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CLAY MATHEMATICS INSTITUTE annual report 2011

#### nominations, proposals, and applications

Nominations for Senior and Research Scholars are considered four times a year at our Scientific Advisory Board (SAB) meetings. Principal funding decisions for Senior Scholars are made at the September SAB meeting. Additional nominations will be considered at other times as funds permit. Clay Research Fellow nominations are considered once a year and must be submitted according to the schedule right:

(\*) Most funding decisions are made by the Scientific Advisory Board at its fall meeting. For the indicated programs, occasional appointments are made at later meetings. However, since most funds are allocated at the fall board meeting, application/nomination by the August date is advisable.

#### **Nomination Deadlines**

Senior Scholars: **August 1** Research Fellows: **October 30** and **November 16** Research Scholars: **August 1**\*

Address all nominations to Nick Woodhouse at president@claymath.org, copied to Naomi Kraker at admin@claymath.org.

#### Workshops at the Mathematical Institute

The Clay Mathematics Institute invites proposals for small workshops, typically ten to twenty persons, to be held at the Mathematical Institute in Oxford, UK. The aim is to bring a small set of researchers together quickly, outside the usual grant and application cycle, when this is likely to result in significant progress. Proposals, which need not be long, will be judged on their scientific merit, probable impact, and potential to advance mathematical knowledge. For more information, or to make a proposal, contact president@claymath.org, copied to admin@claymath.org.

Proposals may also be mailed to: Clay Mathematics Institute Office of the President Mathematical Institute 24-29 St. Giles' Oxford OX1 3LB United Kingdom

#### CLAY MATHEMATICS INSTITUTE

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#### CLAY MATHEMATICS INSTITUTE

#### annual report 2011



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### **Letter from the President**



Dear Friends of Mathematics

This report covers the last full year of James Carlson's presidency. I took over from him in the summer of 2012, when the Clay Mathematics Institute moved its president's office to Oxford, UK.

Jim presided over a period of remarkable activity, during which he did much to consolidate CMI's mathematical program and to build its international reputation. As he explained in our conversations before he handed on the baton: "When I took over, the CMI was best known for establishing the list of seven Prize Problems to celebrate mathematics at the start of the new millennium. It has done a lot of other things as well, in particular in awarding Clay Research Fellowships, which give young mathematicians the time and space to develop over five years. The limiting factors for aspiring mathematicians are talent and time. We cannot do much about talent, but the fellowships do

provide time. Many former fellows have done exceedingly well in their later careers. CMI also supports the Ross and PROMYS programs for talented high school students, and the Independent University of Moscow, which seeks to preserve the great Russian mathematical tradition. Other very valuable programs are the bi-annual summer schools, the CMI workshops and the research scholar program. The latter two are very flexible."

A glance back through earlier annual reports shows what this meant in practice, with an extraordinarily impressive range of conferences, workshops, summer schools, awards, and publications, which Jim, David Ellwood, and the excellent administrative team in Cambridge initiated and organised.

A high point was the first award for the solution of a Millennium Prize Problem to Grigoriy Perelman. The prize problems have had a huge impact in two directions: first in raising awareness of mathematics amongst young students and in encouraging them to enter the discipline. Second in conveying a message to professional mathematicians about the importance of working on really hard long-standing problems. They have also entered the general consciousness and drawn attention to the seven problems in a way that is unusual in our sometimes invisible discipline.

The task of checking Perelman's solution was not only exceedingly difficult and time-consuming, but it was also one that had to be undertaken in the face of intense public interest in the outcome. Jim took on the burden of responsibility for organising the verification process, and carried it through with efficiency and flair—a job very well done. It was, as Jim remarked, "one of the most thorough refereeing jobs in history".

In this annual report you will see that CMI's support for mathematics at the highest level continued unabated in 2011. It now falls to a new team in Oxford to build on the outstanding achievements of the CMI's first decade and to carry forward Landon Clay's vision. The new location offers fresh opportunities to expand and develop the Institute, but ones that are only open because of the foundations laid by my predecessors.

Sincerely,

1. M.S. Woodhouse

N. M. J. Woodhouse President

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# annualmeeting



#### Clay Research Conference 2011 | Cambridge, Massachusetts

The fifth annual Clay Research Conference, an event devoted to recent advances in mathematical research, was held at Harvard University on May 16 and 17, 2011 at the Harvard Science Center (Lecture Hall A).

