[112r]

Ockham Park
Sunday. 11th July ['1841' added by later reader]

Dear Mr De Morgan. I enclose you a paper (marked No 1) from which I think you will see that I now quite understand the real relationship between $\int \frac{x^n dx}{\sqrt{a^2-x^2}}$ and $\int \sqrt{a^2 - x^2} x^{n-2} dx$; & that I [something crossed out] am now aware I wanted to apply to the latter what is not intended to be directly applied to it at all ; & that ['my' inserted] getting both du and dx in, was a complete puzzle & blunder. For where a few lines previously $(-\overline{n-1}) \int \sqrt{a^2 - x^2} x^{n-2} dx$ is substituted for $\int \sqrt{a^2 - x^2} \times (-\overline{n-1}) x^{n-2} dx$, du ceases of course to enter under the Integrated quantity, [something crossed out] since it has been decomposed & otherwise distributed. __

I am still occupied on pages 108, 109, 110, & hope to complete to page 112 during this week. I find this part requires studying with great care. I think you anticipated this. __

I must now thank you very much for your two letters ; & will proceed to notice one or two points [112v] in your replies to my enquiries. I see that in objecting to what I called the division of $\frac{dV}{dx}$, when dV is substituted for $\frac{dV}{dx} dx$, I took a completely wrong view of the matter. It does so happen that the expression (derived from a separate & distinct Theorem) which we may substitute for $\frac{dV}{dx} dx$ coincides in form with what we may call the numerator dV of the diff_co. But the dV that is substituted is not therefore derived from $\frac{dV}{dx}$, at least ['not directly or' inserted] from the decomposition of that which is indecomposible [sic]. __

I return again my former paper (marked No 2.) with a clearer explanation of what I intended to convey by the term equivalent ; a term which it seems I had no business to use in the application which I ['there' inserted] meant to make of it. __

I enclose (marked No 3) my answer to your "Try "to prove the following. It is only when $y = ax$ "(a being constant) that $\frac{dy}{dx} = \frac{y}{x}$" I do not feel quite sure that my proof is a proof. But I think

it is too. __
Now about $v = gt$ and $s = \frac{1}{2}gt^2$; a subject
which troubles me not a little. __
Is the following a correct development of the note in
Useful Knowledge Mechanics? I re-copy the notes first ;
[113r] "$V = \frac{dS}{dT} = gT$. Hence $dS = gT.dT$, which being
"integrated gives $S = \frac{1}{2}g.T^2$"
[something crossed out on two lines]
The Integral of $\frac{dS}{dT}dT$ or of $gT.dT$ will
obviously give us S ; & we know that $\int gT.dT =$
$= \frac{1}{2}gT^2 + C$, (by formula of page 104 of the Calculus).
But it appears to me that the statement
above "Hence $dS = gT.dT$" is an unnecessary
intermediate step :
It is true that $\int \frac{dS}{dT}dT = \int dS$,
that is providing we extend the theorem

$$\int fx\frac{dx}{dt}dt = \int fx.dx$$

to the case when $fx = 1$, which I conclude it is
allowable to do, since 1 may be considered a
function of anything, I imagine ; thro' the formula
$\frac{fx}{fx} = 1$. But tho' true, yet the above ['clause' inserted] appears
to me ['an' inserted] unnecessary introduction. __
I am not sure that I have explained myself well.
With respect to this formula
$\frac{1}{2}gt^2$, & it's [sic] derivation & application ; I have
referred as you desired to pages 27, 28, & have
[113v] fully refreshed my memory upon them. But I
do not feel this helps me much. In this first
place the process is the converse of that I enquired
upon. S is there give, & V is to be derived
from S. My position was ; __ V given, & S to
be derived from V. _
I understand the process of pages 27, 28, considered
as a distinct & separate thing. But I do not
identify it with Differentiation or Integration. ____
I, (knowing by abstract rules & theorems) that
$2x$ is the diff_co of x^2, see that the limit $2t$
which comes out, might be perfectly well expressed
by $\frac{d(t^2)}{dt}$. And that we may put the result
of the Differentiation of t^2, and the result of all
the reasoning of pages 27, 28, indifferently one for
the other. But I only see it as I see that

in the processes $12 \div 4 = 3 \qquad 1 + 2 = 3$ we
might indifferently put the results (3, in both cases)
one for the other. There may, for anything I yet
see or understand, be as little connection between
the abstract process of Differentiation and the
Stone-falling process, as between the above processes
of Division & Addition, which latter tho' their results
agree, cannot be identified, or one made to represent
[114r] the other. __
I apprehend [something crossed out] you will perhaps answer me
here, that I must wait patiently for Chapter 8,
in which (page 143) I see something very like an
explanation of all I want. __ At the same time
I think it better to express fully my difficulties.
I am very anxious to see your Comments on
my two papers ['sent the other day' inserted] upon $\frac{1}{2}gt^2$. For I do not see
where the flaw in them can be ; & yet I suppose
there is one. It is some comfort in the confusions
& puzzles one makes, that they are always
exceedingly amusing to me, after they are cleared
away. And this is at least some compensation
for the plague of them before. _
With many thanks,

Yours most truly
A. A. Lovelace