

Mirror symmetry is a phenomenon arising in string theory in which two very different manifolds give rise to equivalent physics. Such a correspondence has significant mathematical consequences, the most familiar of which involves the enumeration of holomorphic curves inside complex manifolds by solving differential equations obtained from a “mirror” geometry. The inclusion of D-brane states in the equivalence has led to further conjectures involving calibrated submanifolds of the mirror pairs and new (conjectural) invariants of complex manifolds: the Gopakumar Vafa invariants.

This book aims to give a single, cohesive treatment of mirror symmetry from both the mathematical and physical viewpoint. Parts 1 and 2 develop the necessary mathematical and physical background “from scratch,” and are intended for readers trying to learn across disciplines. The treatment is focussed, developing only the material most necessary for the task. In Parts 3 and 4 the physical and mathematical proofs of mirror symmetry are given. From the physics side, this means demonstrating that two different physical theories give isomorphic physics. Each physical theory can be described geometrically, and thus mirror symmetry gives rise to a “pairing” of geometries. The proof involves applying $R \leftrightarrow 1/R$ circle duality to the phases of the fields in the gauged linear sigma model. The mathematics proof develops Gromov-Witten theory in the algebraic setting, beginning with the moduli spaces of curves and maps, and uses localization techniques to show that certain hypergeometric functions encode the Gromov-Witten invariants in genus zero, as is predicted by mirror symmetry. Part 5 is devoted to advanced topics in mirror symmetry, including the role of D-branes in the context of mirror symmetry, and some of their applications in physics and mathematics: topological strings and large N Chern-Simons theory; geometric engineering; mirror symmetry at higher genus; Gopakumar-Vafa invariants; and Kontsevich’s formulation of the mirror phenomenon as an equivalence of categories.

This book grew out of an intense, month-long course on mirror symmetry at Pine Manor College, sponsored by the Clay Mathematics Institute. The lecturers have tried to summarize this course in a coherent, unified text.



For additional information
and updates on this book, visit

www.ams.org/bookpages/cmim-1

ISBN 0-8218-2955-6



9 780821 829554

CMIM/1

www.ams.org

www.claymath.org