RHB70: Analytic Number Theory and Its Integers
July 10-14, 2023

Abstracts of Talks

Brian Conrey (AIM)

Title: Moments of families of L-functions

Abstract: Heath-Brown's many papers on moments of L-functions in families have inspired much subsequent work in this area. In particular his 1979 paper on the fourth moment of the Riemann zeta-function introduced the idea that correlations of shifted divisor functions were an important input into understanding moments beyond the second for zeta.

We will describe joint work with Keating and joint work with Baluyot that lead to an approach for understanding moments of L-functions in families by piecing together in an appropriate way our (conjectural) knowledge of correlations of shifted divisor functions in the case of zeta, or their analogues in the case of other families.

Alexandra Florea (UC Irvine)

Title: Negative moments of the Riemann-zeta function

Abstract: I will talk about joint work with H. Bui regarding negative moments of the Riemann zeta-function. I will explain how to obtain asymptotic formulas in certain ranges for the shift in the zeta-function in the denominator, and will discuss some applications to the question of obtaining cancellation in partial sums of the generalized Mobius function.

Kevin Ford (University of Illinois at Urbana-Champaign)

Title: Toward a theory of prime detecting sieves

Abstract: Given a set of integers, we wish to know how many primes there are in the set. Modern tools allow us to obtain an asymptotic for the number of primes, or at least a lower bound of the expected order, assuming certain strength Type-I information (the distribution of the sequence in progressions) and Type-II information (bilinear sums over the sequence). The methods used previously, especially Harman's sieve, are largely ad-hoc and shed little light on the limitations of the methods. In joint work with James Maynard, we develop a systematic framework for understanding the theoretical limits of these prime detecting sieves. In principle, we can determine whether given Type-I information and Type-II information is sufficient to detect primes in the set, and to produce an optimal lower bound for the count of primes in the set.

Andrew Granville (University of Montreal)

Title: Primes in short intervals

Abstract: Ever since Gauss's notes from his childhood calculations we have wanted to fully understand the distribution of primes in short intervals. We will discuss several aspects of this question, starting from Gauss's data and culminating in a description of Sun-Kai Leung's recent work on correlations between primes in neighbouring intervals.

Ben Green (University of Oxford)

Title: Quadratic forms in 8 prime variables

Abstract: I will discuss the solution of equations $Q(p_1,...,p_8) = N$ in primes, where Q is a quadratic form. The traditional approach to problems of this type, the Hardy-Littlewood circle method, can be made to work for quadratic forms in 9 or more variables, but reaches an obstruction at 8.

The main new idea is to get around this is to involve the Weil representation of the symplectic groups $Sp_8(Z/qZ)$. I will explain what this is, and how it may be used to make progress on the stated problem.

Henryk Iwaniec (Rutgers University)

Title: On k-tuples of integer parts mutually co-prime

Abstract: This is a report on my recent joint work with Jean-Marc Deshouillers. Given a collection of nice smooth functions $F=(f_1,...,f_k)$ on R^{+} we consider the density of integers n such that $[f_1(n)],...,[f_k(n)]$ are pairwise co-prime. This collection covers the Piatetski-Shapiro numbers $[(n+i)^c]$ with $i=1,...,k$ where c is any fixed constant, $1<c<2$.

Jon Keating (University of Oxford)

Title: Joint moments

Abstract: I will review some of the recent progress concerning the joint moments of the Riemann zeta-function and its derivates, in the context of the connection between the zeta function and random matrix theory.

Emmanuel Kowalski (ETH Zurich)

Title: Dix ans après

Abstract: Ten years ago, during the conference organized in honor of Roger Heath-Brown on his 60th birthday, Ph. Michel and myself presented for the first time some of the results concerning the analytic theory of trace functions which we had obtained in a series of works with É. Fouvry.

Ten years later, I will survey the main developments of this story during the intervening years, both from the conceptual and practical point of view, and also discuss some perspective for the next few years.

