Dear Mr De Morgan. It is perhaps unfair of me to write again with a batch of observations & enquiries, before you have had time to reply to the previous one. But I am so anxious to get the present matters off my mind, that I cannot resist dispatching them by this post.

I have two series of observations to send, one relating to the passage from page 107, (line 8 from the bottom), to the last line of page 108; the other to certain former passages in pages 99, 100 & 103, concerning which some questions have suddenly occurred to me quite recently.

I shall begin with pages 107 & 108: I enclose you my development & explanation of
\[ \int x^n \, dx \sqrt{a^2 - x^2} \]
up to
\[ \int x^n \, dx \sqrt{a^2 - x^2} = -x^{n-1} \sqrt{a^2 - x^2} + (n-1)a^2 \int \frac{x^{n-2} \, dx}{\sqrt{a^2 - x^2}} - (n-1) \int \frac{x^n \, dx}{\sqrt{a^2 - x^2}} \]
from which you will judge if I understand it so far. I should tell you that I have not yet begun page 109.

I will now ask two or three questions: 1stly: page 107, [108v] (line 3 from the bottom): “the diff. co of \( a^2 - x^2 \) being \((-2xdx)\) &c”. This surely is incorrect; & you will see that in my development I have written it as I fancy it should be “being = \((-2x)\), &c”

2ndly: page 108, (lines 8, 9, 10 form the top): “By \( \int UdV \) we mean ············ p. 102, where “the values of \( \Delta V \) in the several terms are “different, but comminuent.” I do not see that this is a case of page 102 rather than of page 100; in other words, that the increments in this Integration are “unequal but comminuent”.

3dly: the subtraction in line 15 from the top, of \((n-1)x^{n-2} \times dx\) for \(d.(-x^{n-1})\) appears to me quite inconsistent with the inseparable indivisible nature of a diff. co.

4thly: Lines 9, 10 from the bottom, “We have therefore “&c ········· that of \( \sqrt{a^2 - x^2 x^{n-2} dx} \)”.

Admitted, most fully. But \( \sqrt{a^2 - x^2 x^{n-2} dx} \) does not answer exactly to \( \int v \, dx \) or \( \int \sqrt{v} \, du \), and
therefore it appears to me that this Integration is not strictly an example of lines 5, 6, 7 (from the bottom) of page 107. You will remember that $-x^{n-1}$ was $= 2V$, therefore the $x^{n-2}$ of $(\sqrt{a^2} - x^n x^{n-2})$ is equal to $(-1) \times \frac{2V}{x}$ or $\frac{1}{x} 2V$. So that another factor $\frac{1}{x}$ enters into the expression which was, as I understand it, to answer strictly to $\int vdu$ or $\int \sqrt{v}d2u$.

5thly (line 5 from the bottom) page 108: I think there is an Erratum. Surely $\int (\frac{a^2x^{n-2}dx}{\sqrt{a^2-x^2}} - \frac{x^n dx}{\sqrt{a^2-x^2}})$ ought to be $\int (\frac{a^2x^{n-2}dx}{\sqrt{a^2-x^2}} - \frac{nC}{\Omega} \frac{\Omega^2}{2})$.

I don't know if my pencil Sheet enclosed will be very intelligible, for it is as I wrote it down at the time quite roughly, & without any very great amplitude or method. _

I now proceed to my series of observations relating to former pages, beginning with page 102, (line 10 from the bottom)

"+ less than $nC\frac{\Omega^2}{2}$, or $Ch\frac{\Omega}{2}$;"

now in order to [‘effect’ inserted] the substitution of $Ch\frac{\Omega}{2}$ for $nC\frac{\Omega^2}{2}$ the latter is resolved into $C.n\Omega\frac{\Omega}{2}$, & [‘for’ inserted] $n\Omega$ is substituted $h$. But by the hypothesis & conditions, $h$ must be less than $n\Omega$. Therefore it does not necessarily follow that that which is proved less than $nC\frac{\Omega^2}{2}$, is also less than $Ch\frac{\Omega}{2}$. _

You see my objection. _

2ndly. See Note to page 102 : If the “completion of the [‘first’ inserted] Series” [109v] in this page is unnecessary, surely it is equally unnecessarily in the first Series of page 100 ; for the same observation applies to the latter as to the former, viz : that the additional term is comminuent with $w$. _

3rdly. See page 99 (line 8 from the bottom) :

"$\int_a^x \varphi x .dx = (x - a)a + \frac{(x-a)^2}{2} = \frac{x^2-a^2}{2}$" 8 lines above, & of the limit of the summation for $\varphi x = x$ in the previous page. And therefore it appears to me that it ought to be

$\int_a^x x .dx = (x - a)a + \frac{(x-a)^2}{2} = \frac{x^2-a^2}{2}$

I do not see what business $\varphi x$ has. _
Now at last, I have done troubling you. 
I am very anxious on all these points. 
With many apologies, believe me

Yours very truly

A. A. Lovelace