
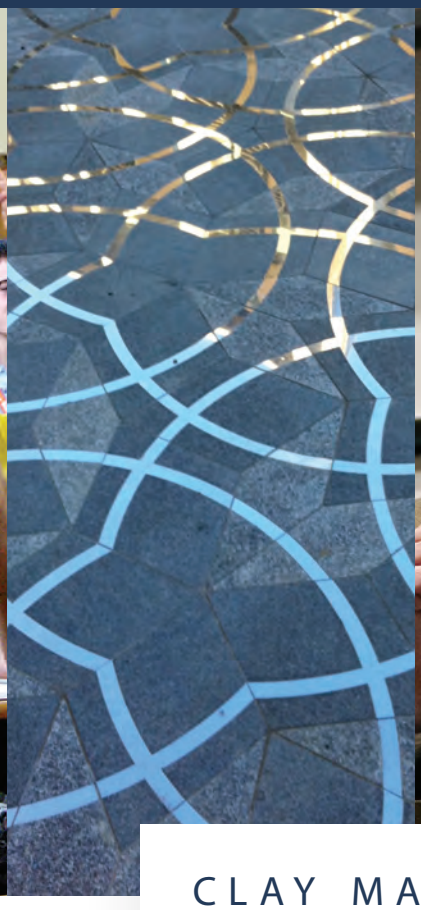

CLAY MATHEMATICS INSTITUTE

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ANNUAL REPORT

2013

Mission

The primary objectives and purposes of the Clay Mathematics Institute are:

- to increase and disseminate mathematical knowledge
- to educate mathematicians and other scientists about new discoveries in the field of mathematics
- to encourage gifted students to pursue mathematical careers
- to recognize extraordinary achievements and advances in mathematical research

The CMI will further the beauty, power and universality of mathematical thought.

The Clay Mathematics Institute is governed by its Board of Directors, Scientific Advisory Board and President. Board meetings are held to consider nominations and research proposals and to conduct other business. The Scientific Advisory Board is responsible for the approval of all proposals and the selection of all nominees.

CONTENTS

LETTER FROM THE PRESIDENT

We have just completed the first full year in which CMI has run its scientific program from offices in the Mathematical Institute at Oxford. In many respects the change of continent has had little impact. CMI remains a US corporation, now with its headquarters in Providence, RI. It continues to organize its financial affairs from the USA and it retains its status as a charitable private operating foundation, with all the opportunities (and constraints) that brings. The mission is unchanged, reflecting Landon Clay's hugely generous vision of supporting mathematical research at the highest level and of encouraging the next generation of world-leading mathematicians.

CMI continues to support the programs in which its has been involved for many years, in some cases at an increased level. PROMYS at Boston University and the Ross Program in Ohio remain central to its work with young mathematicians. It continues to award Clay Research Fellowships to exceptionally talented mathematicians at the outset of their mathematical careers: two were awarded in 2013, to Semyon Dyatlov, to be held at MIT, and Aaron Pixton, to be held at Harvard. It also continues to make annual Research Awards, to appoint Senior Scholars at research institutes in the USA and elsewhere, to be in partnership with the Independent University of Moscow, and to publish Monographs and Proceedings volumes in collaboration with the American Mathematical Society.

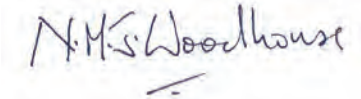
PHOTO COURTESY OF WADHAM COLLEGE

The offices at Bow Street in Cambridge, MA, are no longer available for workshops and other activities, but instead we now have access to the magnificent facilities in the new Andrew Wiles Building in Oxford.

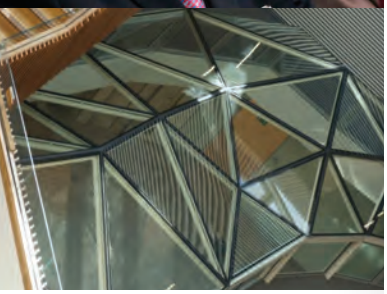
So what has changed? With an improvement in the global economy and a reduction in overheads through buying basic support services from the University of Oxford, more funds are available for the support of mathematics. There is a plan to increase the number of Clay Research Fellowships and to build up international activities. The new 'Enhancement and Partnership Program' adds value to conferences, summer schools, and other activities by paying the costs of distinguished speakers and supporting international participation. It leaves the administration and core costs with the partner organizations, but meets the costs of long-distance travel that may be beyond the reach of local resources. The year saw increasing activity under this heading. It also saw the launch of the CMI-PROMYS International Alliance, under which 11 European high school students joined the six-week summer school in Boston.

The high point of the year was the inauguration of a new format for the Clay Research Conference, which was held in Oxford in October, in conjunction with the opening of the Andrew Wiles Building. By combining the conference itself with four workshops held over the adjacent days, we were able to offer a hugely exciting program of specialist and general lectures, which drew many outstanding mathematicians to join with us in a double celebration.

Sincerely,



N. M. J. Woodhouse



PHOTOS OF AUDIENCE AND CLAYS
BY NICK HALE

ANNUAL MEETING

Clay Research Conference & Workshops

University of Oxford, UK

2013 CLAY RESEARCH CONFERENCE AND THE ANDREW WILES BUILDING

In September 2013, the CMI moved its president's office into the new Andrew Wiles Building at the University of Oxford. The 2013 Research Conference and the opening conference for the new building were held on consecutive days at the beginning of October, the two events being linked not only through Andrew Wiles, but also through the enormously generous philanthropy of Landon and Lavinia Clay, who were the lead donors for the new mathematics building.

The 2013 Research Conference was the first in a new format, which we hope will be repeated in years to come. It was built around four specialist workshops, which came together for a day of colloquium talks and for the presentation of the Clay Research Award. The idea was to provide more than is possible in the CRC itself to attract leading mathematicians and to provide them with the opportunity for discussion in depth of topical questions.

Two of the workshops focused on Millennium Prize Problems, one organized by Eric Allender on the current status of the P *vs* NP problem; the other by Peter Constantin, Gregory Seregin, and Idriss Titi centered on uniqueness and regularity problems for the Navier-Stokes equations, Euler equations, and related hydrodynamical models. Alongside these, Tamas Hausel, Emmanuel Letellier and Fernando Rodriguez Villegas ran a workshop with a broad focus on problems at the interface of physics and number theory. The fourth workshop, organized in conjunction with the Oxford Centre for Quantum Mathematics and Computation, also had a broad reach, covering a number of foundational ideas in mathematics, computer science and physics. Accounts of all four are given later in this annual report.

The CRC and the Opening Conference that ran on from it were delightful events, filling the brand new

Peter Constantin

360-seater auditorium with an enthusiastic and distinguished audience. Peter Constantin opened proceedings with an inspiring lecture on the Navier-Stokes equations, setting the question of the smoothness of solutions for large initial data in the context of a wider set of questions about turbulence and anomalous dissipation. He summarized progress towards the resolution of the central problem of regularity for solutions of the full equations in three dimensions, and as well as towards the solution of related problems for the Navier-Stokes problem under special conditions and for other models. Looking forward, he outlined open questions on boundary effects, on the connection between stability and regularity, and on stochastic approaches—the classical Navier-Stokes equations can be formulated in terms of expected values of a stochastic process, where the questions can be formulated, perhaps more tractably, in terms of what *normally* happens.

In 1220, Fibonnacci set the challenge of finding three rational squares in arithmetic progression with a difference of five. This is the problem of showing that 5 is a *congruent number*—that there is a right-angle triangle with rational sides and area 5. This in turn is equivalent to showing that a certain elliptic curve has positive rank. The problem of proving that 1 is not congruent was solved by Fermat by using his *descent* method.

In 1641, Fermat set the challenge of finding a Pythagorean triple a,b,c (i.e integers such a^2+b^2= c^2) in which c and a+b are squares.

In 1657, Fermat set the challenge of giving a rule to find an infinite number of solutions to x^2-Dy^2=1 where D is a non-square integer (Pell's equation).

(Solutions on page 6)

Lance Fortnow

Lance Fortnow gave a masterful talk on the history and current status of the P *vs* NP problem: 'Can we solve every problem quickly if the solutions are easily verifiable?'. He explained the mathematical robustness of the question in its formulation in terms of Turing machines and the key concept of NP-completeness. Progress has been made on related open questions that arise when one considers other problem classes than NP-complete and other weaker measures of complexity than solvability in polynomial time. Probabilistic and quantum computers offer interesting glimpses of significant new classes. But sadly the almost unimaginable practical benefits of proving that P = NP are matched only by the improbability of finding such a proof, or indeed of settling the question at all in the foreseeable future.

As Fortnow explained, the P *vs* NP problem is not just the only Millennium Problem that is not named after a person (noting, perhaps, that Stephen Cook was in the audience), it is the easiest to explain in simple terms.

The Birch–Swinnerton-Dyer (BS-D) conjecture is in another class. It is said to be hard to understand the statement, harder still to grasp the mathematical context and the deep insights that led Bryan Birch and Peter Swinnerton-Dyer to its formulation. It was fortunate, therefore, that Bryan Birch was available to give a fascinating extended introduction to the next talk, by Fernando Rodriguez Villegas, in which he spoke about how he and Swinnerton-Dyer had arrived at the conjecture. Villegas then talked about the current status of the problem and set out its historical and mathematical context with extraordinary clarity. His story began with the *Arithmetica* of Diophantus and the history of mathematical 'challenges' posed over the centuries from Fibonnacci in the thirteenth to Fermat in the seventeenth. These were much more than elaborate puzzles: they were influential in stimulating the development of key techniques in number theory, not least by Fermat himself.

Fernando Rodriguez Villegas

Villegas followed with an elementary account of the theory of elliptic curves, as far as the Mordell-Weil rank *r*. Moving on to the BS-D conjecture, he gave an account of classical work on Pell's equation, to motivate the conjecture by analogy via Tamagawa numbers, and to formulate it correctly. Great progress has been made using Heegner points, beginning with the work of Gross and Zagier in the 1980's. Combined with results of Kolyvagin, Rubin and Skinner, the conjecture is now essentially known for $r \leq 1$. This Heegner point approach, however, collapses for larger values of *r* (the rank *r* of an elliptic curve may or may not be bounded—an example is known with $r = 28$). New results from Manjul Bhargava and collaborators have established that the average of *r* is positive and less than 0.89. It is now known that the BS-D conjecture is

PHOTOS BY NICK HALE

Edward Witten

Ingrid Daubechies

true for at least 20% of elliptic curves (rank 0 or 1).

In the final full lecture on the first day, Edward Witten took us in a different direction, with a *tour de force,* which he concisely and modestly summarized as follows: 'I describe a gauge theory approach to understanding quantum knot invariants as Laurent polynomials in a complex variable *q*. The two main steps are to reinterpret three-dimensional Chern-Simons gauge theory in four-dimensional terms and then to apply electric-magnetic duality. The variable *q* is associated to instanton number in the dual description in four dimensions.' This hardly does justice to the extraordinary range of ideas and techniques from mathematics and theoretical physics on which he drew in his journey from an elementary starting point in the classical theory of knots. A second lecture in the Number Theory and Physics workshop took the story further, describing how 'Khovanov homology can emerge upon adding a fifth dimension'.

The day finished with the presentation of the Clay Research Award to Rahul Pandharipande and Richard Thomas's introduction to his work (see page 11). The next day, the Clay Research Conference merged into Opening Conference and formal opening by Michael Atiyah of the Andrew Wiles Building. The link between the two conferences was provided by Ingrid Daubechies, who spoke in a joint session. Her playful title, *Animation, teeth and skeletons*, hid the more serious topic of her alternative title, *Distances between surfaces, using discrete differential geometry.* A common problem that crops up in many contexts (the making of *Toy Story* and the classification of common features in the teeth of lemurs being typical examples) is that of quantifying the closeness, or lack of it, of two surfaces represented by discrete information. Do two point clouds correspond to the same surface? If not, how far apart are they? How do you find corresponding points on the two surfaces? There are existing measures, such as the Gromov-Hausdorff distance, but they can be computationally onerous. Daubechies described a new approach based on conformal geometry and optimal mass transport, in which the surface meshes used in discrete approximation are constructed by mid-edge triangulation.

It was an entertaining and thought-provoking lecture, illustrating how serious mathematical questions can arise from what at first seem unlikely sources, in this case the problem of motion capture and of generating visually plausible animations.

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CLAY RESEARCH CONFERENCE WORKSHOPS

The Navier-Stokes Equations and Related Topics

September 29 – October 1, 2013

This three-day workshop brought together world-leading experts in the field of mathematical hydrodynamics to discuss recent developments in, and future directions of, research in this very active area of applied analysis.

Topics included: the interplay between computation and analysis in the study of 3D incompressible Euler and Navier-Stokes equations (Hou); non-blow up criterion involving vorticity direction under the non-slip boundary condition for the 3D Navier-Stokes flow (Giga); fluid models as scaling limits of systems of particles (Saint-Raymond); regularity and mixing for active scalars (Kiselev); long-time behavior of forced critical SQC (Vicol); large time behaviors of the water wave motion (Wu); nonlinear inviscid damping in 2D Euler (Masmoudi); breakdown for fluid interfaces (Cordoba) and the nearly complete solution of the Onsager

WITTEN PHOTO BY NICK HALE
DAUBECHIES PHOTO BY JOHN CAIRNS

conjecture about the energy preserving of the solutions to 3D Euler's equations (De Lellis).

The workshop facilitated an exciting exchange of ideas, with new ideas emerging as a result of the interaction between the participants, who represented the broad mathematical analysis community working on various aspects of hydrodynamics. As a result, there is a better understanding of further directions in the rigorous analysis of PDEs related to mathematical hydrodynamics.

