

Geometry and Dynamics on Moduli Spaces
September 28 – October 2, 2015

Abstracts of Talks

Yves Benoist (Université Paris-Sud)

Title: Harmonic quasiisometries

Abstract: I will explain why a quasiisometric map between rank one symmetric spaces is within bounded distance from a unique harmonic map. This joint work with D. Hulin completes the proof of the Schoen-Li-Wang conjecture.

Simion Filip (University of Chicago)

Title: Hodge theory and Teichmüller dynamics

Abstract: The natural action of $SL(2, \mathbb{R})$ on the moduli space of translation surfaces (i.e. holomorphic 1-forms) exhibits a rich collection of dynamical and geometric properties. Many of these have natural explanations in terms of Hodge theory and the geometry of Riemann surfaces. After providing some background, I will explain some applications to rigidity properties of the dynamics, as well as to Lyapunov exponents. One consequence of these results is that orbit closures, which have an affine structure by work of Eskin, Mirzakhani, and Mohammadi, also have natural algebraic structures.

Pascal Hubert (Université Aix Marseille)

Title: Diffusion in periodic billiards and for Novikov's problem

Abstract: I will explain two situations where diffusion rate can be estimated using Teichmüller flow and Kontsevich-Zorich cocycle. The first one is the wind tree model (a \mathbb{Z}^2 periodic billiard). I will insist on the behavior of chaotic trajectories in Novikov's problem (joint work with A. Avila and A. Skripchenko) in a very special situation first studied by Dynnikov. I will explain how to define a natural measure on the set of chaotic directions in Novikov's problem. This set is a fractal set of zero measure, thus this is a non trivial problem. In both cases, using some results on Lyapunov exponents and following ideas from Zorich and Forni, some estimates on diffusion can be given. For Novikov's problem, I will also give some results on the ergodic properties of the associated foliations.

Maxim Kazarian (Steklov Institute of Mathematics)

Title: Topological recursion for Morita-Mumford classes

Igor Krichever (Columbia University)

Title: Elliptic solutions of matrix KP equation and complete cycles in the moduli spaces of algebraic curves of compact type

François Labourie (Université Paris-Sud)

Title: Coding simple geodesics and applications

Abstract: In a joint work with Ser Peow Tan (NUS-Singapore), we describe a coding for simple geodesics through a cusp on surfaces. This coding is given by a point on a certain combinatorial tree. As an application, and generalizing an idea of Bowditch, we show that hyperbolic metrics (or more generally cross ratios) give rise to certain harmonic measures on this tree and that the McShane—Mirzakhani identities are consequences of a simple Green formula. The coding by itself seems to be of independent interest.

Erwan Lanneau (Institut Fourier)

Title: Finiteness results for Teichmüller curves

Abstract: We will review some finiteness theorems for Teichmüller curves. This will be the occasion to discuss some classification theorems for these curves for some strata in low genera.

Elon Lindenstrauss (Hebrew University)

Title: Effective density of unipotent orbits

Abstract: Raghunathan conjectured that if G is a Lie group, Γ a lattice, p in G/Γ , and U an (ad-)unipotent group then the closure of $U \cdot p$ is homogeneous (a periodic orbit of a subgroup of G). This conjecture was proved by Ratner in the early 90's via the classification of invariant measures; significant special cases were proved earlier by Dani and Margulis using a different, topological dynamics approach. These results from homogeneous dynamics have been highly influential in the study of dynamics on moduli spaces of quadratic and abelian differentials, and in particular motivated the beautiful work of Eskin-Mirzakhani and Eskin-Mirzakhani-Mohammadi in this vein.

The proof of the Raghunathan conjecture given by Ratner as well as the proofs given by Dani and Margulis are not effective, nor do they provide rates --- e.g. if p is generic in the sense that it does not lie on a periodic orbit of any proper subgroup $L < G$ with $U \leq L$, these proofs do not give an estimate (possibly depending on diophantine-type properties of the pair (p, U)) how large a piece of an orbit is needed so that it comes within distance ϵ of any point in a given compact subset of G/Γ .

I will present work in progress with Margulis, Mohammadi and Shah towards giving an effective and quantitative density theorem for orbits of unipotent groups on homogeneous spaces

Howard Masur (University of Chicago)

Title: Counting elliptic elements in the mapping class group

Abstract: In a well-known paper Athreya-Bufetov-Eskin-Mirzakhani gave asymptotics for a counting problem in the mapping class group. They fixed a base point x in Teichmüller space and given another point y counted the number of image points of y under the group action that lie in a ball of radius R about x . In this talk I will discuss the corresponding problem when one restricts to

finite order elements in the group. I will give upper and lower bounds that are of smaller exponential growth. This is joint work with Spencer Dowdall.

