[100r]

Ockham Park
Friday. 19^{th} Feb^y

Dear M^r De Morgan. I have one or two queries to make respecting the "Calculus of Finite Differences" up to page 82.
Page 80, line 4 from the top, "remembering ....
".... that in $\varphi''(x+\theta\omega)$, θ itself <u>is a function</u> "<u>of x and ω</u>, &c" ; Now, neither on examining θ as here used & introduced, nor on referring to the first <u>rise & origin</u> of θ in this capacity, (see page 69), can I discover that it is a function of x and ω here, or a function of the analogous a and h in page 69. I neither see the truth of this assertion, nor do I perceive the importance of it (supposing it <u>is</u> true) to the rest of the argument & demonstration in page 80.

There is also a point of doubt I have relating to the conclusion in lines 15, 16 from [100v] the top of page 79 :
It is very clear that the law for the Co-efficients being proved for u_n, and for Δu_n, follows immediately & easily for u_{n+1}, or $u_n + \Delta u_n$.

But if we now wish to establish it for u_{n+2}, we must prove it true not <u>only</u> for u_{n+1}, but also for Δu_{n+1} :_

To retrace from the beginning : the object in the first half of page 79 evidently is to prove firstly, that <u>any</u> order of u, say u_n <u>can</u> be expressed in term ['of,' inserted] or in a Series of all the Differences of u ; Δu, $\Delta^2 u$, $\Delta^3 u$, ... $\Delta^n u$; _
Secondly, that the Co-efficients for this Series follow the law of those in the Binomial Theorem.

Now the first part is evident from the law of formulation of the Table of Differences ; Since all the Differences Δu, $\Delta^2 u$, $\Delta^3 u$ &c are made out of u, u_1, u_2 &c, it is obvious that by exactly retracing & reversing the process, we can make u, u_1, u_2 &c [101r] out of Δu, $\Delta^2 u$, $\Delta^3 u$ &c.

For the second part of the above ; if we
can [something crossed out] show that the law for the Co-efficients
holds good up to a certain point, say u_4 ;
and also that being true for any one
value, it must be true [something crossed out] for the next
value too ; the demonstration is effected for
all values :
Now the fact is shown that it is true up
to u_4. (I must not here enquire why the
fact is so. That is I suppose not your
arranging, or any part of your affairs).
It is shown that the two parts u_3, Δu_3 of
which u_4 is made up are under this law,
& therefore that u_4 is so. And next it is
shown that any other two parts u_n, Δu_n
being under this law, their sum u_{n+1}
must be so. But this proves nothing
for a continued succession. u_{n+1} being
under this law does not prove that Δu_{n+1}
is under it, & therefore that u_{n+2} is under it.
[101v] There seems to me to be a step or condition
omitted.
I am sorry still to be obliged to trouble
you about $f\,x$, $f'x$, $f''x$, I cannot yet
agree to the assertion that the result would
not be affected by discontinuity or singularity
in $f'x$, $f''x$, &c. The result it is true
would not be directly affected ; but it surely
would be ['indirectly' inserted] affected, inasmuch as the conditions
of page 69, necessary to prove that result,
could not be fulfilled unless we suppose
$f'x$, $f''x$ ..... $f^{(n+1)}x$ continuous &
ordinary as well as $f\,x$. To arrive at
the equation $\frac{\varphi(a+h)}{\psi(a+h)} = \frac{\varphi^{(n+1)}(a+\theta h)}{\psi^{(n+1)}(a+\theta h)}$
page 69, it is a necessary condition that
φx, $\varphi' x$, $\varphi'' x$ .... $\varphi^{(n+1)}x$ be all
continuous & without singularity from $x = a$ to
$x = a + h$. And the $\varphi' x$, $\varphi'' x$ ..... $\varphi^{(n)}x$, $\varphi^{(n+1)}x$
of page 71, could not fulfil this condition
unless $f'x$, $f''x$ ..... $f^{(n)}x$, $f^{(n+1)}x$ did so
[102r] also. _ I fear I am very troublesome about
this. _

I have remarks to make respecting some of
the conclusions of the Chapter on Algebraical
Development ; but they will keep, and
therefore I will delay them, as I think
I have send abundance, & I have also
some questions to put on the last 8 pages
of your "Number & Magnitude" on Logarithms. _

On the Differential Calculus I will only
now further say that on the whole I believe
I go on pretty well with it ; and that
I suppose I understand as much about it,
[something crossed out] as I am intended to do ;
possibly more, for I spare no pains to do
so. __

Now for the Logarithms : I had not till now
read the last pages of your Number & Magnitude,
& there are certain points I do not fully
understand. The last line of the whole, on
the natural logarithms is one. I cannot
[102v] identify the constituent quality of the natural
logarithms there given, with the constituent
qualities I am already acquainted with thro'
other relations & means : I know ['for instance' inserted] that the
natural logarithms must have 2.717281828
for their Base ; that is to say that the
line HL, or A (OK, or V being the linear
unit) should be 2.717281828 V units.
Now I do not see ['but' inserted] that the condition in the
last paragraph of the book is one that
might perfectly consist with any Base whatever.

To prove that I understand
the previous part, at least to a considerable
degree, I enclose a Demonstration I wrote
out of the property to be deduced by the
Student, (see second paragraph of page 79),
& which I believe is quite correct. __

Pray of what use is the Theorem
(page 75, ['& which' inserted] continues in page 76)? I do not see
that it is subservient to anything that
[103r] follows ; and it appears to me, to say the
truth, to be rather a useless & cumbersome
addition to a subject already sufficiently

complicated & cumbersome. _ The passage I mean is from line 13 (from the top) page 75, to the middle of page 76. _

Believe me

Yours very truly

A. A. Lovelace