Organizers
Peter Constantin (Princeton University)
Gregory Seregin (University of Oxford)
Edriss Titi (The Weizmann Institute of Science and University of California, Irvine)

Speakers
Diego Cordoba (ICMAT)
Camillo De Lellis (Universität Zürich)
Yoshikazu Giga (University of Tokyo)
Thomas Yizhao Hou (California Institute of Technology)
Alexander Kiselev (University of Wisconsin)
Nader Masmoudi (New York University)
Laure Saint-Raymond (ENS Paris)
Vlad Vicol (Princeton University)
Siju Wu (University of Michigan)

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New Insights into Computational Intractability

September 30 – October 4, 2013

Computational complexity theory deals with the central mystery of computation: What is feasible to compute, and what computational tasks will forever remain out of reach? Although the main open questions (such as the Millennium Problem of P *vs* NP) continue to elude us, exciting progress has been reported on a number of fronts recently.

This workshop brought together many leading researchers in the field of computational complexity theory, providing a forum where they were able to share their perspectives on the new insights that have been obtained. Talks included presentations on the GCT chasm (Mulmuley), geometric complexity theory (Bürgisser), real algebraic geometry and computational complexity (Miltersen), meta-computational problems as a link between algorithms and lower bounds (Impagliazzo), QBF algorithms and circuit lower bounds (Santhanam), algorithms for circuits and circuits for algorithms (Williams), the #CSP dichotomy (Dyer), the complexity of approximate counting (L. Goldberg), phase transitions and computational intractability (Jerrum), PCPs and direct products (Dinur), structure in satisfiability (Jeavons), communication complexity and information complexity (Pitassi), computational complexity and physics (Aaronson), monotone computation and economics (Koutsoupias), data privacy (Vadhan), polynomial identity testing for small depth circuits (Agrawal), the complexity of "path-following" algorithms for equilibria and related problems (P. Goldberg), the monotone duality problem (Gottlob), the comparator circuit value problem (Cook), reverse Newman's theorem in interactive information complexity (Buhrman), and complexity classes via algorithmic information theory (Allender).

Solutions to problems on page 4:

Fibonacci:
(31/12)^2, (41/12)^2, (49/12)^2

Fermat:
a= 4565486027761,
b= 1061652293520,
c=4687298610289

Many participants provided enthusiastic feedback to the organizer, commenting on the "fantastic conference" with a "nice mix of people and interesting talks". Several speakers reported on receiving valuable comments on their presentations from other participants, leading to promising avenues for further investigations. Student participants were especially appreciative of the opportunity to interact with leading researchers in the field, and of the financial support that allowed them to attend the workshop.

Organizer
Eric Allender (Rutgers University)

Speakers
Scott Aaronson (Massachusetts Institute of Technology)
Eric Allender (Rutgers University)
Manindra Agrawal (IIT Kanpur)
Harry Buhrman (CWI, Amsterdam)
Peter Bürgisser (Technische Universität Berlin)
Stephen Cook (University of Toronto)
Irit Dinur (The Weizmann Institute of Science)
Martin Dyer (University of Leeds)
Lance Fortnow (Georgia Institute of Technology)
Leslie Ann Goldberg (University of Oxford)
Paul Goldberg (University of Oxford)
Georg Gottlob (University of Oxford)
Russell Impagliazzo (University of California, San Diego)
Peter Jeavons (University of Oxford)
Mark Jerrum (Queen Mary, University of London)
Elias Koutsoupias (University of Oxford)
Peter Bro Miltersen (Aarhus University)
Ketan Mulmuley (University of Chicago)
Toniann Pitassi (University of Toronto)
Rahul Santhanam (University of Edinburgh)
Salil Vadhan (Harvard University)
Ryan Williams (Stanford University)

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Number Theory and Physics

September 30 – October 4, 2013

This workshop explored the interactions between number theory and physics, specifically focussing on four main points of contact: *L*-function, random matrix theory and quantum chaos; number theory, representation theory and Donaldson-Thomas invariants; Feynman diagrams and multi-zeta values; and the geometric and arithmetic Langlands program. The topics, of current interest and activity, share a significant intersection with both number theory and physics, and strongly influence and enrich one another.

Each morning an expository talk addressed one of these topics: Brown discussed quantum field theory and arithmetic, Bhargava talked about representation theory and rational points on elliptic and hyperelliptic curves, Witten gave a presentation on Khovanov homology and guage theory and Okounkov discussed membranes and sheaves.

The afternoon sessions were devoted to more specialized talks, including: random matrices and L-functions (Keating); periods and special functions of renormalizable quantum fields (Kreimer); primes, polynomials over finite fields, and matrix integrals (Rudnick); topological recursion and applications (Borot); Feynman diagrams and twistor diagrams (Hodges); DT-invariants for Hitchin systems (Kontsevich); motivic TQFT (Gukov); non-abelian zeta functions and stable pairs on curves via wall-crossing (Reineke); and relative cohomology of the Hitchin fibration (Laumon).

The talks were broad, with an emphasis on the interplay between number theory and physics, and the speakers were encouraged to include bold speculations and their vision for the future of the subject.

Organizers
Tamás Hausel (EPFL)
Emmanuel Letellier (University of Caen)
Fernando Rodriguez Villegas (University of Texas at Austin and ICTP)

Speakers
Manjul Bhargava (Princeton University)
Gaetan Borot (Université de Genève)
Francis Brown (Institut de Mathématiques de Jussieu)
Philip Candelas (University of Oxford)
Sergei Gukov (California Institute of Technology)
Andrew Hodges (University of Oxford)
Jon Keating (University of Bristol)
Maxim Kontsevich (Institut des Hautes Études Scientifiques)
Dirk Kreimer (Humboldt University)
Gérard Laumon (Paris Sud University, Orsay)
Andrei Okounkov (Columbia University)
Marcus Reineke (Bergische Universität Wuppertal)
Zeev Rudnick (Tel Aviv University)
Edward Witten (Institute for Advanced Study)

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Quantum Mathematics and Computation

October 1 – 4, 2013

In the autumn of 2011, a group of Oxford mathematics and computer science faculty recognized the deep and unexploited connections between their mutual research interests in algebra, topology, and quantum computer science, and formulated a vision for a centre to unify their efforts to understand the mathematical and computational nature of quantum phenomena. This workshop marked the launch of the new Centre for Quantum Mathematics and Computation (QMAC), bringing together world experts to take stock of contemporary foundational ideas in mathematics, computer science, and physics.

Particular attention was devoted to questions concerning the theory and mathematical application of local topological quantum field theory, and an investigation of how modern mathematical perspectives on quantum theory could be advanced through an understanding of the geometry of type theory. The presentations included talks on: spans and the categorified Heisenberg algebra (Baez), homotopy type theory and univalent foundations (Awodey), entangled photons in the large (Zeilinger), stochastic Lambda-calculi

(Scott), functional interpretations of type theory (Hyland), singular support of l-adic sheaves (Beilinson), operationalism, probabilities, agency and composition (Hardy), the logic of quantum mechanics (Coecke), geometry in a symmetric monodial category (Kremnitzer), contextual semantics (Abramsky), and matter, topology, and computation (Simon). In addition to formal presentations and research activities, the workshop incorporated a number of informal events, such as problem sessions and research and outreach planning activities.

The workshop brought researchers and students from diverse technical backgrounds into a dynamic conversation about the fundamental questions of quantum mathematics and computation, sparking new connections (such as between categorical and simplicial methods and univalent type theory, and between representation theory and condensed matter classification), publicizing crucial problems and approaches, and forging future scientific collaborations.

Organizers
Samson Abramsky (University of Oxford)
Bob Coecke (University of Oxford)
Michael Collins (University of Oxford)
Christopher Douglas (University of Oxford)
Yakov Kremnitzer (University of Oxford)
Steve Simon (University of Oxford)
Ulrike Tillmann (University of Oxford)
Jamie Vicary (University of Oxford)

Speakers
Samson Abramsky (University of Oxford)
Steve Awodey (Carnegie Mellon University)
John Baez (University of California, Riverside)
Alexander Beilinson (University of Chicago)
Bob Coecke (University of Oxford)
Lucian Hardy (Perimeter Institute)
Martin Hyland (University of Cambridge)
Yakov Kremnitzer (University of Oxford)
Dana Scott (Carnegie Mellon University)
Steve Simon (University of Oxford)
Vladimir Voevodsky (Institute for Advanced Study)
Anton Zeilinger (University of Vienna)

Rahul Pandharipande

Recognizing extraordinary achievement

Clay Research Awards

The Clay Millennium Prize problems have given the Clay Mathematics Institute a high public profile. Less well known outside of the mathematical world, but widely appreciated within it, are the annual Clay Research Awards. These celebrate the outstanding achievements of the world's most gifted mathematicians.

In 2013 the Clay Research Award was presented to Rahul Pandharipande for his recent outstanding work in enumerative geometry, specifically for his proof in a large class of cases of the MNOP conjecture that he formulated with Maulik, Okounkov, and Nekrasov.

Since its inception in 1999, 31 individuals have been recognized for the major contemporary breakthroughs they have made in mathematical research. The standard has been very high. Four of the ten Fields Medals awarded since the foundation of the CMI have gone to people who had previously received Clay Research Awards. Indeed, six of the 31 Clay awardees are also Fields Medallists.

PHOTOS BY NICK HALE

Richard Thomas

The MNOP Conjecture

by Richard Thomas

Rahul Pandharipande (ETH, Zürich) was given the 2013 Clay Research Award for his recent outstanding work in enumerative geometry, specifically for his proof with Aaron Pixton of the famous MNOP conjecture for most Calabi-Yau 3-folds. The conjecture relates two methods of counting curves in an algebraic variety, one given by Gromov-Witten theory and the other by Donaldson-Thomas invariants, and was formulated by Rahul a decade ago with Davesh Maulik, Nikita Nekrasov and Andrei Okounkov.

The MNOP conjecture has become one of the most active areas of both geometry and mathematical string theory in the last decade. It is a magical conjecture, relating Gromov-Witten theory (counting maps of curves into algebraic varieties) to a sheaf theory counting embedded curves (more generally subschemes) of the variety. It is philosophically natural and pleasing that the two ways of counting holomorphic curves (via parametrised curves, or unparametrised curves cut out by equations, as the slogan goes) should be equivalent, but the formula relating them is quite extraordinary and not really direct, involving an analytic continuation.

Clay Research Awardees

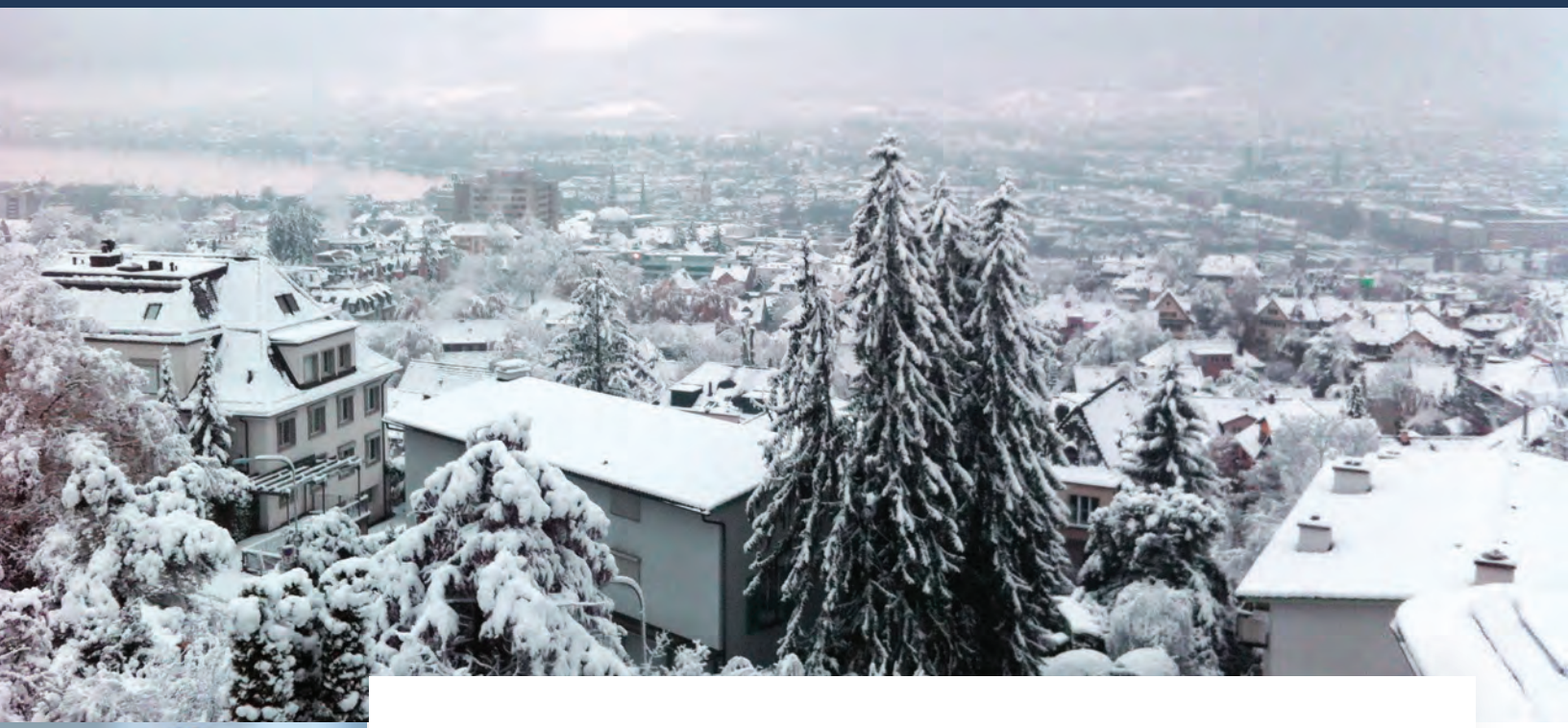
2013	Rahul Pandharipande
2012	Jeremy Kahn and Vladimir Markovic
2011	Yves Benoist and Jean-François Quint Jonathan Pila
2009	Jean-Loup Waldspurger Ian Agol, Danny Calegari and David Gabai
2008	Cliff Taubes Claire Voisin
2007	Alex Eskin Christopher Hacon and James McKernan Michael Harris and Richard Taylor
2005	Manjul Bhargava Nils Dencker
2004	Ben Green Gérard Laumon and Ngô Bao Châu
2003	Richard Hamilton Terence Tao
2002	Oded Schramm Manindra Agrawal
2001	Edward Witten Stanislav Smirnov
2000	Alain Connes Laurent Lafforgue
1999	Andrew Wiles

With Maulik, Alexei Oblomkov and Okounkov he proved it for toric varieties; a tour de force combining clever geometric degeneration arguments with combinatorics and representation theory. This gave the faint hope of proving the conjecture for Calabi-Yau 3-folds that can be degenerated to unions of toric varieties glued over their boundaries. However, this would require a stronger MNOP conjecture including all the necessary gluing data. This became possible with the development of "stable pairs", a theory he developed with Thomas. It runs parallel to DT theory but with certain advantages which make the theory more efficient. In particular the formulae involved in the MNOP conjecture become more transparent and comprehensible. The stable pairs theory also made contact with exciting new developments in "wall-crossing formulae" due to Joyce and Kontsevich-Soibelman, allowing Bridgeland and Toda to prove that stable pair invariants and DT invariants are equal.

