CMI-HIMR Integrable Probability Summer School
July 27 – 31, 2020

Mini Course Abstracts

Hugo Duminil (IHÉS)

Title: On the free energy of the six-vertex model

Abstract: In this series of lectures, we will explain how to compute the free energy of the six-vertex model with a given density of up arrows, and how one can relate the behaviour of this free energy to localization/delocalization of the corresponding height-function, and rotation invariance of the model in the delocalized regime.

Rick Kenyon (Yale University)

Title: Limit shapes and variational principles

Abstract: We discuss the notion of limit shape for random height models in 2+1 dimensions, giving basic examples and ideas of proofs. We discuss the analysis of the resulting variational problems, in particular the connection between the conformal structure and the surface tension. This is based on joint work with Istvan Prause.

Greta Panova (USC)

Title, talk 1: Algebraic Combinatorics basics
Title, talks 2 and 3: Algebraic Combinatorics meets Probability: Statistical Mechanics and Asymptotics

Abstract: Algebraic Combinatorics sits at the intersection of Combinatorics and Representation Theory and Algebra, using algebraic methods to answer combinatorial questions and employs tools from discrete mathematics to study problems in group theory and representation theory.
In this minicourse we will start with introducing basic tools and theorems from Algebraic Combinatorics which have seen wide applications in Statistical Mechanics: symmetric function theory, representation theory of the symmetric group, the RSK algorithm and nonintersecting lattice paths etc. We will then see how we can use them to derive probabilistic results like limit behavior in dimer models (mainly lozenge tilings). We will also discuss the application of probabilistic methods in enumerate asymptotically combinatorial objects.

Fabio Toninelli (TU Vienna)

Title: (2+1)-dimensional growth models and the AKPZ universality class

Abstract: I will discuss 2+1-dimensional growth models and in particular the so-called Anisotropic KPZ (AKPZ) universality class. This comprises models that have the same growth and roughness exponents as the two-dimensional stochastic heat equation with additive noise (2d-SHE).
I will review recent rigorous results both on discrete growth models in this class, and on the AKPZ equation itself. The latter is an anisotropic variant of the two-dimensional KPZ stochastic PDE (with noise regularization). In the physics literature, the AKPZ equation was conjectured to have the same scaling limit as the 2d-SHE, but we will show that this is not entirely true. Indeed, while the 2d-SHE is invariant under diffusive rescaling, this does not hold (not even asymptotically on large

scales) for the AKPZ equation, and logarithmic corrections to the scaling are needed instead. [The last part is based on joint work with G. Cannizzaro and D. Erhard]

Michael Wheeler (Melbourne)

Title: An invitation to the q-Whittaker polynomials

Abstract: The q-Whittaker polynomials are a family of symmetric functions that can be obtained as a degeneration of the famous Macdonald polynomials. They have played an important role in integrable probability, notably via the framework of Macdonald processes and their connection to the q-TASEP.

The aim of these lectures will be to give two combinatorial formulas for the q-Whittaker polynomials, using the theory of integrable vertex models. These formulas look completely different, but both of them exhibit the q-positivity of the q-Whittaker polynomials in an explicit way. In reaching this goal, we will pass through a number of important landmarks in the theory, including the coloured stochastic six-vertex model, fusion, and the Yang--Baxter equation.