Ockham Park  
Sunday. 11th July ['1841 added by later reader]  

Dear Mr De Morgan. I enclose you a paper (marked No 1) from which I think you will see that I now quite understand the real relationship between $\int \frac{x^n}{\sqrt{a^2-x^2}}\,dx$ and $\int \sqrt{a^2-x^2}x^{n-2}\,dx$; & that I [something crossed out] am now aware I wanted to apply to the latter what is not intended to be directly applied to it at all; & that ['my' inserted] getting both $du$ and $dx$ in, was a complete puzzle & blunder. For where a few lines previously $(-n-1)\int \sqrt{a^2-x^2}x^{n-2}\,dx$ is substituted for $\int \sqrt{a^2-x^2} \times (-n-1)x^{n-2}\,dx$, $du$ ceases of course to enter under the Integrated quantity, [something crossed out] since it has been decomposed & otherwise distributed. 

I am still occupied on pages 108, 109, 110, & hope to complete to page 112 during this week. I find this part requires studying with great care. I think you anticipated this. 

I must now thank you very much for your two letters; & will proceed to notice one or two points [112v] in your replies to my enquiries. I see that in objecting to what I called the division of $dV/dx$, when $dV$ is substituted for $\frac{dV}{dx}\,dx$, I took a completely wrong view of the matter. It does so happen that the expression (derived from a separate & distinct Theorem) which we may substitute for $\frac{dV}{dx}\,dx$ coincides in form with what we may call the numerator $dV$ of the diff co. But the $dV$ that is substituted is not therefore derived from $\frac{dV}{dx}$; at least ['not directly or' inserted] from the decomposition of that which is indecomposible [sic]. 

I return again my former paper (marked No 2.) with a clearer explanation of what I intended to convey by the term equivalent; a term which it seems I had no business to use in the application which I ['there' inserted] meant to make of it. 

I enclose (marked No 3) my answer to your "Try to prove the following. It is only when $y = ax$ 

"(a being constant) that $\frac{dy}{dx} = \frac{ay}{x}$" I do not feel quite sure that my proof is a proof. But I think..."
it is too.

Now about \( v = gt \) and \( s = \frac{1}{2}gt^2 \); a subject
which troubles me not a little.

Is the following a correct development of the note in
Useful Knowledge Mechanics? I re-copy the notes first ;
[113r] "\( V = \frac{ds}{dT} = gT \). Hence \( dS = gTdT \), which being
"integrated gives \( S = \frac{1}{2}gT^2 \)"
[something crossed out on two lines]
The Integral of \( \frac{ds}{dT} \) or of \( gTdT \) will
obviously give us \( S \); & we know that \( \int gTdT =
\frac{1}{2}gT^2 + C \), (by formula of page 104 of the Calculus).

But it appears to me that the statement
above "Hence \( dS = gTdT \)" is an unnecessary
intermediate step:
It is true that \( \int \frac{ds}{dT}dT = \int dS \),
that is providing we extend the theorem
\[ \int f(x)dx = \int f(x)dx \]
to the case when \( f(x) = 1 \), which I conclude it is
allowable to do, since 1 may be considered a
function of anything, I imagine ; thro’ the formula
\( \frac{f(x)}{f(x)} = 1 \). But tho’ true, yet the above [‘clause’ inserted] appears
to me [‘an’ inserted] unnecessary introduction. __
I am not sure that I have explained myself well.

With respect to this formula
\( \frac{1}{2}gt^2 \), & it’s [sic] derivation & application ; I have
referred as you desired to pages 27, 28, & have
[113v] fully refreshed my memory upon them. But I
do not feel this helps me much. In this first
place the process is the converse of that I enquired
upon. \( S \) is there give, & \( V \) is to be derived
from \( S \). My position was ; __ \( V \) given, & \( S \) to
be derived from \( V \). __
I understand the process of pages 27, 28, considered
as a distinct & separate thing. But I do not
identify it with Differentiation or Integration. ____
I, (knowing by abstract rules & theorems) that
2x is the diff.co of \( x^2 \), see that the limit 2t
which comes out, might be perfectly well expressed
by \( \frac{d(x^2)}{dt} \). And that we may put the result
of the Differentiation of \( t^2 \), and the result of all
the reasoning of pages 27, 28, indifferently one for
the other. But I only see it as I see that
in the processes $12 \div 4 = 3 \quad 1 + 2 = 3$ we
might indifferently put the results (3, in both cases)
one for the other. There may, for anything I yet
see or understand, be as little connection between
the abstract process of Differentiation and the
Stone-falling process, as between the above processes
of Division & Addition, which latter tho' their results
agree, cannot be identified, or one made to represent
[114r] the other.

I apprehend [something crossed out] you will perhaps answer me
here, that I must wait patiently for Chapter 8,
in which (page 143) I see something very like an
explanation of all I want. At the same time
I think it better to express fully my difficulties.

I am very anxious to see your Comments on
my two papers ['sent the other day' inserted] upon $\frac{1}{2}gt^2$. For I do not see
where the flaw in them can be; & yet I suppose
there is one. It is some comfort in the confusions
& puzzles one makes, that they are always
exceedingly amusing to me, after they are cleared
away. And this is at least some compensation
for the plague of them before.

With many thanks,

Yours most truly

A. A. Lovelace