This left formulating and proving the stronger version of the MNOP conjecture (including "relative conditions" and "descendents") for stable pairs and Gromov-Witten invariants. In a series of papers, Pandharipande and his student Pixton did just that. The work is an amalgam of algebraic geometry (degeneration and special geometries), localisation, combinatorics, representation theory, number theory, magical manipulations and identities, nested inductions and so on. The upshot is a complete proof for Calabi-Yau 3-folds admitting appropriate degenerations, including probably all known Calabi-Yau 3-folds, and perhaps all of them full stop.

The final result could perhaps be compared to a proof of the Witten conjecture relating Seiberg-Witten and Donaldson invariants. It is a high point of the modern interaction between geometry and string theory, but one in which Rahul beat the string theorists to it!

PHOTO BY NICK HALE



The earliest memory I have about mathematics is my grandmother teaching me how to factor integers into primes during the monsoons in Mumbai.

PHOTO COURTESY OF ETH ZÜRICH

PROFILE

Interview with Rahul Pandharipande

2013 Clay Research Awardee

What first drew you to mathematics? What are some of your earliest memories of mathematics?

I was interested in mathematics from a young age. I grew up in a mathematical family, my father was a theoretical physicist and my grandfather was a mathematician. The earliest memory I have about mathematics is my grandmother teaching me how to factor integers into primes during the monsoons in Mumbai. As a child, most of my mathematical education happened at home, but I remember learning about straight edge and compass constructions in a geometry class in school. I was fascinated by the mix of geometry and algebra involved there. That the subject was completely solved by Galois theory came as a bit of a disappointment.

Could you talk about your mathematical education? What experiences and people were especially influential?

I went to a high school on the campus of the University of Illinois (in Urbana). There were several interesting mathematical personalities among the teachers and many very strong students. All the students had the possibility of taking courses at the University—I took many and had finished much of the standard mathematics curriculum before graduating from high school. As an undergraduate at Princeton, two courses made a big difference. In my second year, Bill Thurston taught a geometry class which met for an afternoon once a week in his house, near the mathematics department. He had an amazing camera in his head—to see someone interact with geometry like that was an inspiration. An algebraic geometry course by Nick Katz in my third year marked the beginning of my interest in the subject. He guided my early study and explained how much there was to learn.

Did you have a mentor? Who helped you develop your interest in mathematics, and how?

In graduate school at Harvard my advisor was Joe Harris. I learned a lot of classical techniques in algebraic geometry: curves, surfaces, invariant theory (with careful study of many particular examples). While this has not been the main subject of my research, the knowledge has been of constant use to me. In my years as a post-doc at the University of Chicago in the group of Bill Fulton, there was a lot of activity related to algebraic cycles and intersection theory. Those were great years for me and have influenced almost all of my subsequent mathematical research. I have moved around a lot and learned from many different people since then, especially my collaborators and students.

The assertion that mathematical knowledge and insights play a significant role in a wide range of human affairs is hardly controversial. However the motivation behind mathematical research often comes from a different source: the joy of exploration and discovery.

From your own experience at high school, are there any aspects of mathematics education that you would like to see changed?

Since I took courses at the University, I did not have a usual high school experience. I was fortunate to be surrounded by many mathematically knowledgeable people.

Can you describe your research in accessible terms?

Most of my research concerns the algebraic geometry of moduli spaces. Algebraic geometry is the study of the zero sets of polynomial equations in several variables. The subject has a central role in mathematics with connections to number theory, representation theory, and topology. Moduli questions in algebraic geometry concern the behavior of the solution sets as the coefficients of the polynomials vary. At the end of the 20th century, several fundamental links between the algebraic geometry of moduli spaces and path integrals in quantum field theory were made. The subject today uses insights and techniques with origins in both mathematics and physics.

One of the simplest objects in algebraic geometry is a curve (the solution of one polynomial equation in two variables). The moduli space of curves is perhaps the oldest and most interesting moduli space in mathematics. Its study can be traced back to papers of Riemann. I have been interested for a long time in the cohomology (and cycle theory) of the moduli of curves—loosely speaking "what are the holes in the space?". These questions can sometimes be formulated as integration problems over the moduli space. A discovery made with D. Maulik, N. Nekrasov, and A. Okounkov more than a decade ago was a conjectural relationship of such integrals to the cohomology of a completely different moduli space of sheaves. This equivalence has many applications to concrete integration questions. With Aaron Pixton, we showed how to prove this Gromov-Witten/Donaldson-Thomas conjecture in many new cases by degeneration methods.

How do you think mathematics benefits culture and society?

For a long time now, we have been living in a society directly underpinned by numbers, patterns and algorithms. The assertion that mathematical knowledge and insights play a significant role in a wide range of human affairs is hardly controversial. However the motivation behind mathematical research often comes from a different source: the joy of exploration and discovery. There is an immense and beautiful mathematical landscape with all sorts of peaks, passes and hidden valleys.

Summary of 2013 Research Activities

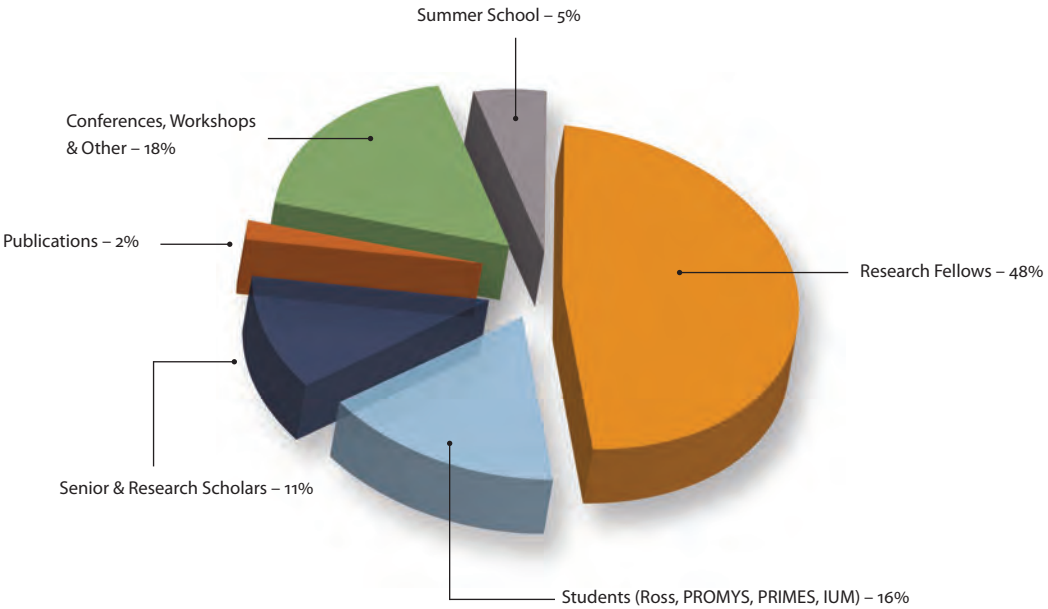
The activities of CMI researchers and research programs are sketched on the following pages. Researchers and programs are selected by the Scientific Advisory Board (see inside front cover).

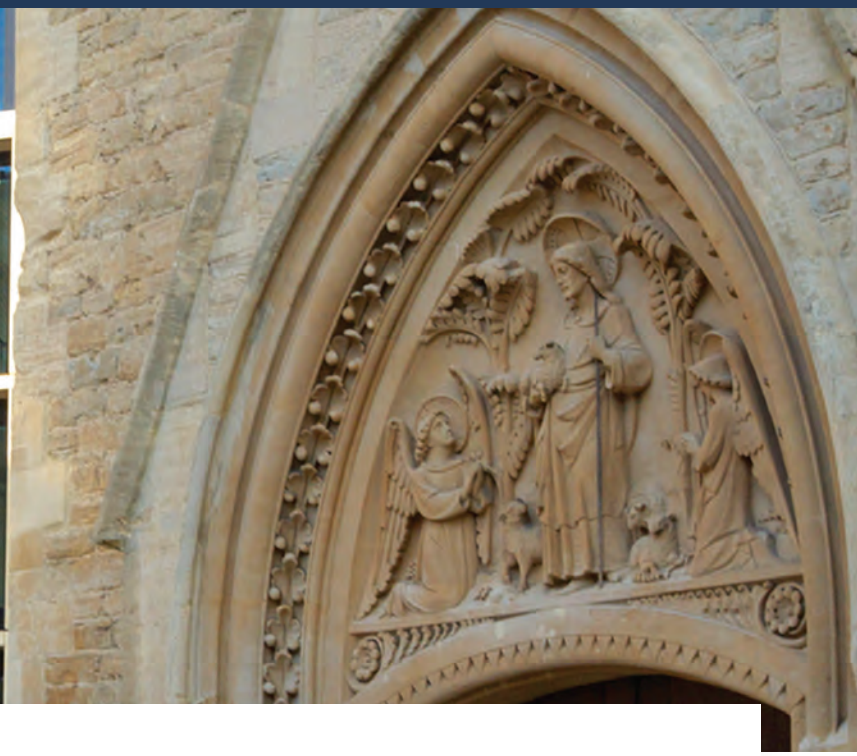
PROGRAM ALLOCATION

Estimated number of persons supported by CMI in selected scientific programs for calendar year 2013

- Research Fellows, Research Awardees, Senior Scholars, Research Scholars: 16
- CMI Workshops: 203
- PROMYS/Ross/PRIMES Faculty and Participants: 52
- Participants attending Conferences and Joint Programs: >250
- Independent University of Moscow: 95

RESEARCH EXPENSES FOR FISCAL YEAR 2013





Researchers

CLAY RESEARCH FELLOWS

Top: Semyon Dyatlov
Above: Aaron Pixton

Semyon Dyatlov received his PhD from the University of California, Berkeley under the supervision of Maciej Zworski. He applies the methods of microlocal analysis and dynamical systems to problems in scattering theory, quantum chaos, and mathematical general relativity. One of his research interests is scattering resonances, which are complex numbers generalizing the concept of bound states to open systems. Resonances appear in particular when studying long-time behavior of linear waves on noncompact manifolds, or decay of classical correlations for chaotic flows. In his thesis, Semyon develops a new microlocal framework to describe asymptotics of resonances and phase space concentration of associated resonant states under dynamical assumptions motivated by the wave equations on rotating black holes. Semyon received his BSc from Novosibirsk State University in 2008. He was a member of MSRI in Fall 2013 and is currently at MIT.

Aaron Pixton received his PhD in 2013 from Princeton University under the supervision of Rahul Pandharipande. His research is in enumerative algebraic geometry. The topics he has worked on recently include the tautological ring of the moduli space of curves, moduli spaces of sheaves on 3-folds, and Gromov-Witten theory.

Semyon Dyatlov and Aaron Pixton joined CMI's 2013 group of research fellows: Tim Austin (New York University), Ivan Corwin (Columbia University), Sucharit Sarkar (Princeton University), Peter Scholze (University of Bonn) and Jack Thorne (Harvard University).