Kaisa Matomäki (University of Turku)

Title: Detecting primes in multiplicatively structured sequences

Abstract: I will discuss a new sieve set-up which allows one to find primes in sequences that have good type I information and a suitable multiplicative structure. While Type I information is not sufficient for detecting primes with a classical sieve, it is possible to detect them in cases where a multiplicative structure allows one to combine Type I information from different scales.

Among other things, the method gives a new L-function free proof of Linnik's theorem. The talk is based on on-going joint work with Jori Merikoski and Joni Teräväinen.

Sarah Peluse (University of Michigan)

Title: Bounds for sets without L-shapes

Abstract: I will discuss the difficult problem of proving reasonable bounds in the multidimensional Szemerédi theorem and describe a proof of such bounds for sets lacking nontrivial "L-shapes", i.e., the configuration (x,y), $(x,y+z)$, $(x,y+2z)$, $(x+z,y)$.

Lillian Pierce (Duke University)

Title: A polynomial sieve: beyond separation of variables

Abstract: Many problems in number theory can be framed as questions about counting solutions to a Diophantine equation, say, within a certain “box”. If there are very few, or very many variables, certain methods gain an advantage, but sometimes there is extra structure that can be exploited as well. For example: let f be a given polynomial with integer coefficients in n variables. How many values of f are a perfect square? A perfect cube? Or, more generally, a value of a different polynomial, say g(y)? We will survey sieve methods that can exploit this type of structure, and in particular explain how a new polynomial sieve method, in joint work with Dante Bonolis, allows greater flexibility, so that the variables in the polynomials f and g can “mix.”

Maksym Radziwill (Caltech)

Title: The Fyodorov-Hiary-Keating conjecture

Abstract: The Fyodorov-Hiary-Keating conjecture describes the distribution of the local maximum of the Riemann zeta-function. In other words, pick a typical t between 0 and T, then the conjecture describes the fluctuations of the local maximum of zeta(1/2 + i u) in a unit interval around t. The choice of a unit interval is immaterial and this can be enlarged or shrunk as needed.

In recent work with Arguin and Bourgade we establish tightness, thus for 99% of t's the local maximum is of size (\log t) (\log \log t)^{-3/4}.

I will describe the main ideas from the proof. Those are motivated by work of Bramson on branching random walks (and those in turn are motivated by the KPP equation in PDE's). In particular the proof shows that whenever the local maximum is achieved at 1/2 + iu, say, then the partial sums of \log \zeta at that point have to evolve in a very specific and rigid way.

Per Salberger (Chalmers University)

Title: On Heath-Brown's determinant method

Abstract: We present some recent Diophantine estimates obtained by a global version of Heath-Brown's *p*-adic determinant method.

Damaris Schindler (University of Göttingen)

Title: Generalised quadratic forms over totally real number fields

Abstract: We introduce a new class of generalised quadratic forms over totally real number fields, which is rich enough to capture the arithmetic of arbitrary systems of quadrics over the rational numbers. We explore this connection through a version of the Hardy-Littlewood circle method over number fields. This is joint work with Tim Browning and Lillian Pierce.

K. Soundararajan (Stanford University)

Title: Divisibility properties of the character values of the symmetric group.

Abstract: The values taken by characters of the symmetric group have long been known to be integers. But only recently A. Miller made the surprising empirical observation that most of the character values are even integers, and indeed most seem to be divisible by any given prime number. Miller's conjecture that most character values are even was established by Sarah Peluse in 2020, and subsequently she and I have shown that almost all entries in the character table of the symmetric group are divisible by any fixed integer. I will discuss some aspects of the proof, which combines combinatorial ideas together with an understanding of the structure of random partitions of integers.

T. D. Wooley (Purdue University)

Title: Beyond Vinogradov's mean value theorem

Abstract: Essentially optimal estimates have been obtained for mean values of Vinogradov's exponential sum as a consequence of the decoupling method (by Bourgain, Demeter and Guth), and the efficient congruencing method (by the speaker). Such work makes essential use of the fact that the system of Diophantine equations associated with these mean values is translation-dilation invariant. We report on progress for systems which are not translation-dilation invariant.