Gauge Theory and Topology
July 24-28, 2023

Abstracts of Talks

Mohammed Abouzaid (Columbia University)

Title: Floer homotopy as a bordism theory

Abstract: Floer homotopy has a well-deserved reputation as an abstruse subject, whose very formulation requires enormous technical machinery from stable homotopy theory. I will explain new work, joint with Blumberg, which develops such a theory from the perspective of bordism, and which leads, for example, so a definition of Floer homotopy groups that does not require one to first know anything about stable homotopy. The starting point of the construction is a description of bordism of manifolds given in terms of Morse theory. The new approach makes structural properties of Floer homotopy transparent.

Ali Daemi (Washington University in St Louis)

Title: The knot complement problem for nullhomotopic knots

Abstract: In one of his many important contributions to low-dimensional topology, Kronheimer, in a joint work with Tom Mrowka, proved the Property P conjecture. This conjecture states that any non-trivial Dehn surgery on a knot in the 3-dimensional sphere is not simply connected. A Kirby Problem proposed by Boileau states that for any nullhomotopic knot K in a 3-manifold Y, the dual knot in a non-trivial Dehn surgery on K is not nullhomotopic. In particular, this can be regarded as a generalization of the Property P conjecture to arbitrary 3-manifolds. In this talk, I will discuss a program aimed at proving this conjecture and explain how this program can be applied to some families of 3-manifolds.

Kristen Hendricks (Rutgers University)

Title: Naturality issues in involutive Heegaard Floer homology

Abstract: Heegaard Floer homology is an invariant of 3-manifolds, and knots and links within them, introduced by P. Oszváth and Z. Szabó in the early 2000s. Because of its relative computability by the standards of gauge and Floer theoretic invariants, it has enjoyed considerably popularity. However, it is not immediately obvious from the construction that Heegaard Floer homology is natural, that is, that it assigns to a basepointed 3-manifold a well-defined module over an appropriate base ring rather than an isomorphism class of modules, and well-defined cobordism maps to 4-manifolds with boundary. This situation was improved in the 2010s when A. Juhász, D. Thurston, and I. Zemke showed naturality of the various versions of Heegaard Floer homology. In this talk we consider involutive Heegaard Floer homology, a refinement of the theory introduced by C. Manolescu and I in 2015, whose definition relies on Juhász-Thurston-Zemke naturality but which is itself not obviously natural even given their results. We prove that involutive Heegaard Floer homology is a natural invariant of basepointed 3-manifolds together with a framing of the basepoint, and has well-defined maps associated to cobordisms, and discuss some consequences and implications. This is joint work with J. Hom, M. Stoffregen, and I. Zemke.

Nigel Hitchin (University of Oxford)

Title: ALE spaces revisited

Abstract: In his thesis Peter Kronheimer gave a beautiful construction of asymptotically Euclidean 4-manifolds which provides a comprehensive view of their global structure and moduli but leaves open a more detailed local description of the actual metric. In this talk we show how to use Penrose's twistor construction to convert the problem into finding the nodal rational curves in a linear system on a compactification of the 4-manifold to a concrete algebraic surface.

Andras Juhász (University of Oxford)

Title: The unknotting number, hard unknot diagrams, and Reinforcement Learning

Abstract: We have developed a Reinforcement Learning agent based on the IMPALA architecture that often finds minimal unknotting trajectories for a knot diagram up to 200 crossings. We have used this to determine the unknotting number of 57k knots. We then took diagrams of connected sums of such knots with oppositely signed signatures, where the summands were overlaid. The agent has found unknotting trajectories involving several crossing changes that result in hyperbolic knots. Based on this, we have shown that, given knots K and K' that are not 2-bridge, there is a diagram of their connected sum and u(K) + u(K') unknotting crossings such that changing any one of them results in a prime knot. As a by-product, we have obtained a dataset of 2.6 million distinct hard unknot diagrams; most of them under 35 crossings. Assuming the additivity of the unknotting number, we can determine the unknotting number of 43 at most 12-crossing knots for which the unknotting number is unknown. This is joint work with Taylor Applebaum, Sam Blackwell, Alex Davies, Thomas Edlich, and Marc Lackenby

Hokuto Konno (University of Tokyo)

Title: Exotic Dehn twists on 4-manifolds

Abstract: A self-diffeomorphism of a smooth manifold is said to be exotic if it is topologically isotopic to the identity but smoothly not. We provide the first examples of exotic diffeomorphisms (in a relative sense) of contractible 4-manifolds, more generally of definite 4-manifolds. Such examples are given as Dehn twists along certain Seifert homology 3-spheres, which also give new examples of exotic diffeomorphisms that survive after one stabilization, and the smallest closed 4-manifold known to support an exotic diffeomorphism. The proof uses families Seiberg-Witten theory over RP^2. This is joint work with Abhishek Mallick and Masaki Taniguchi.