Research Scholars

Roman Travkin
2012-2015
University of Chicago

Kiran Kedlaya
2013-2014
University of California
at San Diego

Senior Scholars

Toby Stafford (MSRI)
January – May 2013
Noncommutative Algebraic Geometry and Representation Theory

Richard Schoen (PCMI)
June 30 – July 20, 2013
Geometric Analysis

Gerhard Huisken (PCMI)
June 30 – July 20, 2013
Geometric Analysis

Robert Griffiths (CRM)
July – October 2013
Biodiversity and Evolution

Cedric Villani (MSRI)
August – December 2013
Optimal Transport: Geometry and Dynamics

Vincent Moncrief (MSRI)
September – November 2013
Mathematical General Relativity

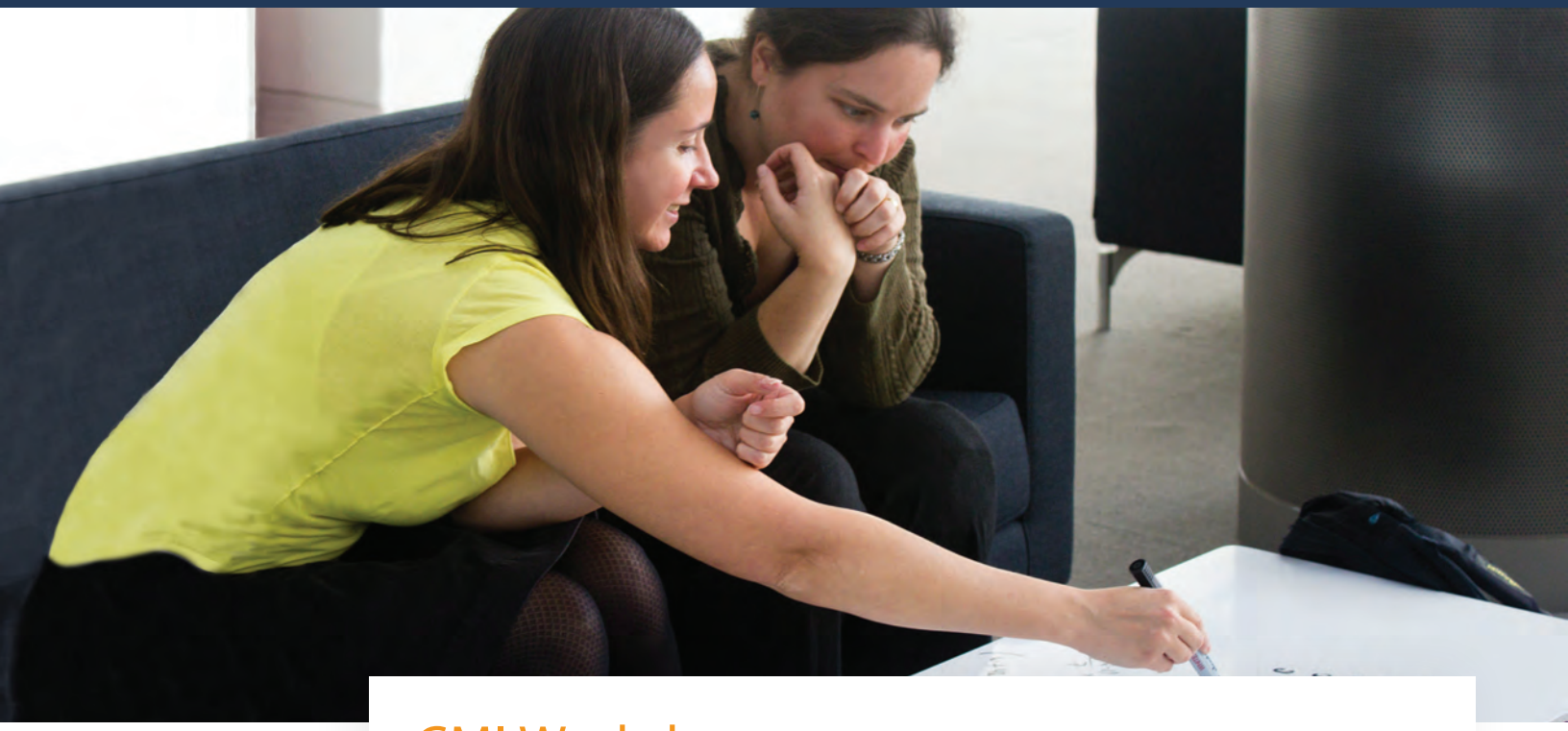


PHOTO COURTESY OF JENNIFER BALAKRISHNAN

CMI Workshops

CMI conducts a program of workshops at the Mathematical Institute in Oxford, UK which bring together a small set of researchers quickly, outside the usual grant and application cycle, when this is likely to result in significant progress.

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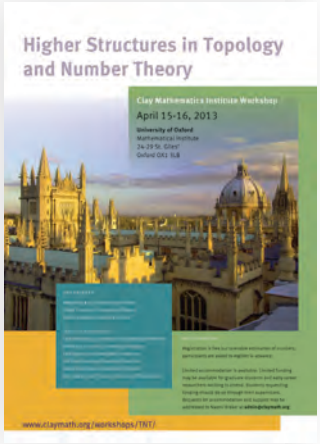
Higher Structures in Topology and Number Theory



April 15-16, 2013

This event was a satellite of the *Grothendieck-Teichmüller Groups, Deformations and Operads* program taking place at the Isaac Newton Institute in Cambridge, UK between January and April 2013. It brought together research communities working on realization of higher algebraic structures in number theory and topology, as well as geometry and mathematical physics.

Don Zagier

PHOTO BY HUDSTON

The workshop featured eight lectures by mathematicians working at the interface of topology, number theory, and geometry, unified by varying degrees of contact with higher algebraic structures. The breadth of the mathematics contained in this short sequence is indicated by the beginning lecture of Ezra Getzler on *Higher Stacks in Analytic Geometry* and the concluding one that featured Don Zagier speaking on the *Arithmetic of Quantum Knot Invariants*. The intervening presentations covered: the double trace (Henriques); Etale homotopy and diophantine equations (Schlank); Galois action on knots (Furusho); pro-algebraic l-adic homotopy groups (Pridham); tropical geometry and scheme theory (Giansiracusa); and the Grothendieck-Teichmüller group and the stable symplectic category (Kitchloo).

The high level of sophistication of the lectures was in keeping with the highly abstract theme of the workshop. On the other hand, the opportunity for people viewing the same essential structures from a variety of angles and manifestations to meet and discuss their perspectives, and to consider opportunities for future collaboration proved invaluable.

Organizers
Minhyong Kim (University of Oxford)
Ulrike Tillmann (University of Oxford)
Bruno Vallette (University of Nice)

Speakers
Hidekazu Furusho (University of Nagoya)
Ezra Getzler (Northwestern University)
Jeff Giansiracusa (Swansea University)
André Henriques (Universiteit Utrecht)
Nitu Kitchloo (Johns Hopkins University)
Jonathan Pridham (University of Cambridge)
Tomer Schlank (Massachusetts Institute of Technology)
Don Zagier (Max Planck Institute for Mathematics and Collège de France)

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Developing Exceptional Talent in Mathematics

August 16 – 18, 2013

The workshop brought together invitees from the US and Europe to share their extensive experience, accumulated wisdom, questions and sometimes divergent opinions relating to the identification, encouragement, and training of exceptionally gifted young mathematicians. It was attended by representatives of a number of European programs for young mathematicians, as well as alumni of PROMYS and others with personal experience of work with talented high school students.

It was hugely useful to hear about different approaches to the problem that is common to the diverse education systems from which the attendees were drawn, that of nurturing the talent and enriching the mathematical lives of the small proportion of highly exceptional young mathematicians within an educational environment that has to cater for the needs of all. PROMYS, in which the CMI has been a partner for many years, stands on one end of the spectrum, in offering an extended period of immersion in mathematics over six weeks of the summer. At the other are programs such as MATHEON in Berlin, which focuses on the school curriculum, teacher training, and the development of specialized classes in close cooperation with the Berlin universities. There is also a divergence of emphasis, between those that offer collaborative work on open-ended 'research problems' and those that develop talent through training for competitions, notably the International Olympiad. But they are not mutually exclusive: the International Tournament for Young Mathematicians (ITYM), for example, is a competition based on teamwork and problems of a 'research character'.

The divergence, sometimes more of perception than anything deeper, prompted lively debate. Some of the questions touched on the underlying philosophy: do competitions promote the habits of mind needed for later success? Do 'immersion' programs do no more than accelerate the talented along a path that they would follow in any case? Is it important to follow the Cambridge Mathematics Education Project in drawing a distinction between acceleration and enrichment? Others touched on more practical issues concerning the use of resources, and on concerns that programs might compete unproductively in fundraising and recruitment.

A central problem is the meaning of 'exceptional'. First, there is the simple numerical problem of definition, highlighted by Martin Andler, who runs the French program Animath, and illustrated by his table showing

percentages of the French population. In the context of the general population everyone who uses mathematics in a serious way in his or her working life is 'exceptional'.

Fields Medals	Research Mathematicians	Academic Researchers in Quantitative Science	STEM Teachers, Engineers, Scientists	High School Graduates in STEM Subjects	High School Graduates
0-0.00002%	0.01%	0.10%	10.00%	20.00%	66.00%

A program designed for future potential Fields Medallists may not appear to have much in common with one aimed at enriching the education of the next generation of teachers, scientists and engineers. But, as Owen Patashnick (a PROMYS alumnus) remarked, the methods based on proof and collaboration on hard problems can also work with weaker students. He recounted the successful application of the PROMYS approach in teaching remedial students struggling to achieve college admission. The focus should perhaps be less on exceptional talent than on exceptional motivation, a point made strongly by Po-Shen Loh, the Lead Coach for the US International Olympiad Team.

Part of the discussion was around the specific issue of whether the PROMYS model could work in a European context. There are clearly practical obstacles, notably the problem of finding an appropriate six-week period during the summer that fits with all the different academic schedules of European countries. But the duration of the Boston summer schools is not their defining feature, nor is their focus on number theory. The distinctive elements are the engagement of alumni, the return visits in subsequent years as participants progress through high school, college, and, in some cases, their professional careers; and the focus on collaborative exploration ahead of lectures. Another PROMYS alumna, Claudia Scheimbauer, remarked on the important lessons she had taken from the program, that you do not have to be told what to do and that there was more than one way to do mathematics. Much of this philosophy, as well as the emphasis on proof, also finds expression in other programs.

Perhaps the most important elements of PROMYS are the leadership of the program and its success in selection of participants. All the programs represented at the workshop depend critically on the willingness of talented people to give freely and copiously of their time and expertise. The central problem in maintaining successful programs and developing new ones is in identifying leaders. There is no shortage of demand for participation.

Joshua Greene, a PROMYS alumnus now working outside the academic community, summarized his impressions as outside observer: "Several questions stand out: what do we mean by mathematical talent, how do we identify talent, what does it mean to develop that talent, and why do program leaders volunteer so much in this effort?". There is no single metric which captures the variety of mathematical ability. There is no one true form of mathematical talent and no one right way to nurture it. Above all, it is clear that the mathematical community owes a huge debt to program leaders with a passion for public service in the mathematical community, often driven by their own experience in enrichment programs.

Organizers
David Conlon (University of Oxford)
Joshua Greene (COMAC Capital LLP)
Jürg Kramer (Humboldt University of Berlin)
Dierk Schleicher (Jacobs University)

Glenn Stevens (Boston University)
Nicholas Woodhouse (Clay Mathematics Institute)

Programs and organizations represented at the workshop

Animath: Association pour l'Animation Mathématique. www.animath.fr

ARK Schools: www.arkschools.org

CMEP: Cambridge Mathematics Education Project. www.maths.cam.ac.uk/cmep

EGMO: European Girls' Mathematical Olympiad. https://www.egmo.org

ESTALMAT: Estímulo Talento Matemàtico. www.estalmat.org

ISSMYS: International Summer School of Mathematics for Young Students. www.issmys.eu

King's College London Maths School: www.kcl.ac.uk/mathsschool/Home.aspx

MATHEON: Mathematik für Schlüsseltechnologien. www.matheon.de

NRICH: nrich.maths.org/frontpage

Oxford Masterclasses: www.wadham.ox.ac.uk/news/2013/august/developing-exceptional-talent-in-maths

PROMYS: Program in Mathematics for Young Scientists. www.promys.org

UKMT: UK Mathematics Trust. www.ukmt.org.uk

Westminster School: https://www.westminster.org.uk/

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Mixed Hodge Modules and Their Applications

August 19 – 23, 2013

The theory of mixed Hodge modules represents a vast generalization of classical Hodge theory. It was introduced by Morihiko Saito in two long papers in 1988 and 1990, building on the foundations created by many people during the 1970s and 1980s, especially in the study of variations of mixed Hodge structures and their degenerations, perverse sheaves, and *D*-module theory. Over the years, new applications of mixed Hodge modules came to light, but in spite of its power, the theory is not as widely known as it should be.

The goal of the workshop was to make Saito's theory more accessible by bringing together a group of people to discuss both the fundamentals of the theory and its applications. While a handful of participants were experts in this topic (including Saito himself), the majority consisted of experts in other fields, who used some aspects of the theory and who wanted to get a better understanding of its inner workings and of the existing applications.

Presentations on the first day provided an overview of the main notions in the theory: Sabbah gave an introduction to pre-Hodge modules, Arapura gave an overview of polarized Hodge modules, and Mochizuki presented an overview of mixed Hodge modules. Morihiko Saito himself gave a talk on Tuesday discussing a new simplifying approach to the definition of mixed Hodge modules in the algebraic case, which he recently developed.

The remaining talks fell into two categories: talks discussing some of the main results in the theory of mixed Hodge modules, giving either full proofs or detailed accounts of the ingredients needed for the proof, and talks describing the way in which these results and, more generally, mixed Hodge modules have been used. Saito's participation in the workshop was particularly helpful to clarify various subtle points, both during talks and in private conversations between the talks.

