by you), why $\frac{dy}{dx}$ can only $= \frac{y}{x}$ when y is either $= x$, or $= ax$ (a being Constant)
Your proof is perfect, but still I do not see that mine was not sufficient, tho' derived from much more general grounds. __

My argument was as follows : Given us $\frac{dy}{dx} = \frac{y}{x}$, what conditions must be fulfilled in order to make this equation possible? _ Firstly : I see that [116v] since $\frac{dy}{dx}$ means a Differential Co-efficient, which from it's [*sic*] nature (being a Limit) is a constant & fixed thing, $\frac{y}{x}$ must also be a constant & fixed quantity. That is y must have to x a constant Ratio which we may call a. __

This seems to me perfectly valid. And surely a Differential Co-efficient is as fixed & invariable in it's [*sic*] nature as anything under the sun can be.
To be sure you may say that there is a different Differential Co-efficient for every different initial value of x taken to start from, thus :

$$\frac{d(x^2)}{dx} = 2x \qquad \text{if } x = a,\ \frac{d(x^2)}{dx} = 2a$$

$$\text{if } x = b,\ \frac{d(x^2)}{dx} = 2b$$

And this is perhaps what invalidates my argument above. __

4^thly: In the two papers folded together & marked 2, which I also again return for reference, I perfectly see that tho' mathematically correct. I was completely wrong in my application. But my proofs do apply to any two different & independent velocities, whatever of two different bodies, or of the same body moving at two different uniform ratio [*sic*] at different epochs. __

Thus my paper (marked upon it 1st Paper) proves [117r] the following : that the Spaces moved over at two different times, in virtue of the Velocity acquired at the end of each of those times, (the impelling cause being supposed to cease at the end respectively of each time fixed on), would be to each other as the squares of the times fixed on. _ But I perfectly see that this is quite a different & independent consideration from that of the Space actually moved over by a body impelled by an accelerating force, & how wholly inapplicable my ['former' inserted] view of it was. __

I have been especially studying this subject of [something crossed out] Accelerating Force, & believe that I now understand it very completely. I found I could not rest upon it at all, until I made the whole of the subject out entirely to my satisfaction : _ I enclose you (marked 4) the first of a Series of papers I am making out in the different parts ['of' inserted] it. This one is the more general development of the particular case of Gravitation in pages 27, 28 ; & my more especial object in it has been the identification of the results arrived at in this real application, with the Mathematical Differential Co-efficient. __

I have worked most earnestly & incessantly at the Application of the Differential & Integral Calculus to the [117v] subject of Accelerating Force, & Accelerated Motion, during the last 2 or 3 weeks. It has interested me beyond everything. After making out (according to my own notions) the two papers on $v = \frac{ds}{dt}$, and $s = \int v dt$, (the first of which I now send, & the Second you will have in a day or two), I attacked your Chapter 8, pages 144, 145, worked out all the

Formulae there ; & had excessive trouble with my
t<u>hird</u> paper on $t = \int \frac{ds}{v}$, (now successfully terminated);
and I am now on $f = \frac{dv}{dt}$, page 146. ____
You will perhaps not approve my having thus <u>run a little riot</u>, & <u>anticipated</u>. But I think it has done me great good. And I am anxious to know if I may read the rest of this Chapter 8, <u>before</u> reading Chapter 7 on Trigonometrical Analysis ; & if I am likely to understand it all without having read Chapter 7.
I shall probably write again tomorrow ; or if not, <u>certainly</u> I shall on Tues^{dy}. _
We are very anxious to know if there is no time between the 1st Nov^{r} & the middle of Feb^{y}, when you & M^{rs} De Morgan (& family) would come & stay here for as long as you <u>can</u> & <u>would</u> like. We should be delighted if you would remain <u>2 or 3 week</u>s.
[118r] And if this should be impossible for y<u>ou</u>, perhaps still you would bring M^{rs} De Morgan & the children here, & remain a few days ; having t<u>hem</u> to stay longer.

We b<u>oth of us</u> assure you that it would be <u>no inconvenience</u> whatever to us ; but rather contrary the greatest pleasure. And I am certain it would do M^{rs} De Morgan good to be here for a time. __ Pray consider my proposal ; __ at any rate for her & the children, if y<u>our own</u> avocations should make it impossible for you even.
Believe me

Yours most truly

A. A. Lovelace