Maggie Miller (Stanford University)

Title: Splitting spheres in S^4

Abstract: A 2-component link L is split if its components lie in disjoint balls. The boundary of either of these balls is called a splitting sphere. In the 3-sphere, 2-component split links have unique splitting spheres, meaning any two splitting spheres for L are isotopic in S^3-L. In this talk, we’ll discuss why this fails in dimension 4: many 2-component split links of surfaces in the 4-sphere do not have unique splitting spheres. (In fact, many unlinks have non-unique splitting spheres.) This is joint work with Mark Hughes and Seungwon Kim.

Tom Mrowka (MIT)

Title: Thirty years of working with Peter

Abstract: This talk will give some reminisces about the 30 years of our collaboration. Much of this work centers on Yang-Mills theory for connections with singularities. Periodically it has given insight into important questions in low dimensional topology. More recently it holds out promise for applications to other fields. I’ll try to give a flavor of some of the arc of this work.

Hiraku Nakajima (University of Tokyo)

Title: Coulomb branches and singular monopole moduli

Abstract: I will review a mathematical definition of Coulomb branches of 3d SUSY gauge theories given my joint work with Braverman and Finkelberg. It uses a language of affine Grassmannian. I would like to compare it with another approach, based on singular monopole moduli spaces, by physicists, Ito, Okuda, Taki and subsequent works by Yoshida and his collaborators.

Peter Ozsváth (Princeton University)

Title: Constructions in bordered Floer homology

Abstract: Bordered Floer homology, introduced by Lipshitz, D. Thurston, and me, allows one to express the U=0-specialized Heegaard Floer homology of a closed three-manifold Y in terms of a suitable decomposition of Y. This theory should have a generalization to the case of unspecialized Heegaard Floer homology. I will describe aspects of this generalization in the case of three-manifolds with torus boundary. This is joint work with Lipshitz and Thurston.

Lisa Piccirillo (MIT)

Title: Building closed exotic manifolds by hand

Historically, closed exotic 4-manifolds are built using cut and paste constructions, and their gauge theoretic invariants are computed using gluing formulae. In this talk, I'll define some new smooth 4-manifold invariants which we can compute by hand using a (Dehn) surgery formula. Armed with this I'll build explicit closed exotic 4-manifolds out of elementary handle cobordisms. We'll see some new phenomena, like closed exotic definite 4-manifolds (with fundamental group $\mathbb{Z}/2$), closed 4-manifolds with homologically essential square 0 spheres and nonvanishing invariants, and instances when knot surgery on an Alexander polynomial 1 knot can change the smooth structure. We'll try not to work too hard. This is joint work with Adam Levine and Tye Lidman.

Arunima Ray (MPIM Bonn)

Title: Surfaces in 4-manifolds

Abstract: I will describe a general procedure to homotope an immersed positive genus surface in a simply connected 4-manifold to a locally flat embedding. This is a special case of a surface embedding theorem, joint with Daniel Kasprowski, Mark Powell, and Peter Teichner.

Daniel Ruberman (Brandeis University)

Title: Homotopy properties of diffeomorphism groups and spaces of positive scalar curvature on 4-manifolds

Abstract: We show that the higher homotopy groups of the diffeomorphism group of a smooth 4-manifold can have infinitely generated free abelian summands. The same applies to the homology of the Torelli subgroup and of its classifying space. We show that the homotopy groups of the space of positive scalar curvature metrics on a large connected sum of S^2 x S^2's can have infinitely generated free abelian subgroups. This is all joint work with Dave Auckly.