Carlos Matheus (Université Paris 13)

Title: Quaternionic orthogonal groups in the monodromy of Teichmüller curves

Abstract: A recent work of Filip gives a list of all possible monodromies of (the blocks of) the Kontsevich-Zorich cocycle over the closure of any $SL(2, \mathbb{R})$ -orbit of any translation surface. As it turns out, this list builds on the classification of monodromies of variations of Hodge structures of weight one over quasiprojective varieties. Since the Kontsevich-Zorich cocycle is a particular case of variations of Hodge structures of weight one, it is not clear that all monodromy groups in Filip's list actually show up in the context of the $SL(2, \mathbb{R})$ -action on moduli spaces of translation surfaces. In this talk, we discuss a joint work with Filip and Forni about the realization of quaternionic orthogonal groups in the Kontsevich-Zorich cocycle over certain Teichmüller curves.

Martin Möller (Goethe Universität)

Title: Compactification of strata of Abelian differentials

Kasra Rafi (University of Toronto)

Title: Geometry and dynamics of the Thurston metric on Teichmüller space

Abstract: Teichmüller space can be equipped with a metric using the hyperbolic structure of a Riemann surface, as opposed to the conformal structure that is used to define the Teichmüller metric. This metric, which is asymmetric, was introduced by Thurston and has not been studied as extensively as Teichmüller metric or the Weil-Petersson metric. However, it equips Teichmüller space with a distinctive and rich structure. We give a survey of some recent results and discuss some open problems and conjectures.

Igor Rivin (Temple University)

Title: Random 3-manifolds

Abstract: We will discuss the "expected" behavior of 3-dimensional manifolds obtained by a mapping torus of a random map or a random Heegaard splitting (à la Dunfield-Thurston).

Corinna Ulcigrai (University of Bristol)

Title: Cutting sequences for Bouw-Möller surfaces

Abstract: The cutting sequence of a linear trajectory on a translation surface encodes the sequence of labels of sides of a polygonal presentation of the surface which are hit by the trajectory. In the case of a torus, cutting sequences of linear trajectories in a square coincide with the well known Sturmian sequences and can be characterized as limit of products of two substitutions, a characterization known as S-adic. We gave a similar characterization for cutting sequences on the

octagon and regular $2n$ -gons in joint work with J. Smillie. We consider here the family of Veech translation surfaces discovered by Bouw-Möller surfaces, obtained (as described by Hooper) by glueing a collection of semi-regular polygons. We provide for these sequences a full S -adic characterization governed by a continued fraction-like map. As for the square and $2n$ -gons, we exploit renormalization and a combinatorial notion of derivation. One of the novelties is that we describe intermediate steps to the action of Veech group elements and we crucially exploit as a technical tool Hooper diagrams. This is joint work with Diana Davis and Irene Pasquinelli.

Amie Wilkinson (University of Chicago)

Title: The Weil-Petersson metric, from a dynamical point of view

Alex Wright (Stanford University)

Title: $SL(2, \mathbb{R})$ -Invariant suborbifolds in the moduli spaces of Abelian differentials

Jean-Christophe Yoccoz (College de France)

Title: Diophantine conditions for interval exchange map

Abstract: We discuss diophantine conditions which arise in relation with Birkhoff sums, the cohomological equation and the linearization problem for interval exchange maps. This is related to several past and current collaborations with S. Marmi, P. Moussa and C. Ulcigrai.

Petr Zograf (Steklov Institute of Mathematics)

Title: Large genus asymptotics of the Weil-Petersson volumes of moduli spaces of curves

Abstract: The talk is based on a joint work with M. Mirzakhani and deals with the large genus behavior of the Weil-Petersson volumes of moduli spaces of n -pointed complex algebraic curves. The diverging factor in the large genus asymptotics is explicitly described, and, modulo a universal multiplicative constant, a complete asymptotic expansion of the volumes in the inverse powers of genus (with coefficients that are polynomials in n) is obtained. This can be done by analyzing various recursions for the more general intersection numbers of tautological classes on moduli spaces.

Dmitri Zvonkine (Jussieu)

Title: Homology classes of strata in spaces of differentials

Abstract: The main character of this talk is a compactified space of holomorphic or meromorphic differentials with zeros of prescribed multiplicities. We will present several results and conjectures on a family of related questions:

- Find the homology class of this locus in the space of all holomorphic or meromorphic differentials.
- Find the homology class of its image in the moduli space of curves.
- Find an analog of rubber maps and of double ramification cycles for differentials.

This is work in progress with Felix Janda, Rahul Pandharipande, Aaron Pixton and Adrien Sauvaget.