Conference speakers were Manjul Bhargava (Princeton University), Yves Benoist (CNRS-Université de Paris Sud 11), Mihalis Dafermos (University of Cambridge), Alex Eskin (University of Chicago), Jonathan Pila (University of Oxford), Jean-François Quint (CNRS-Université de Paris 13), Peter Sarnak (Princeton University) and Alex Wilkie (University of Manchester). Abstracts appear on the following pages.

On the afternoon of May 16, the **Clay Research Awards** were presented to **Jonathan Pila** for his resolution of the André-Oort Conjecture in the case of products of modular curves and to **Yves Benoist** and **Jean-François Quint** for their spectacular work on stationary measures and orbit closures for actions of non-abelian groups on homogeneous spaces.





Yves Benoist



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Jonathan Pila

# abstracts



#### Jean-François Quint, CNRS-Université de Paris 13

Stationary measure on finite volume homogeneous spaces (I)

Consider a Lie group G, a finite volume quotient X of G, a subgroup H of G, and a probability measure m on H whose support is compact and generates H. We study the dynamics of H on X when the Zariski closure of the adjoint group of H is semisimple. We describe the H-orbit closures in X and various equidistribution results on X. To this end, we classify the m-stationary probability measures on X.

#### Yves Benoist, CNRS-Université de Paris Sud 11

Stationary measure on finite volume homogeneous spaces (II)

Consider a group H of matrices with integer coefficients acting on a torus X, and a probability measure m on H whose support is finite and generates H. We study the dynamics of H on X when the Zariski closure of H is semisimple. We develop new tools in order to classify the m-stationary probability measures on X: the positive m-unstability of the diagonal, the horocyclic flow, and the exponential drift. To this end, we check various equidistribution results for the associated random walk.

#### Alex Eskin, University of Chicago

The SL(2, **R**) action on moduli space of Riemann surfaces

I discuss some applications of the ideas introduced by Y. Benoist and J.F. Quint in their breakthrough work on stationary measures on homogeneous spaces to the seemingly unrelated problem of counting periodic billiard trajectories in polygons, all of whose angles are rational multiples of *p*. The connection is via the study of the ergodic theory of the SL(2, **R**) action on the moduli space of compact Riemann surfaces. This is joint work with M. Mirzakhani.



Alex Eskin

#### Alex Wilkie, University of Manchester

The relevance of logic to transcendental number theory: a motivated account

My talk describes the role that a certain branch of mathematical logic, known as o-minimality, plays in the work of Jonathan Pila. I begin by stating a fundamental result of Pila and Bombieri giving a bound (in terms of height) on



Alex Wilkie

#### Jonathan Pila, University of Oxford

Diophantine geometry via o-minimality

the number of rational values that a sufficiently differentiable function (in several real variables) can take at rational arguments. However, in order to apply this result to interesting (usually analytic) subsets of euclidean space one needs to represent the subset as a finite union of sets, each being the range of a sufficiently differentiable function with bounded derivatives. To do this one proceeds by induction on dimension, but the problem is to control the logical complexity of the definitions of the sets that one encounters along the way. The key point is that one never has to leave the collection of sets logically definable in an o-minimal structure. My aim is to explain this remark.

Starting with a 1989 joint paper with Enrico Bombieri, I studied rational points on algebraic and (certain) non-algebraic sets, culminating in a general theorem, joint with Alex Wilkie, concerning the distribution of rational points in a set in real space that is "definable in an o-minimal structure over the real numbers". This is a model-theoretic notion of geometric "tameness". Bounded semi-analytic sets are examples of such sets. Employing a strategy proposed by Umberto Zannier, we used this result to give a new proof of the Manin-Mumford conjecture (Raynaud's theorem) about torsion points on subvarieties of abelian varieties. Further applications of this general strategy to diophantine problems in the Andre-Oort-Manin-Mumford circle of problems and the broader "unlikely intersection" conjectures of Zilber and Pink have been made by Masser-Zannier, by me, and by others. I describe these problems, indicate the main elements involved in carrying out this strategy, and sketch an unconditional proof of the Andre-Oort conjecture for products of modular curves. I discuss further applications and questions.

#### Mihalis Dafermos, University of Cambridge

Recent progress in mathematical general relativity

General relativity has witnessed a resurgence of activity in the last few years from the point of view of the analysis of hyperbolic partial differential equations. I review recent progress on long-standing open problems in the field, including the formation, uniqueness and stability of black holes, and the structure of singularities in gravitational collapse.



