

CMI-HIMR Summer School
Unlikely Intersections in Diophantine Geometry
22-26 August 2022

Abstracts

Gal Binyamini

Title: Counting rational points on transcendental sets

Abstract: I'll discuss the Pila-Wilkie theorem in its various forms that will be needed in the other courses. I will then go over the proof, reviewing some basic tools from o-minimality along the way. Later I'll talk about Wilkie's conjecture sharpening the Pila-Wilkie theorem, and various results in this direction with a focus on the theory of Pfaffian functions.

Laura Capuano

Title: Unlikely intersections and applications to Diophantine problems

Abstract: The Zilber-Pink conjectures on unlikely intersections deal with intersections of subvarieties of a (semi)abelian variety or, more in general, of a Shimura variety, with "special" subvarieties of the ambient space. These conjectures generalize many classical results such as Faltings' Theorem (Mordell Conjecture), Raynaud's Theorem (Manin-Mumford Conjecture) and André-Oort Conjecture, and have been studied by several authors in the last two decades. Most of the proofs of results in this area follows the well-established Pila-Zannier strategy, first introduced by the two authors in 2008 to give an alternative proof of Raynaud's theorem. The strategy combines counting theorems on rational points of bounded height in o-minimal structures (Pila-Wilkie's theorem and subsequent generalizations) with other diophantine ingredients. After a general introduction to these problems, I will present results of unlikely intersections in semi-abelian varieties and in families of abelian varieties, and I will present applications of these results to other problems of Diophantine nature.

Paola D'Aquino

Title: Exponential polynomials, their zero sets and E-ideals

Abstract: I will review some results on algebraic properties of the ring of exponential polynomials, also in connection with Zilber's axioms on the complex exponential field.

Gabriel Dill

Title: Functional Transcendence in Diophantine Geometry

Abstract: In this minicourse, we will get to know various settings, where functional transcendence properties known as Ax-Lindemann or Ax-Schanuel hold. We will then see a sketch of how Gao's mixed Ax-Schanuel theorem in functional transcendence has been used by Dimitrov-Gao-Habegger to obtain new upper bounds on the number of rational points on a smooth projective curve of genus > 1 over a number field, which was also the subject of Habegger's ICM 2022 talk.

Gareth Jones

Title: A very short introduction to o-minimality.

Abstract: I'll introduce some ideas from model theory that will be used in some of the other talks. In particular, I'll discuss definability in o-minimal expansions of the real field.

Jonathan Pila

Title: Ax-Schanuel and exceptional integrability

Abstract: In joint work with Jacob Tsimerman we study when the primitive of a given algebraic function can be constructed using primitives from some given finite set of algebraic functions, their inverses, algebraic functions, and composition. When the given finite set is just $\{1/x\}$ this is the classical problem of "elementary integrability". This problem connects somewhat surprisingly with unlikely intersections and Ax-Schanuel properties.