[106r]

Ockham Park
Ripley
Surrey
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Dear M^r^ De Morgan. You are perhaps surprised that I have not sooner troubled you again. And you may think it a very bad reason to give, that I have done nothing. We returned here on Tuesday, & now I am working away famously, & hope I have before me 7 or 8 months of ditto. You left me at page 106. I remember your enquiry if I were sure that I understood $\int_b^{b+k} fx \times \frac{dx}{dt}$ as developped [*sic*] in pages 102, 103. I answered confidently, that I did. I now enclose you my own development of this Integration, that we may be quite certain of my comprehension of [something crossed out] it. On the other page of my sheet, is the application of it to $\int udv = uv - \int vdu$ (page 105); & to $\int_a^x \frac{1}{v}\frac{dv}{dx}dx$ (page 107).

I have now two questions to propose.
I differ from you in my development of $\int \frac{1}{1-x}dx$
(see page 107)
[106v] I cannot see why the Constant C is omitted in this more than in $\int \frac{1}{1+x}dx$.
I subjoin my development: Let $v = 1 - x$
$\int \frac{1}{1-x}dx = \int \frac{1}{1-x} \times -(-1)dx$ (which is only another way of writing $\int \frac{1}{1-x}.1.dx$)
And as $\frac{dv}{dx}$ or $\frac{d(1-x)}{dx} = -1$, we may in the above substitute $\int \frac{1}{1-x}dx = \int \frac{1}{1-x} \times -\left(\frac{d(1-x)}{dx}\right) dx$

Or $\int \frac{1}{v}dx = \int \frac{1}{v} \times -\frac{dv}{dx}dx$
$= \int \frac{1}{v}\frac{dv}{dx}.(-1)dx$ which by
$\int budx = b\int udx$ (see page 105) is $= (-1)\int \frac{1}{v}\frac{dv}{dx}dx$
or $= -\int \frac{1}{v}\frac{dv}{dx}dx$
Now since by line 4, $\int \frac{1}{v}\frac{dv}{dx}dx = \int \frac{1}{v}dv =$
$= \log v + C$, it follows that
(— this same expression) must $= -(\log v + C)$
$= -(\log(1-x) + C) = -\log(1-x) - C$
$= \log \frac{1}{1-x} - C$
[107r] Now how do you get rid of $(-C)$? __

My second question is unconnected with any of your books. _ But I think I may venture to trouble you with it. _ In the two equations,

$V = gT$ (1)

$S = \frac{1}{2}g.T^2$ (2)

which you will at once recognise, I want to know how (2) is derived from (1). __
Will you refer to Mechanics (in the Useful Knowledge Library), page 10, Note, which is as follows, "Let S be the space described by the "falling body. $V = \frac{dS}{dT} = gT$. Hence $dS = gT\,dT$, "which being integrated gives $S = \frac{1}{2}g.T^2$."
Now can I ['as yet' inserted] understand this application of Differentiation & Integration? __
I conclude that $\frac{dS}{dT}$ here means

Diff. co of S with respect to T, S being (by Definition & Hypothesis) a function of T, & of V
I know that $V = gT$

And that $V = \frac{S}{T}$ But I neither see how $V = \frac{dS}{dT}$, nor how the subsequent Integration applies. [107v] The object, I need not say, is the solution of S.
I mean to work very hard at my Chapter on Integration &c, now. __ And I hope this summer & autumn will see me progressing at no small rate.
How is the Baby? __ And does M[rs] De Morgan enjoy Highgate? I ['am' inserted] enjoying the country not a little, I assure you.

Yours most truly

A. A. Lovelace