Organizers
Mircea Mustaţă (University of Michigan)
Claude Sabbah (Centre de Mathématiques Laurent Schwartz)
Christian Schnell (Stony Brook University)

Speakers
Pramod Achar (Louisiana State University)
Donu Arapura (Purdue University)
Patrick Brosnan (University of Maryland)
Nero Budur (University of Notre Dame)
Mark de Cataldo (Stony Brook University)
Florian Ivorra (Université de Rennes)
Laurentiu Maxim (University of Wisconsin)
Takuro Mochizuki (Kyoto University)
Mircea Mustaţă (University of Michigan)
Chris Peters (Université de Grenoble I)
Mihnea Popa (University of Illinois, Chicago)
Thomas Reichelt (Universität Mannheim)
Claude Sabbah (Centre de Mathématiques Laurent Schwartz)
Morihiko Saito (Kyoto University)
Christian Schnell (Stony Brook University)
Jorg Schürmann (Universität Münster)
Christian Sevenheck (Universität Mannheim)
Balazs Szendroi (University of Oxford)
Kari Vilonen (Northwestern University)

The Mathematics of CCC: Mathematical Physics with Positive Lambda

September 11 – 13, 2013

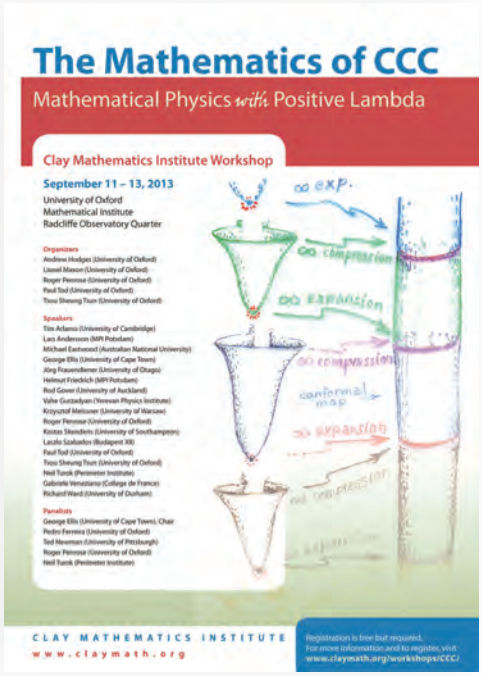
Conformal cyclic cosmology (CCC) proposes that the Big Bang was not the universe's origin, but a conformal continuation of an earlier *aeon*, where aeons occur sequentially, each starting with a big bang and ending with an exponential expansion driven by positive Λ. There is no early inflation, exponential expansion of the previous aeon serving instead. Emissions from previous-aeon black-hole encounters have observational implications, apparently supported in recent analyses of the CMB. Equations governing the transition from aeon to aeon imply creation of a dark material acquiring mass early in the new aeon, related to Higgs-type conformal symmetry-breaking.

This workshop brought together experts in the fields of relativity, geometry, partial differential equations, observational cosmology, and particle physics to explore the many mathematical and astrophysical issues raised by CCC, including the extension of Einstein's equations to the crossover between aeons leading to

interesting aspects of PDEs, geometrical aspects of conformal geometry, galactic dynamics in the very far future, the nature of gravitational wave emission from black-hole encounters, conventional cosmology, energy conservation in general relativity with positive Λ, and particle physics at very high energy, and also at very low energy, in the presence of positive Λ.

It was found useful to be able to gather together experts, and not just speakers, in these very different fields to explore various aspects of CCC. The interchange of ideas was much enhanced by the discussion panel during one of the evenings of the meeting.

Organizers
Andrew Hodges (University of Oxford)
Lionel Mason (University of Oxford)
Roger Penrose (University of Oxford)
Paul Tod (University of Oxford)
Tsou Sheung Tsun (University of Oxford)



Speakers
Tim Adamo (University of Cambridge)
Lars Andersson (MPI Postdam)
Michael Eastwood (Australian National University)
George Ellis (University of Cape Town)
Jörg Frauendiener (University of Otago)
Helmut Friedrich (MPI Potsdam)
Rod Gover (University of Auckland)
Vahe Gurzadyan (Yerevan Physics Institute)
Claude LeBrun (Stony Brook University)
Krzysztof Meissner (University of Warsaw)
Roger Penrose (University of Oxford)
Kostas Skenderis (University of Southampton)
Laszlo Szabados (Budapest XII)
Paul Tod (University of Oxford)
Tsou Sheung Tsun (University of Oxford)
Neil Turok (Perimeter Institute)
Gabriele Veneziano (Collège de France)

Computational Number Theory, Geometry and Physics (Sage Days 53)

September 23 – 29, 2013

The goal of the workshop was to improve the toric varieties module in the Sage computer algebra system to make it an effective test bed for computational experiments in number theory, geometry, and mathematical physics. Held the week before the Clay Research Conference workshop on Number Theory and Physics, the workshop served as a computational lead-in to the research conference.

Participants aimed to implement the computation of zeta functions of toric varieties, hypersurfaces, and complete intersections to vastly extend Sage's zeta function computing capabilities beyond curves; improve functionality for p-adics; and extend cohomology computations from toric line bundles to hyper-

cohomology of complexes of (sums of) toric line bundles.

The first day of the workshop offered an introduction to Sage aimed at graduate students, postdocs, and faculty, starting with an introduction to Python and Sage that did not assume any previous knowledge, and ending with a hands-on example of the development workflow. In the spirit of the Sage Days series, this workshop placed emphasis on joint and rapid code development with daily status reports by the participants.

Organizers
Jennifer Balakrishnan (University of Oxford)
Volker Braun (University of Oxford)

Speakers
Jennifer Balakrishnan (University of Oxford)
Amnon Besser (Ben Gurion University)
Volker Braun (University of Oxford)
Francois Escriva (Vrije University)
Jean-Pierre Flori (ANSSI)
Shaun Harker (Rutgers University)
Kiran Kedlaya (University of California, San Diego)
Jan Keitel (LMU Munich)
Simon King (Universitaet Jena)
Andrey Novoseltsev (University of Alberta)
Adriana Salerno (Bates College)
Jan Tuitman (Katholieke Universiteit Leuven)
John Voight (Dartmouth College)
Ursula Whitcher (University of Wisconsin, Eau Claire)

PHOTO COURTESY OF RINA SERGEEVA

CMI Outreach

As part of its mission to increase and disseminate mathematical knowledge, and to encourage gifted students to pursue mathematical careers, CMI partners with the established outreach programs PROMYS at Boston University, Ross Program at Ohio State and PRIMES at MIT.

In addition to maintaining the existing support for these programs, CMI is looking to build on it. By providing funding to extend the international reach of the programs CMI already supports and by identifying new partners, CMI is seeking to make available funding where it will make a real difference (see articles about PROMYS International Alliance and Modern Mathematics on the following pages).

To further the Institute's mission, the CMI Outreach Committee was established in 2013. Members David Bressoud (MacAlester), Frances Kirwan (Oxford), Marcus du Sautoy (Oxford), Günther M. Ziegler (FU Berlin) and Nick Woodhouse (CMI) will evaluate and give weight to proposals for outreach activities that may be taken to CMI's Scientific Advisory Board. The Committee welcomes proposals for new programs or the expansion of existing ones.

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CMI-PROMYS International Alliance

by Glenn Stevens

The CMI-PROMYS International Alliance launched in 2013 with a successful summer program that extended and strengthened the longstanding partnership of the Clay Mathematics Institute and the Program in Mathematics for Young Scientists (PROMYS). The goals of the first year were to build a bridge from PROMYS at Boston University to the Oxford Masterclasses in the UK, and to establish the foundations on which to create, in Europe, a sister program to PROMYS. To this end, we actively recruited European students to participate in both the US and UK programs as CMI-PROMYS International Alliance Scholars.

Thanks to strong support from established European programs and an energetic group of PROMYS alumni in Europe, we were able to overcome the challenges of a short recruitment period, receiving 90 exceptionally

PHOTO COURTESY OF WADHAM COLLEGE

PROMYS Oxford

strong applications from 15 European countries. Twelve talented high school students were invited to the program with full support from CMI, 11 of whom were able to attend. The inaugural group of CMI-PROMYS International Alliance Scholars represented Germany, Greece, the Netherlands, Poland, Romania, Spain, Turkey and the United Kingdom. These young scholars participated in PROMYS at Boston University before travelling to the UK for an additional week at the Oxford Masterclasses in Combinatorics at Wadham College, University of Oxford.

PROMYS is a six-week residential summer program for mathematically gifted high school students. Since its beginnings in 1989, PROMYS has been sustained and enriched by two guiding principles: (1) an emphasis on mathematical habits of mind that support independence and creativity in facing unfamiliar mathematical challenges and (2) a belief that mathematics is a deeply human activity best experienced within a richly interacting and mutually supportive community of learners, including high school students, undergraduate and graduate students, teachers, and experienced mathematical researchers. PROMYS aims to create an authentic experience of doing mathematics within a community of mathematicians having various levels of experience and expertise, every one of whom is actively engaged in significant mathematical activity appropriate to his of her individual level of expertise.

PHOTO COURTESY OF BOSTON UNIVERSITY

PROMYS Boston

In Boston, the International Alliance Scholars joined a group of 47 other first-year PROMYS participants, 25 returning participants, 24 counsellors (undergraduates in top mathematics programs), seven university faculty, six research mentors, and numerous visiting mathematicians and alumni. For six weeks they wrestled with significant mathematics through individual and collective efforts on the daily number theory problem sets. Most joined an exploration lab of their choosing to work on more open-ended problems, and presented their results to the rest of the program during the last week. Depending on experience and interest, many participated in an advanced seminar on *Representation Theory, Geometry and Symmetry* or *Wavelet Transformations* offered by PROMYS faculty. The Scholars worked closely with one another and formed good working relationships with many of the non-CMI participants. For many of these talented individuals, this was the first time in their lives that they found themselves tackling mathematics that was beyond their immediate grasp, were held to exacting standards of rigor and precision, met others with the same level of talent and passion, and worked side by side with other, more experienced, individuals who were actively engaged in developing their own careers as research mathematicians and scientists.

Immediately following the Boston program, the 11 Scholars, and one experienced PROMYS student from the US, flew to the UK to participate in the *Oxford Masterclasses in Combinatorics*. During this one-week program,

two short lectures each day were followed by problems sets concerning the material. The students worked in pairs to prepare short presentations on topics such as *The Four Colour Theorem* and *The Regularity Lemma* for the final afternoon of the program. In addition, three PROMYS counsellors who accompanied the students helped them throughout the week with their problem sets and projects; the counsellors were invaluable in helping to maintain a close continuity between the two programs. Jet lag aside, the students and counsellors moved easily from the PROMYS environment in Boston to the Masterclasses setting in Oxford, suggesting that a program like PROMYS could, with proper care, take root and flourish there.

Our experience this year has led us to believe in the possibility of creating a new program at Oxford and doing this in a way that will lead to the existence of two programs, one in Oxford and one in Boston, each of which is at least as strong as the current program in Boston. The very successful nature of the recruitment efforts proves that there is significant demand for an immersion-based program like PROMYS in Europe. The Scholars themselves indicated that they would like to help achieve these goals, with a number voicing interest in returning as advanced students and counsellors.

- *I've had an amazing summer in Boston and Oxford and really like the idea of a* PROMYS-*like program in Europe. Can I come back?*
- *It gives you a whole new way of looking at math and approaching problems. Also, it made me discover that I can actually do those problems. I wouldn't have thought that I could do that before* PROMYS.
- *I'm sure I want to study mathematics Th[ese] two experiences developed my research skills and made me think deeply on each topic. I think this is the difference between real mathematics and mathematics learnt in school.*
- *I know a lot of people who want to participate in this kind of summer program. A European* PROMYS *would be perfect!*

Over the years, we have come to appreciate the crucial role played by PROMYS alumni in the long-term success of the program. Feedback from the first cohort of CMI-PROMYS Scholars suggests that the seeds of a similarly active alum community in Europe have already been planted, thus promising a vital resource for the future success of a European PROMYS.

Modern Mathematics: International Summer School for Students

by Dierk Schleicher

The "Modern Mathematics" summer schools are a unique opportunity for top-level mathematicians of today and tomorrow to interact in a fruitful and substantial way. For ten days, some 100 young mathematicians from around the globe meet with international leading mathematicians, through formal presentations and informal interaction and communication. Three summer schools have taken place to date, under similar names with similar concepts: in Bremen, Germany (2011), Lyon, France (2012), and again in Bremen (2013). We very much hope to have started a lasting tradition; the next two events are planned for Lyon (2014) and Bremen (2015).

In all three years, we received several hundred applications from highly promising individuals; selecting the most suited ones was a difficult challenge. Candidates were asked to describe their mathematical background, indicate previous participation in mathematical events, and explain their mathematical interests

by describing their preferred mathematical theorems and proofs and explaining what they liked about them. The only formal requirement is English language competence. No previous experience with any competitions or olympiads is required; although these were seen as one possible positive indicator of talent, the students were free to present their talent and interest in their preferred way. The main criterion is not "past achievement" but "potential of development during the summer school".

The core of the summer school is a high-level lecture program by leading mathematicians of different mathematical interests, backgrounds, personalities, and styles. Common to all of the contributions is that they try to address an audience with a minimum of technical prerequisites, but develop a subject in surprising and inspiring ways, aiming at deep mathematical insights or connections between superficially unrelated ideas. Speakers are asked to present top-level mathematics in such a way as to get the participants hooked and involved. Among the highlights from the past three years are:

- Nalini Anantharaman (Paris), 2013, *Spectral Geometry*
- John Conway (Princeton): 2011, 2012, 2013, *Combinatorial Game Theory, Fractran, The Free Will Theorem, Lexicodes, Sphere Packings,* and more
- Gerhard Frey (Essen), 2013, *Elliptic Curves in Theory and Practice*
- Matthias Görner (Pixar Inc.), 2013, *Surfaces From Two Sides (The Mathematics of Making Movies)*
- Olga Holtz (Berlin), 2013, *Zero Localization of Polynomials*
- Yves Meyer (Paris), 2012, *Quasicrystals in Nature and in Mathematics*
- Yair Minsky (Yale), 2013, *Manifolds, their Geometry and Topology*
- Vlad Vicol (Princeton), 2013, *Mathematical Aspects of Incompressible Fluid Dynamics*
- Cédric Villani (Paris), 2012, *When the Earth was Too Young for Darwin*
- Jean-Christophe Yoccoz (Paris), 2012, *Translation Surfaces and Their Geodesics*
- Wendelin Werner (Paris), 2011, *Four Lectures on Probability*
- Don Zagier (Bonn/Paris), 2011, 2012, 2013, *Various Topics Related to Number Theory*
- Günter Ziegler (Berlin), 2011, 2013, *Discrete Geometry; New Proofs for THE BOOK*

Each lecturer gives at least one plenary talk, followed by one or several parallel lectures, so that participants can choose which topic they want to learn about in more depth. We encourage the participants to be selective with the lectures and not overdo spending time in too many presentations in order to have time to digest—but most of the participants don't want to miss an opportunity and attend as many talks as at all possible.

