Dear Mr De Morgan. You are perhaps surprised that I have not sooner troubled you again. And you may think it a very bad reason to give, that I have done nothing. We returned here on Tuesday, & now I am working away famously, & hope I have before me 7 or 8 months of ditto. You left me at page 106. I remember your enquiry if I were sure that I understood $\int b^k f x \times \frac{dx}{dt}$ as developed [sic] in pages 102, 103. I answered confidently, that I did. I now enclose you my own development of this Integration, that we may be quite certain of my comprehension of [something crossed out] it. On the other page of my sheet, is the application of it to $\int udv = uv - \int vdu$ (page 105); & to $\int_a^b \frac{1}{v} \frac{dv}{dx} dx$ (page 107).

I have now two questions to propose. I differ from you in my development of $\int 1 \frac{1}{1-x} dx$ (see page 107)

I cannot see why the Constant $C$ is omitted in this more than in $\int 1 \frac{1}{1-x} dx$. I subjoin my development: Let $v = 1 - x$

$\int 1 \frac{1}{1-x} dx = \int 1 \frac{1}{1-x} \times -(-1) dx$ (which is only another way of writing $\int 1 \frac{1}{1-x} 1 dx$)

And as $\frac{dv}{dx}$ or $\frac{d(1-x)}{dx} = -1$, we may in the above substitute $\int 1 \frac{1}{1-x} dx = \int 1 \frac{1}{1-x} \times -\left(\frac{d(1-x)}{dx}\right) dx$

Or $\int 1 \frac{1}{v} dx = \int 1 \frac{1}{1-x} \times \frac{dv}{dx} dx$

$= \int 1 \frac{dv}{dx} (-1) dx$ which by $\int budx = b \int udx$ (see page 105) is $= (-1) \int 1 \frac{dv}{dx} dx$

Now by line 4, $\int 1 \frac{dv}{dx} dx = \int 1 \frac{dv}{v} = log v + C$, it follows that

$= -(log(1-x) + C) = -log(1-x) - C$

$= log \frac{1}{1-x} - C$

[107r] Now how do you get rid of $(-C)$?
My second question is unconnected with any of your books. But I think I may venture to trouble you with it. In the two equations,

\[ V = gT \quad (1) \]

\[ S = \frac{1}{2} gT^2 \quad (2) \]

which you will at once recognise, I want to know how (2) is derived from (1). Will you refer to Mechanics (in the Useful Knowledge Library), page 10, Note, which is as follows, “Let \( S \) be the space described by the ‘falling body. \( V = \frac{dS}{dT} = gT \). Hence \( dS = gT \, dT \), “which being integrated gives \( S = \frac{1}{2} gT^2 \).” Now can I ‘as yet’ inserted understand this application of Differentiation & Integration? I conclude that \( \frac{dS}{dT} \) here means Diff. co of \( S \) with respect to \( T \), \( S \) being (by Definition & Hypothesis) a function of \( T \), & of \( V \) I know that \( V = gT \)

And that \( V = \frac{S}{T} \). But I neither see how \( V = \frac{dS}{dT} \), nor how the subsequent Integration applies.

[107v] The object, I need not say, is the solution of \( S \). I mean to work very hard at my Chapter on Integration &c, now. And I hope this summer & autumn will see me progressing at no small rate.

How is the Baby? And does M’ns De Morgan enjoy Highgate? I ‘am’ inserted] enjoying the country not a little, I assure you.

Yours most truly

A. A. Lovelace