[108r]

Ockham
Sunday. 6th July

Dear Mr De Morgan. It is perhaps unfair of me to write again with a batch of observations & enquiries, before you have had time to reply to the previous one. But I am so anxious to get the present matters off my mind, that I cannot resist dispatching them by this post. __

I have two series of observations to send, one relating to the passage from page 107, (line 8 from the bottom), to the last line of page 108; the other to certain former passages in pages 99, 100 & 103, concerning which some questions have suddenly occurred to me quite recently.
I shall begin with pages 107 & 108: I enclose you my development & explanation of $\int \frac{x^n dx}{\sqrt{a^2-x^2}}$ up to $\int \frac{x^n dx}{\sqrt{a^2-x^2}} = -x^{n-1}\sqrt{a^2-x^2} + (n-1)a^2 \int \frac{x^{n-2}dx}{\sqrt{a^2-x^2}} - (n-1)\int \frac{x^n dx}{\sqrt{a^2-x^2}}$ from which you will judge if I understand it so far. I should tell you that I have not yet begun page 109.
I will now ask two or three questions : 1stly: page 107,
[108v] (line 3 from the bottom): "the diff. co of $a^2 - x^2$ being $(-2xdx)$ &c". This surely is incorrect ; & you will see that in my development I have written it as I fancy it should be "being = $(-2x)$, &c"
2ndly: page 108, (lines 8, 9, 10 form the top) : "By $\int UdV$
"we mean ············· p. 102, where
"the values of ΔV in the several terms are
"different, but comminuent." I do not see that this is a case of page 102 rather than of page 100 ; in other words, that the increments in this Integration are "unequal but comminuent". __
3dly: the subtraction in line 15 from the top, of $(n-1)x^{n-2} \times dx$ for $d.(-x^{n-1})$ appears to me quite inconsistent with the inseparable indivisible nature of a diff. co. __
4thly: Lines 9, 10 from the bottom, "We have therefore
"&c ··········· that of $\sqrt{a^2-x^2}x^{n-2}dx$".
Admitted, most fully. But $\int \sqrt{a^2-x^2}x^{n-2}dx$ does not answer exactly to $\int vdx$ or $\int \sqrt{v}d2u$, and

therefore it appears to me that this Integration is not strictly an example of lines 5, 6, 7 (from the bottom) of page 107. You will remember that $-x^{n-1}$ was $= 2V$, therefore the x^{n-2} of $(\sqrt{a^2 - x^2}x^{n-2})$ is equal to $(-1) \times \frac{2V}{x}$ or $\frac{-1}{x}.2V$. So that another factor $\frac{-1}{x}$ enters into the [109r] expression which was, as I understand it, to answer strictly to $\int vdu$ or $\int \sqrt{v}d2u$

5thly (line 5 from the bottom) page 108: I think there is an Erratum. Surely $\int \left(\frac{a^2x^{n-2}}{\sqrt{a^2-x^2}} - \frac{x^n dx}{\sqrt{a^2-x^2}} \right)$ ought to be $\int \left(\frac{a^2x^{n-2}dx}{\sqrt{a^2-x^2}} - \frac{x^n dx}{\sqrt{a^2-x^2}} \right)$

I don't know if my pencil Sheet enclosed will be very intelligible, for it is as I wrote it down at the time quite roughly, & without any very great amplitude or method. __

I now proceed to my series of observations relating to former pages, beginning with page 102, (line 10 from the bottom)

"+ less than $nC\frac{\Omega^2}{2}$, or $Ch\frac{\Omega}{2}$";

now in order to ['effect' inserted] the substitution of $Ch\frac{\Omega}{2}$ for $nC\frac{\Omega^2}{2}$ the latter is resolved into $C.n\Omega.\frac{\Omega}{2}$, & ['for' inserted] $n\Omega$ is substituted h. But by the hypothesis & conditions, h must be less than $n\Omega$. Therefore it does not necessarily follow that that which is proved less than $nC\frac{\Omega^2}{2}$, is also less than $Ch\frac{\Omega}{2}$. _ You see my objection. __

2ndly. See Note to page 102 : If the "completion of the ['first' inserted] Series" [109v] in this page is unnecessary, surely it is equally unnecessarily in the first Series of page 100 ; for the same observation applies to the latter as to the former, viz : that the additional term is comminuent with w. __

3dly. See page 99 (line 8 from the bottom) :

"$\int_a^x \varphi x.dx = (x-a)a + \frac{(x-a)^2}{2} = \frac{x^2-a^2}{2}$"

This is another form of $\int_a^{a+h} xdx = ha + \frac{h^2}{2}$ 8 lines above, & of the limit of the summation for $\varphi x = x$ in the previous page. And therefore it appears to me that it ought to be

$$\int_a^x x.dx = (x-a)a + \frac{(x-a)^2}{2} = \frac{x^2-a^2}{2}$$

I do not see what business φx has. __

Now at last, I have done troubling you. __
I am very anxious on all these points. ____
With many apologies, believe me

Yours very truly

A. A. Lovelace