However, the purpose of the school is not to have participants just sit and watch, but to become involved in mathematics. Equally important to the lecture program is the proximity of instructors and participants. Many lecturers attend each others' talks and even tutorials, contributing their points of view and discussing with students not only *their* own field, but the larger context of the topics presented. In addition, lectures are supplemented by tutorials, group work, and mathematical activities, as well as informal activities (sports, games, music) and organised excursions. The freedom offered allows creativity to find its way and is one of the underlying reasons for the success of the summer school.

From the outset, the summer schools were developed with the intention of having a lasting impact beyond the 10 days of the event. To this end, all lectures were videotaped and are freely accessible. Other legacies of the summer schools include an interview with John Conway, published in *Notices of the American Mathematical Society*, several joint publications by distinguished lecturers (John Conway and Don Zagier) with young participants and a special issue of *American Mathematical Monthly*. But most importantly, the

IMAGE COURTESY OF JANA KLEINEBERG

lasting impact should happen in the minds and lives of the young participants. A sampling of student feedback indicates that we might have been successful.

> *I came back with better English, better background in math, a different person, open-minded, and with friends I would never forget … Thank you for showing me I don't have to be limited. I can go farther. I'd never imagine I could study at IMPA, and now, I'm going to study there for two months!*

> *[T]he "Modern Mathematics Summer School for Students" is the best thing that I could experience at this time. It greatly enhanced my horizon, gave me fresh inspiration and motivation, I met wonderful people that are as crazy for mathematics as I am, and the 11 days were not only great fun but will influence for a long time my view on mathematics and on the people that do mathematics, at a time when I was rather frustrated by my own mathematical environment.*

> *I almost don't dare to list what I've taken along from Bremen … most important and impressive to me are the encounters of the past weeks: encounters with future mathematicians, their dreams, their ideas; encounters with established mathematicians, their experience, their advice, their stories.*

Enhancement and Partnership

CMI's Enhancement and Partnership Program, introduced in 2012, aims to add value to activities that have already been planned, particularly by increasing international participation. In accordance with CMI's mission and its status as an operating foundation, its funding is used to enhance mathematical activities organized by, or planned in partnership with, other organizations. In 2013 CMI partnered in 16 initiatives, often by funding a distinguished international speaker or supporting participants from outside the host country.

SIMPLICITY: IDEALS OF PRACTICE IN MATHEMATICS AND THE ARTS

April 3 – 5, 2013 | City University of New York, NY

IMA CONFERENCE ON MATHEMATICS IN FINANCE

April 8 – 9, 2013 | University of Edinburgh, Edinburgh, UK

WOMEN IN MATHEMATICS DAYS

April 18 – 19, 2013 | Isaac Newton Institute, Cambridge, UK

CONFORMAL GEOMETRY AND GEOMETRIC PDEs

May – July 2013 | Centre de Recerca Mathemàtica, Bellaterra, Spain

ICMS SUMMER SCHOOL AND WORKSHOP

RICCI CURVATURE: LIMIT SPACES AND KÄHLER GOEMETRY

July 1 – 13, 2013 | University of Edinburgh, Edinburgh, UK

LMS-EPSRC INSTRUCTIONAL COURSE:
MODERN NONLINEAR PDE METHODS IN FLUID DYNAMICS
July 8 – 12, 2013 | University of Reading, Reading, UK

CIMPA RESEARCH SCHOOL:
CURRENT TRENDS IN COMPUTATIONAL METHODS FOR PDEs
July 8 – 19, 2013 | Indian Institute of Science, Bangalore, India

4TH EUROPEAN SET THEORY CONFERENCE
July 15 – 18, 2013 | Barcelona, Spain

SUMMER GRADUATE SCHOOL ON MATHEMATICAL GENERAL RELATIVITY
July 29 – August 9, 2013 | Cortona, Italy

WOMEN IN TOPOLOGY
August 18 – 23, 2013 | Banff International Research Station, Banff, Canada

GELFAND CENTENNIAL CONFERENCE:
A VIEW OF 21ST CENTURY MATHEMATICS
August 28 – September 2, 2013 | Massachusetts Institute of Technology, Cambridge, MA

GEOMETRY AND DYNAMICS OF INTEGRABLE SYSTEMS
September – November 2013 | Centre de Recerca Mathemàtica, Bellaterra, Spain

MOSTOWSKI 100
October 11 – 13, 2013 | University of Warsaw, Warsaw, Poland

WOMEN IN NUMBERS
October 14 – 18, 2013 | CIRM, Luminy, France

EXTERIOR DIFFERENTIAL SYSTEMS AND LIE THEORY
December 9 – 13, 2013 | Fields Institute, Toronto, Canada

REPRESENTATION THEORY AND APPLICATIONS TO COMBINATORICS, GEOMETRY AND QUANTUM PHYSICS
December 13 – 19, 2013 | Independent University of Moscow, Moscow, Russia

Publications

SELECTED ARTICLES BY RESEARCH FELLOWS

Tim Austin

Partial difference equations over compact Abelian groups, I: modules of solutions, submitted. arXiv: 1305.7269

Partial difference equations over compact Abelian groups, II: step-polynomial solutions, submitted. arXiv: 1309.3577

Ivan Corwin

Macdonald processes, with Alexei Borodin, to appear in *Probability Theory and Related Fields*. arXiv:1111.4408

Tropical combinatorics and Whitakker functions, with Neil O'Connell, Timo Seppäläinen and Nikolaos Zygouras, to appear in *Duke Mathematical Journal.* arXiv: 1110:3489

Semyon Dyatlov

Dynamical zeta functions for Anosov flows via microlocal analysis, with Maciej Zworski, submitted. arXiv: 1306.4203

Asymptotics of linear waves and resonances with applications to black holes, submitted. arXiv: 1305.1723

Aaron Pixton

Relations on Mbar_{g,n} via 3-spin structures, with Rahul Pandharipande and Dimitri Zvonkine, submitted. arXiv: 1303.1043

Relations in the tautological ring of the moduli space of curves, with Rahul Pandharipande, submitted. arXiv: 1301.4561

Sucharit Sarkar

A refinement of Rasmussen's s-invariant, with Robert Lipshitz, to appear in *Duke Mathematical Journal*. arXiv:1206.3532

A Steenrod square on Khovanov homology, with Robert Lipshitz, to appear in *Journal of Topology*. arXiv:1204.5776

A Khovanov homotopy type, with Robert Lipshitz, to appear in *Journal of the American Mathematical Society*. arXiv: 1112:3932

Peter Scholze

The pro-étale topology for schemes, with Bhargav Bhatt, submitted. arXiv: 1309.1198

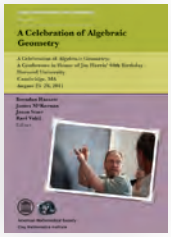
On torsion in the cohomology of locally symmetric varieties, submitted. arXiv: 1306.2070

Jack Thorne

Level raising and symmetric power functoriality I, II, with Laurent Clozel. www.math.harvard.edu/~thorne/

On the rigid cohomology of certain Shimura varieties, with Michael Harris, Kai-Wen Lan and Richard Taylor. www.math.harvard.edu/~thorne/

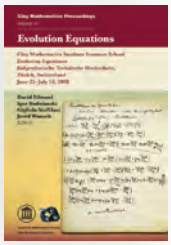
BOOKS

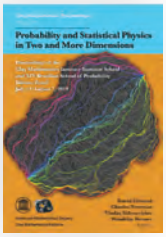


A Celebration of Algebraic Geometry

Editors: Brendan Hassett, James McKernan, Jason Starr and Ravi Vakil
CMI/AMS, 2013, 599 pp., softcover, ISBN: 0-8218-8983-4. List Price: $149. AMS Members: $119.20. Order Code: CMIP/18.

This volume resulted from the conference held in honor of Joe Harris' 60th birthday. Harris is famous around the world for his lively textbooks and enthusiastic teaching, as well as for his seminal research contributions. The articles are written in this spirit: clear, original, engaging, enlivened by examples, and accessible to young mathematicians. The articles focus on the moduli space of curves and more general varieties, commutative algebra, invariant theory, enumerative geometry both classical and modern, rationally connected and Fano varieties, Hodge theory and abelian varieties, and Calabi-Yau and hyperkähler manifolds. Taken together, they present a comprehensive view of the long frontier of current knowledge in algebraic geometry.





Evolution Equations

Editors: David Ellwood, Igor Rodnianski, Gigliola Staffilani and Jared Wunsch
CMI/AMS, 2013, 572 pp., softcover, ISBN: 0-8218-6861-6. List Price: $149. AMS Members: $119.20. Order Code: CMIP/17.

This volume is a collection of notes from lectures given at the 2008 Clay Mathematics Institute Summer School, held in Zurich, Switzerland. The lectures were designed for graduate students and mathematicians within five years of the PhD and the main focus of the program was on recent progress in the theory of evolution equations. Such equations lie at the heart of many areas of mathematical physics and arise not only in situations with a manifest time evolution (such as nonlinear wave and Schrödinger equations) but also in the high energy or semi-classical limits of elliptic problems.

Topics in Noncommutative Geometry

Editor: Guillermo Cortiñas
CMI/AMS, 2012, 276 pp., softcover, ISBN: 0-8218-6864-0. List Price: $79. AMS Members: $63.20. Order Code: CMIP/16.

This volume contains the proceedings of the third Luis Santaló Winter School held at FCEN in 2010. Topics included in this volume concern noncommutative geometry in a broad sense, encompassing various mathematical and physical theories that incorporate geometric ideas to the study of noncommutative phenomena. It explores connections with several areas, including algebra, analysis, geometry, topology and mathematical physics.

Probability and Statistical Physics in Two and More Dimensions

Editors: David Ellwood, Charles Newman,Vladas Sidoravicius and Wendelin Werner
CMI/AMS, 2012, 467 pp., softcover, ISBN: 0-8218-6863-2. List Price: $114. AMS Members: $91.20. Order Code: CMIP/15.

This volume is a collection of lecture notes for six of the ten courses given in Búzios, Brazil by prominent probabilists at the 2010 CMI Summer School, "Probability and Statistical Physics in Two and More Dimensions" and at the XIV Brazilian School of Probability. Together, these notes provide a panoramic, state-of-the-art view of probability theory areas related to statistical physics, disordered systems and combinatorics.

Grassmannians, Moduli Spaces and Vector Bundles

Editors: David A. Ellwood, Emma Previato
CMI/AMS, 2011, 180 pp., softcover, ISBN: 0-8218-5205-1. List Price: $55. AMS Members: $44. Order Code: CMIP/14.

This collection of cutting-edge articles on vector bundles and related topics originated from a CMI workshop, held in October 2006, that brought together a community indebted to the pioneering work of P. E. Newstead, visiting the United States for the first time since the 1960s. Moduli spaces of vector bundles were then in their infancy, but are now, as demonstrated by this volume, a powerful tool in symplectic geometry, number theory, mathematical physics, and algebraic geometry. This volume offers a sample of the vital convergence of techniques and fundamental progress taking place in moduli spaces at the outset of the twenty-first century.

On Certain *L*-Functions

Editors: James Arthur, James W. Cogdell, Steve Gelbart, David Goldberg, Dinakar Ramakrishnan, Jiu-Kang Yu
CMI/AMS, 2011, 647 pp., softcover, ISBN: 0-8218-5204-3. List Price: $129. AMS Members: $103.20. Order Code: CMIP/13.

This volume constitutes the proceedings of the conference organized in honor of the 60th birthday of Freydoon Shahidi, who is widely recognized as having made groundbreaking contributions to the Langlands program. The articles in this volume represent a snapshot of the state of the field from several viewpoints. Contributions illuminate various areas of the study of geometric, analytic, and number theoretic aspects of automorphic forms and their L-functions, and both local and global theory are addressed.

Motives, Quantum Field Theory, and Pseudodifferential Operators

Editors: Alan Carey, David Ellwood, Sylvie Paycha, Steven Rosenberg
CMI/AMS, 2010, 349 pp., softcover. ISBN: 0-8218-5199-3. List price: $89. AMS Members: $71.20. Order Code: CMIP/12.

This volume contains articles related to the conference "Motives, Quantum Field Theory, and Pseudodifferential Operators" held at Boston University in June 2008, with partial support from the Clay Mathematics Institute, Boston University, and the National Science Foundation. There are deep but only partially understood connections between the three conference fields, so this book is intended both to explain the known connections and to offer directions for further research.

Quanta of Maths; Proceedings of the Conference in honor of Alain Connes

Editors: Etienne Blanchard, David Ellwood, Masoud Khalkhali, Matilde Marcolli, Henri Moscovici, Sorin Popa
CMI/AMS, 2010, 675 pp., softcover, ISBN: 0-8218-5203-5. List price: $129. AMS Members: $103.20. Order Code: CMIP/11.