Rosa Sena-Dias (IST Lisbon)

Title: Einstein metrics from the Calabi ansatz via Derdziński duality

Abstract: One of the main sources of examples of Einstein metrics has been Kähler geometry. Yet we know that several Kähler manifolds do not carry Kähler-Einstein metrics. It is therefore natural to relax the Kähler condition in looking for Einstein metrics. In the 80's, Derdziński obtained a local construction for 4d conformally Kähler, Einstein metrics using extremal Kähler metrics. Together with G. Oliveira, we set out to use Derdziński's methods on a class of extremal Kähler metrics arising from an ansatz due to Calabi. In this talk, I shall report on our findings. I will start by reviewing Derdziński's results and the Calabi anstaz. Then I shall explain how our construction includes Einstein fillings of S^3/Z_m. I also discuss a cone-angle Einstein deformation of the Page metric on $CP^2\sharp CP^2$. If time permits, I shall describe a limiting procedure yielding an asymptotically hyperbolic Einstein metric not arising from the Derdziński construction. This is joint work with G. Oliveira.

Steven Sivek (MPIM Bonn)

Title: Homology RP^3s and SL(2,C)

Abstract: Building on non-vanishing theorems of Kronheimer and Mrowka in instanton homology, Zentner proved that if Y is a homology 3-sphere other than S^3, then its fundamental group admits a homomorphism to SL(2,C) with non-abelian image. In this talk, I'll explain how to prove the same result if H_1(Y) = Z/2 and Y is different from RP^3, and discuss how one might generalize this to some other rational homology spheres as well. This is joint work with Sudipta Ghosh and Raphael Zentner.

Zoltan Szabó (Princeton University)

Title: Knot Floer homology and Pong algebras

Abstract: In this talk I will describe some recent joint work with Peter Ozsvath on differential graded algebras and knot Floer homology. The bordered Floer homology of Lipshitz, Ozsvath and Thurston is an invariant of three-manifolds with surface boundary, and can be used to study Heegaard Floer homology for closed three-manifolds. Adaptation of these methods, developed in a joint work with Peter Ozsvath, allows effective computation for a version of knot Floer homology over the ring Z/2[U,V]/(U·V=0)

In the talk I will describe some new constructions, some interesting problems and recent advances that are related to extending these methods.

Joshua Wang (Harvard University)

Title: Progress towards the colored sl(N) homology of T(2,m)

Abstract: A precise prediction for the colored sl(N) homology of any 2-bridge knot or link can be made based on a conjectural relationship with SU(N) instanton homology. It is predicted to be isomorphic to the ordinary cohomology of an associated closed manifold consisting of SU(N) representations of the fundamental group of its complement. I will explain some recent progress towards verifying the prediction for the torus knot or link T(2,m) and other applications partially based on joint work with Mike Willis.

Yi Xie (Peking University)

Title: Kontsevich's characteristic classes and the diffeomorphism groups of 4-manifolds

Abstract: In 2018, Watanabe disproved the 4-dimensional Smale conjecture by showing that the diffeomorphism group of a 4-dimensional disk relative to its boundary is non-contractible. In Watanabe's proof he used a version of Kontsevich's characteristic classes to detect non-trivial smooth families of disk bundles. In this talk we will show that Kontsevich's characteristic classes only depend the formal smooth structure, i.e. a lift of the tangent microbundle to a vector bundle. As an application, we will prove that the diffeomorphism group of an arbitrary compact 4-manifold (with or without boundary) is not homotopy equivalent to its homeomorphism group. This is joint work with Jianfeng Lin.

Ian Zemke (Princeton University)

Title: Perspectives on the Heegaard Floer surgery formulas
Abstract: In this talk, we will describe perspectives and generalizations of the Heegaard Floer surgery formulas of Manolescu, Ozsvath and Szabo. We will describe a "local" version of the surgery formula which naturally generalizes their surgery formula to arbitrary knots and links in any 3-manifold. This is purely a statement in terms of the Fukaya category. We will describe additional applications of this formula, and time permitting will describe the relation with the bordered theory of Lipshitz, Ozsvath and Thurston.

Raphael Zentner (University of Regensburg)

Title: SL(2,C)-character varieties of knots and maps of degree 1

Abstract: We ask to what extent the SL(2,C)-character variety of the fundamental group of the complement of a knot in S^3 determines the knot. Our methods use results from group theory, classical 3-manifold topology, but also geometric input in two ways: The geometrisation theorem for 3-manifolds, and instanton gauge theory. In particular this is connected to SU(2)-character varieties of two-component links, a topic where much less is known than in the case of knots. This is joint work with Michel Boileau, Teruaki Kitano and Steven Sivek.