The work of Alain Connes has cut a wide swath across several areas of mathematics and physics. Reflecting its broad

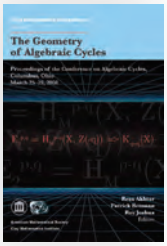
spectrum and profound impact on the contemporary mathematical landscape, this collection of articles covers a wealth of topics at the forefront of research in operator algebras, analysis, noncommutative geometry, topology, number theory and physics.

Homogeneous Flows, Moduli Spaces and Arithmetic

Editors: Manfred Einsiedler, David Ellwood, Alex Eskin, Dmitry Klein, Elon Lindenstrauss, Gregory Margulis, Stefano Marmi, Jean-Christophe Yoccoz

CMI/AMS, 2010, 438 pp., softcover, ISBN: 0-8218-4742-2. List price: $99. AMS Members: $79.20. Order Code: CMIP/10.

This book contains a wealth of material concerning two very active and interconnected directions of current research at the interface of dynamics, number theory and geometry. Examples of the dynamics considered are the action of subgroups of SL(n,R) on the space of unit volume lattices in Rn and the action of SL (2, R) or its subgroups on moduli spaces of flat structures with prescribed singularities on a surface of genus >= 2.

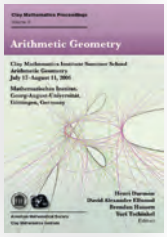


The Geometry of Algebraic Cycles

Editors: Reza Akhtar, Patrick Brosnan, Roy Joshua

CMI/AMS, 2010, 187 pp., softcover, ISBN: 0-8218-5191-8. List Price: $52. AMS Members: $41.60. Order Code: CMIP/9.

The subject of algebraic cycles has its roots in the study of divisors, extending as far back as the nineteenth century. Since then, and in particular in recent years, algebraic cycles have made a significant impact on many fields of mathematics, among them number theory, algebraic geometry, and mathematical physics. The present volume contains articles on all of the above aspects of algebraic cycles.

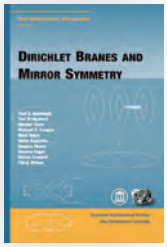


Arithmetic Geometry

Editors: Henri Darmon, David Ellwood, Brendan Hassett, Yuri Tschinkel.

CMI/AMS 2009, 562 pp., softcover. ISBN:0-8218-4476-8. List price: $119. AMS Members: $95.20. Order Code: CMIP/8.

This book is based on survey lectures given at the 2006 CMI Summer School at the Mathematics Institute of the University of Göttingen. It introduces readers to modern techniques and outstanding conjectures at the interface of number theory and algebraic geometry.

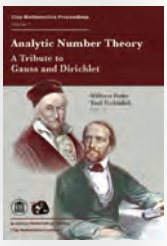


Dirichlet Branes and Mirror Symmetry

Editors: Michael Douglas, Mark Gross.

CMI/AMS 2009, 681 pp., hardcover. ISBN: 0-8218-3848-2. List price: $109. AMS Members: $87.20. Order Code: CMIM/4.

The book first introduces the notion of Dirichlet brane in the context of topological quantum field theories, and then reviews the basics of string theory. After showing how notions of branes arose in string theory, it turns to an introduction to the algebraic geometry, sheaf theory, and homological algebra needed to define and work with derived categories. The physical existence conditions for branes are then discussed, culminating in Bridgeland's definition of stability structures. The book continues with detailed treatments of the Strominger-Yau-Zaslow conjecture, Calabi-Yau metrics and homological mirror symmetry, and discusses more recent physical developments.

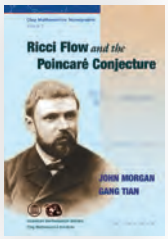


Analytic Number Theory: A Tribute to Gauss and Dirichlet

Editors: William Duke, Yuri Tschinkel.

CMI/AMS, 2007, 265 pp., softcover. ISBN: 0-8218-4307-9. List Price: $50. AMS Members: $40. Order Code: CMIP/7.

This volume contains the proceedings of the Gauss–Dirichlet Conference held in Göttingen from June 20–24 in 2005, commemorating the 150th anniversary of the death of Gauss and the 200th anniversary of Dirichlet's birth. It begins with a definitive summary of the life and work of Dirichlet by J. Elstrodt and continues with thirteen papers by leading experts on research topics of current interest within number theory that were directly influenced by Gauss and Dirichlet.



Ricci Flow and the Poincaré Conjecture

Authors: John Morgan, Gang Tian.

CMI/AMS, 2007, 521 pp., hardcover. ISBN: 0-8218-4328-1. List price: $71. AMS Members: $56.80. Order Code: CMIM/3.

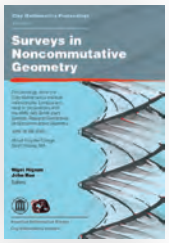
This book presents a complete and detailed proof of the Poincaré conjecture. This conjecture was formulated by Henri Poincaré in 1904 and had remained open until the work of Grigory Perelman. The arguments given in the book are a detailed version of those that appear in Perelman's three preprints.

The Millennium Prize Problems

Editors: James Carlson, Arthur Jaffe, Andrew Wiles.

CMI/AMS, 2006, 165 pp., hardcover. ISBN: 0-8218-3679-X. List Price: $30. AMS Members: $24. Order Code: MPRIZE.

This volume gives the official description of each of the seven problems as well as the rules governing the prizes. It also contains an essay by Jeremy Gray on the history of prize problems in mathematics.

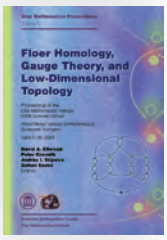


Surveys in Noncommutative Geometry

Editors: Nigel Higson, John Roe.

CMI/AMS, 2006, 189 pp., softcover. ISBN: 0-8218-3846-6. List Price: $50. AMS Members: $40. Order Code: CMIP/6.

In June of 2000, a summer school on noncommutative geometry, organized jointly by the American Mathematical Society and the Clay Mathematics Institute, was held at Mount Holyoke College in Massachusetts. The meeting centered around several series of expository lectures that were intended to introduce key topics in noncommutative geometry to mathematicians unfamiliar with the subject. Those expository lectures have been edited and are reproduced in this volume.

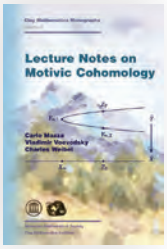


Floer Homology, Gauge Theory, and Low-Dimensional Topology

Editors: David Ellwood, Peter Ozsváth, András Stipsicz, Zoltán Szábo.

CMI/AMS, 2006, 297 pp., softcover. ISBN: 0-8218-3845-8. List price: $66. AMS Members: $52.80. Order Code: CMIP/5.

This volume grew out of the summer school that took place in June of 2004 at the Alfréd Rényi Institute of Mathematics in Budapest, Hungary. It provides a state-of-the-art introduction to current research, covering material from Heegaard Floer homology, contact geometry, smooth four-manifold topology, and symplectic four-manifolds.

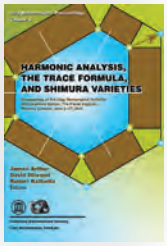


Lecture Notes on Motivic Cohomology

Authors: Carlo Mazza, Vladimir Voevodsky, Charles Weibel.

CMI/AMS, 2006, 216 pp., softcover. ISBN: 0-8218-5321-X. List Price: $47. AMS Members: $37.60. Order Code: CMIM/2.S.

This book provides an account of the triangulated theory of motives. Its purpose is to introduce the reader to motivic cohomology, to develop its main properties, and finally to relate it to other known invariants of algebraic varieties and rings such as Milnor K-theory, étale cohomology, and Chow groups.

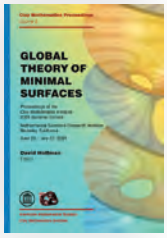


Harmonic Analysis, the Trace Formula and Shimura Varieties

Editors: James Arthur, David Ellwood, Robert Kottwitz.

CMI/AMS, 2005, 689 pp., softcover. ISBN: 0-8218-3844-X. List Price: $131. AMS Members: $104.80. Order Code: CMIP/4.

The subject of this volume is the trace formula and Shimura varieties. These areas have been especially difficult to learn because of a lack of expository material. This volume aims to rectify that problem. It is based on the courses given at the 2003 Clay Mathematics Institute Summer School at Fields Institute, Toronto. Many of the articles have been expanded into comprehensive introductions, either to the trace formula or to the theory of Shimura varieties, or to some aspect of the interplay and application of the two areas.

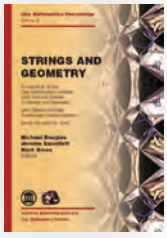


Global Theory of Minimal Surfaces

Editor: David Hoffman.

CMI/AMS, 2005, 800 pp., softcover. ISBN: 0-8218-3587-4. List Price: $131. AMS Members: $104.80. Order Code: CMIP/2

This book is the product of the 2001 CMI Summer School held at MSRI. The subjects covered include minimal and constant-mean-curvature submanifolds, geometric measure theory and the double-bubble conjecture, Lagrangian geometry, numerical simulation of geometric phenomena, applications of mean curvature to general relativity and Riemannian geometry, the isoperimetric problem, the geometry of fully nonlinear elliptic equations, and applications to the topology of three-manifolds.

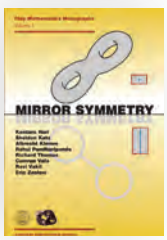


Strings and Geometry

Editors: Michael Douglas, Jerome Gauntlett, Mark Gross.

CMI/AMS, 2004, 376 pp., softcover. ISBN: 0-8218-3715-X. List Price: $76. AMS Members: $60.80. Order Code: CMIP/3.

This volume is the proceedings of the 2002 Clay Mathematics Institute Summer School held at the Isaac Newton Institute for Mathematical Sciences in Cambridge, UK. It contains a selection of expository and research articles by lecturers at the school and highlights some of the current interests of researchers working at the interface between string theory and algebraic geometry. The topics covered include manifolds of special holonomy, supergravity, supersymmetry, D-branes, the McKay correspondence and the Fourier-Mukai transform.



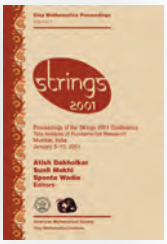
Mirror Symmetry

Editors: Cumrun Vafa, Eric Zaslow.

CMI/AMS, 2003, 929 pp., hardcover. ISBN: 0-8218-2955-6. List Price: $137. AMS Members: $109.60. Order Code: CMIM/1

This thorough and detailed exposition develops mirror symmetry from both mathematical and physical perspectives and will be particularly useful for those wishing to advance their understanding by exploring mirror symmetry at the interface of mathematics and physics. This one-of-a-kind volume offers the first comprehensive exposition

on this increasingly active area of study. It is carefully written by leading experts who explain the main concepts without assuming too much prerequisite knowledge.



Strings 2001

Editors: Atish Dabholkar, Sunil Mukhi, Spenta R. Wadia.
CMI/AMS, 2002, 489 pp., softcover. ISBN: 0-8218-2981-5. List Price: $86. ASM Members: $68.80. Order Code: CMIP/1.

This multi-authored book summarizes the latest results across all areas of string theory from the perspective of world-renowned experts, including Michael Green, David Gross, Stephen Hawking, John Schwarz, Edward Witten and others. The book comes out of the "Strings 2001" conference, organized by the Tata Institute of Fundamental Research (Mumbai, India), the Abdus Salam ICTP (Trieste, Italy), and the Clay Mathematics Institute (Cambridge, MA, USA). Individual articles discuss the study of D-branes, black holes, string dualities, compactifications, Calabi-Yau manifolds, conformal field theory, noncommutative field theory, string field theory, and string phenomenology. Numerous references provide a path to previous findings and results.

To order print copies of these books, please visit www.ams.org/bookstore. PDF versions are posted on CMI's Online Library six months after publication and can be found at www.claymath.org/node/262.

DIGITAL LIBRARY

CMI's Digital Library includes facsimiles of significant historical mathematical books and manuscripts, collected works and seminar notes.

Euclid's Elements, Constantinople, 888 AD (Greek). MS at the Bodleian Library
The oldest extant manuscript and printed editions of Euclid's Elements, in Greek (888 AD) and Latin (1482 AD), respectively. High resolution copies of the manuscript are available for study at the Bodleian Library, Oxford University. **www.claymath.org/euclids-elements**

Riemann's 1859 Manuscript
The manuscript in which Riemann formulated his famous conjecture about the zeroes of the zeta function.
www.claymath.org/publications/riemanns-1859-manuscript

Klein Protokolle
The "Klein Protokolle," comprising 8600 pages in 29 volumes, record the activity of Felix Klein's seminar in Göttingen for the years 1872-1912. **www.claymath.org/publications/klein-protokolle**

James Arthur Archive
James Arthur attended the University of Toronto as an undergraduate, and received his PhD at Yale University in 1970, where his advisor was Robert Langlands. He has been a University Professor at the University of Toronto since 1987. Almost all of Arthur's professional career has been dedicated to exploring the analogue for general reductive groups of the trace formula for SL2 first proved by Selberg in the mid 1950s. This has proved to be enormously complex in its details, but also extraordinarily fruitful in its applications. With help from Bill Casselman at the University of British Columbia, this website presents the author's complete published work in an easily accessible set of searchable PDFs.
www.claymath.org/publications/collected-works-james-g-arthur

Notes of Talks at the I. M. Gelfand Seminar
The notes presented here were taken by a regular participant at the celebrated Monday evening mathematical seminar conducted by Israel Moiseevich Gelfand at Moscow State University. Mikhail Aleksandrovich Shubin, who began attending in September 1964 as a fourth-year student in the mathematics department of Moscow State University, took notes over 25 years and, even more remarkably, managed to keep all his notes. With the financial support of the Clay Mathematics Institute, Shubin's notes have been scanned for all to appreciate. The entire project would not have been possible without the involvement of M. A. Shubin, S. I. Gelfand, and the assistance of the Moscow Center of Continuous Mathematical Education. **www.claymath.org/publications/notes-talks-imgelfand-seminar**

Nominations, Proposals and Applications

FELLOWS AND SCHOLARS

Nominations for Senior and Research Scholars are considered four times a year at our Scientific Advisory Board (SAB) meetings. Principal funding decisions for Senior Scholars are made at the September SAB meeting. Additional nominations will be considered at other times as funds permit. Clay Research Fellow nominations are considered once a year and must be submitted according to the schedule below. Address all nominations to Nick Woodhouse at president@claymath.org, copied to Naomi Kraker at admin@claymath.org.

Nomination Deadlines

- Senior Scholars: August 1
- Research Fellows: October 30
- Research Scholars: August 1*

WORKSHOPS AT THE MATHEMATICAL INSTITUTE

The Clay Mathematics Institute invites proposals for small workshops, typically ten to twenty persons, to be held at the Mathematical Institute in Oxford, UK. The aim is to bring a small set of researchers together quickly, outside the usual grant and application cycle, when this is likely to result in significant progress. Proposals, which need not be long, will be judged on their scientific merit, probable impact, and potential to advance mathematical knowledge. For more information, or to make a proposal, contact president@claymath.org, copied to admin@claymath.org.

ENHANCEMENT AND PARTNERSHIP

The Clay Mathematics Institute invites proposals under its Enhancement and Partnership Program. The aim is to enhance activities that are already planned, particularly by funding international participation. The program is broadly defined, but subject to the general principles: CMI funding will be used in accordance with the Institute's mission and its status as an operating foundation to enhance mathematical activities organized by or planned in partnership with other organizations; it will not be used to meet expenses that could be readily covered from local or national sources; and all proposals will be judged by the CMI's Scientific Advisory Board. For more information, visit www.claymath.org/programs/enhancement-and-partnership-proposals. Enquiries about eligibility should be sent to president@claymath.org and proposals should be sent to admin@claymath.org.

Nominations and proposals may also be mailed to:

Clay Mathematics Institute
Office of the President
Andrew Wiles Building
Radcliffe Observatory Quarter
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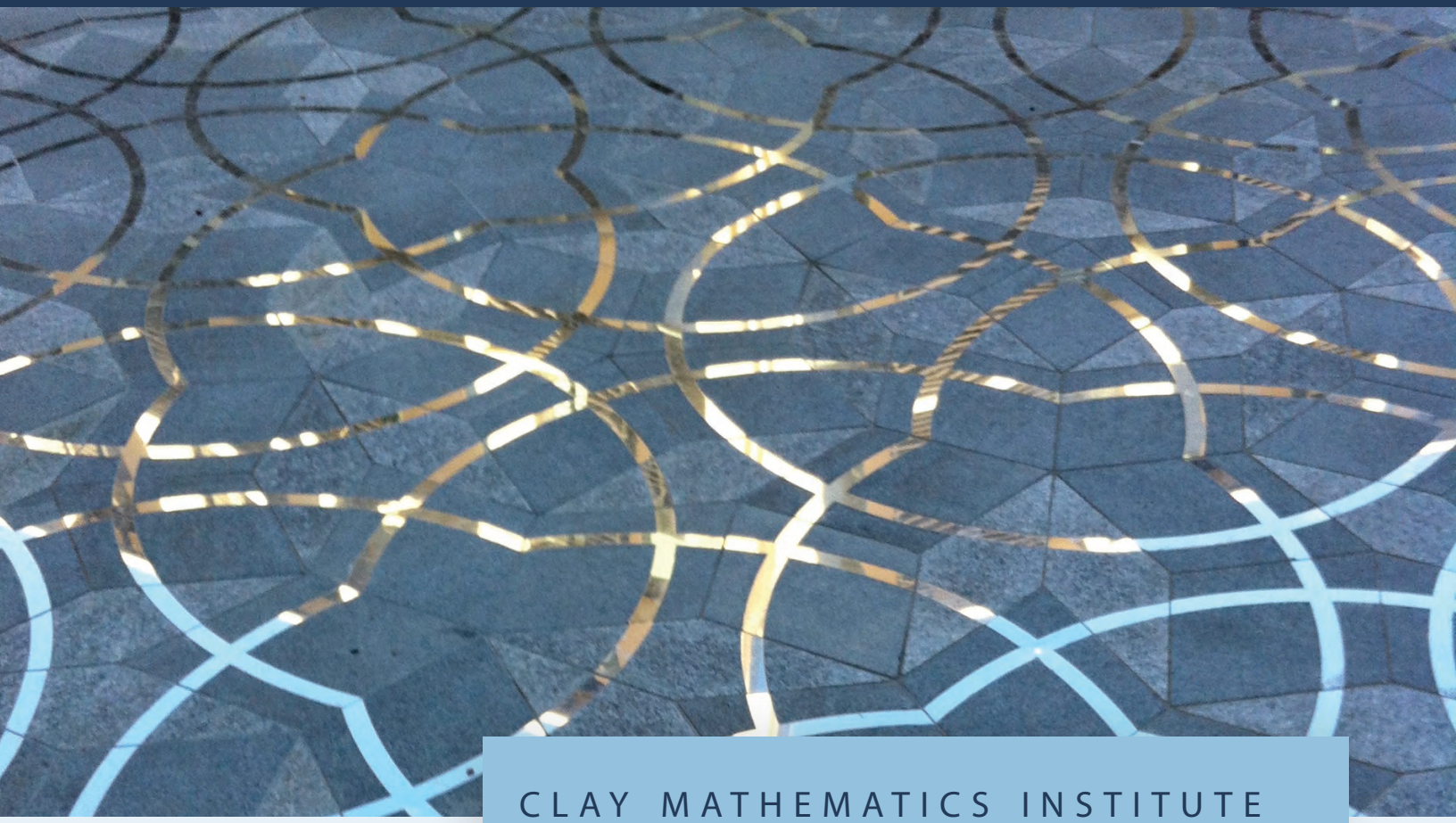
(*) Most funding decisions are made by the Scientific Advisory Board at its Fall meeting. For the indicated programs, occasional appointments are made at later meetings. However, since most funds are allocated at the Fall board meeting, application/nomination by the August date is advisable.

2014 Institute Calendar

Jan 1 – Dec 31	PRIMES	Massachusetts Institute of Technology, Cambridge, MA
Jan 1 – Dec 31	Independent University of Moscow	Moscow, Russia
Jan - July	Central Configurations, Periodic Orbits and Beyond in Celestial Mechanics	CRM, Bellaterra, Spain
Jan 20 – 24	Representation Theory Days in Patagonia	Punta Arenas, Chile
Feb – April	Senior Scholar Ehud Hrushovski, *Model Theory, Arithmetic Geometry and Number Theory*	MSRI, Berkeley, CA
Feb 17 – 21	Perfectoid Spaces and their Applications	MSRI, Berkeley, CA
March – May	Senior Scholar Lars Hesselholt, *Algebraic Topology*	MSRI, Berkeley, CA
March 17	SET for Britain 2014	London, UK
April 7 – 10	British Mathematical Colloquium	Queen Mary University of London, London, UK
April 7 – 11	Geometry and Fluids workshop	University of Oxford, Oxford, UK
April 12 – 15	Applications of Automorphic Forms in Number Theory and Combinatorics	Louisiana State University, Baton Rouge, LA
April 20 – 24	Women in Numbers	Banff International Research Station, Banff, Canada
May 12 – 16	Rational and Integral Points on Higher-dimensional Varieties	Massachusetts Institute of Technology, Cambridge, MA
May 19 – 23	Representation of Reductive Groups	Massachusetts Institute of Technology, Cambridge, MA
May 26 – 30	From Macdonald Processes to Hecke Algebras and Quantum Integrable Systems	Institut Henri Poincaré, Paris, France
June – July	Senior Scholars Yves Beniost and Alex Eskin, *Dynamics of Group Actions and Number Theory*	Isaac Newton Institute, Cambridge, UK
June 2 – 6	Extremal and Probabilistic Combinatorics Workshop	University of Oxford, Oxford, UK
June 2 – 7	CIME: Partial Differential Equations and Geometric Measure Theory	Cetaro, Cosenza, Italy
June 2 – 14	Algebraic Geometry and Number Theory	Galatasaray University, Istanbul, Turkey
June 2 – 27	Summer School in Probability	University of British Columbia, Vancouver, BC, Canada
June 16 – 20	12th Workshop on Interactions between Dynamical Systems and Partial Differential Equations (JISD 2014)	Universitat Politècnica de Catalunya, Barcelona, Spain
June 16 – 27	Summer School on Quiver Hecke Algebras and Conference on Geometric Representation	Institut d'Etudes Scientifiques de Cargese, Corsica
June 16 – July 25	Ross Program	Ohio State University, Columbus, OH
June 23 – 27	Algebra, Geometry and Physics	IHES, Bures-sur-Yvette, France
June 23 – 27	Apollonian Circle Packings Summer School	Institute Mittag-Leffler, Stockholm, Sweden
June 23 – 27	Manifolds, *K*-theory and Related Topics	Inter University Centre Dubrovnik, Croatia
June 23 – 27	Conference in Honor of Richard Stanley	Massachusetts Institute of Technology, Cambridge, MA
June 23 – 27	Strings 2014	Princeton University, Princeton, NJ
June 27	Non-Western Mathematics	University of Oxford, Oxford, UK
June 29 – July 3	International Conference on Formal Power Series and Algebraic Combinatorics	DePaul University, Chicago, IL
June 29 – July 19	Senior Scholars L. Mahadevan and Felix Otto, *Materials*	PCMI, Park City, UT

June 29 – Aug 9	PROMYS	Boston University, Boston, MA
June 30 – July 4	Groups, Numbers and Dynamics Workshop	Isaac Newton Institute, Cambridge, UK
June 30 – July 5	LMS CMI Research School: Automorphic Forms and Related Topics	Bristol University, Bristol, UK
June 30 – July 25	CMI Summer School *Periods and Motives: Feynman amplitudes in the 21st century*	ICMAT, Madrid, Spain
July – Dec	Senior Scholars Simon Brendle and Nader Masmoudi, *Thematic Program on Variational Problems in Physics, Economics and Geometry*	Fields Institute, Toronto, Canada
July 5 – 10	Symmetries and Correspondences in Number Theory, Geometry, Algebra and Quantum Computing	University of Oxford, Oxford, UK
July 7 – 11	LMS CMI Research School: An Invitation to Geometry and Topology via G2	Imperial College, London, UK
July 7 - 11	Building Bridges: 2nd EU/US Workshop on Automorphic Forms and Related Topics	Bristol University, Bristol, UK
July 14 – 18	30th International Colloquium on Group Theoretical Methods in Physics	University of Ghent, Belgium
July 14 – 18	Analytic Number Theory and its Applications	Perrotis College, Thessaloniki, Greece
July 27 – Aug 1	37th Conference on Stochastic Processes and their Applications	Universidad de Buenos Aires, Buenos Aires, Argentina
Aug 10 – 16	Oxford Masterclasses in Geometry (PROMYS)	Wadham College, University of Oxford, Oxford, UK
Aug 17 – 22	UKMT Summer School for Girls	Balliol College, University of Oxford, Oxford, UK
Aug 23 – 26	Holomorphic Dynamics in One and Several Variables	Gyeong-ju, Korea
Aug 25 – 29	LMS CMI Research School: Algebraic Lie Theory and Representation Theory	University of Glasgow, Glasgow, UK
Sept 1 – 5	Algebraic Varieties: Bundles, Topology, Physics (VBAC 2014)	Freie Universitat Berlin, Berlin, Germany
Sept 19 – 21	Harmonic Analysis and Partial Differential Equations	University of Chicago, Chicago, IL
Sept 22 – 25	New Geometric Structures in Scattering Amplitudes	University of Oxford, Oxford, UK
Sept 22 – 26	LMS CMI Research School: Bounded Gaps Between Primes	University of Oxford, Oxford, UK
Sept 28 – Oct 3	Clay Research Conference and Workshops	University of Oxford, Oxford, UK
Sept 28 – Oct 2	Advances in Probability: Integrability, Universality and Beyond Workshop	University of Oxford, Oxford, UK
Sept 29 – Oct 3	Analytic Number Theory Workshop	University of Oxford, Oxford, UK
Sept 29 – Oct 3	Functional Transcendence around Ax-Schanuel Workshop	University of Oxford, Oxford, UK
Sept 29 – Oct 3	Symplectic Topology Workshop	University of Oxford, Oxford, UK
Oct 1	Clay Research Conference	University of Oxford, Oxford, UK
Oct 26	Michael Sipser Colloquium	Massachusetts Institute of Technology, Cambridge, MA
Fall 2014	Senior Scholar Pierre Colmez, *New Geometric Methods in Number Theory and Automorphic Forms*	MSRI, Berkeley, CA
Fall 2014	Senior Scholars Joseph Bernstein and Ngô Bao Châu, *Geometric Representation Theory*	MSRI, Berkeley, CA
Fall 2014	Senior Scholar Vladimir Voevodsky, *Type Theory and its Applications*	University of Oxford, Oxford, UK
Fall 2014	Senior Scholar Rick Durrett, *Cancer and its Environment*	MBI, Ohio State University, Columbus, OH
Nov 17 – 21	Categorical Structures in Harmonic Analysis	MSRI, Berkeley, CA
Dec 1 – 5	Automorphic Forms, Shimura Varieties, Galois Representations and L-functions	MSRI, Berkeley, CA
Dec 11 – 20	Foundations of Computational Mathematics	Universidad de la Republica del Uruguay, Montevideo, Uruguay



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