Mihalis Dafermos

#### abstracts

#### Peter Sarnak, Princeton University

Thin integer matrix groups and the affine sieve

Infinite index subgroups of groups like SL(n, Z), which are Zariski dense in SL(n) arise in geometric diophantine problems (e.g. Apollonian packings) and in problems connected with monodromy groups. One of the key features in diophantine applications is a super form of strong approximation connected with congruence graphs associated to these groups. We review the recent developments by a number of authors and discuss the applications to the general affine sieve.



Peter Sarnak

#### Manjul Bhargava, Princeton University

The average rank of elliptic curves

A rational elliptic curve may be viewed as the set of solutions to an equation of the form  $y^2=x^3+Ax+B$ , where A and B are rational numbers. It is known that the rational points on this curve possess a natural abelian group structure, and the Mordell-Weil theorem states that this group is always finitely generated. The rank of a rational elliptic curve measures how many rational points are needed to generate all the rational points on the curve. There is a standard conjecture—originating in work of Goldfeld and Katz-Sarnak—that states that the average



rank of all elliptic curves should be 1/2; however, it has not previously been known that the average rank is even finite! In this lecture, we describe recent work that shows that the average rank is finite (in fact, we show that the average rank is less than one). This is joint work with Arul Shankar.

Manjul Bhargava

## achievement



Jonathan Pila

#### **Clay Research Awards**

#### The Andre-Oort Conjectures

#### by Peter Sarnak

The conjectured finiteness properties for torsion points lying on subvarieties of products of the multiplicative group of complex numbers (Lang Conjectures) and on subvarieties of Abelian Varieties (Manin-Mumford Conjectures) have been understood for some time and by a number of different methods. The analogue of these Conjectures for Shimura varieties, in which torsion points are replaced by "Complex Multiplication points" are known as the Andre-Oort Conjectures and until recently they had for the most part resisted all attempts. Andre established the Conjecture for a product of two modular curves, and if one assumes the Generalized Riemann Hypothesis then the Conjecture is known in most cases. In his paper "o-minimality and the Andre-Oort Conjecture for any number of curves. His novel techniques have led him and others to the solution of numerous further cases of the Conjecture including that of the moduli spaces  $A_a$  of Abelian Varieties of dimension g with g<7.

Pila's proof is based in part on a generalization of his result with Bombieri (1989) which gives sharp upper bounds for the number rational points of growing height on a transcendental plane curve. The subtle higher dimensional generalization was formulated by Pila in 2004 and proved by him and Wilkie in 2006. Their proof makes essential use of o-minimal structures and in particular a parametrization of the definable sets in this theory which is ideally suited for carrying out the analysis in this real analytic setting. Their very general sharp upper bounds for the number of rational points of growing height lying on the trancendental part of an analytic set is a fundamental result in real diophantine analysis. Pila's recent work, and also his earlier treatment together with Zannier of the Manin Mumford Conjectures, show that his results with Wilkie when combined with transecdence theorems of Ax-Lindemann type and lower bounds for the sizes of Galois orbits, lead to the solution of some long standing finiteness problems in diophantine geometry (it is the lower bound that is not known for  $A_g$  when g is at least 7). Pila, Zannier and others have also applied it to problems of unlikely intersections in diophantine analysis.

## activities

#### Researchers, Workshops & Conferences | Summary of 2011 Research Activities

The activities of CMI researchers and research programs are sketched below. Researchers and programs are selected by the Scientific Advisory Board (see inside front cover).

#### **Clay Research Fellows**

**Peter Scholze**, born 1987 in Dresden, Germany, obtained his PhD in 2012 under supervision of Michael Rapoport at the University of Bonn. After working about the cohomology of Shimura varieties and the Langlands program, his PhD thesis was about a theory of perfectoid spaces, which gives a method to compare objects in mixed characteristic with objects in equal characteristic *p*, with applications to *p*-adic Hodge theory and the weight-monodromy conjecture. Peter was appointed as a Clay Research Fellow for a term of five years beginning July 2011.

Peter Scholze joined CMI's 2011 group of research fellows: **Mohammed Abou**zaid (MIT), **Tim Austin** (Brown University), **Adrian Ioana** (UCSD), **Davesh Maulik** (Columbia University), **Sophie Morel** (Harvard University), **Sucharit Sarkar** (Columbia University) and **Xinyi Yuan** (Princeton University).



Peter Scholze

#### **Research Scholars**

Daniel Allcock • (University of Texas at Austin) • 2010 – 2011 • Kyoto University

#### **Senior Scholars**

Henryk Iwaniec • MSRI • January 10 – May 20, 2011 • Program of Arithmetic Statistics
Barry Mazur • MSRI • January 10 – May 20, 2011 • Program of Arithmetic Statistics
Keith Ball • MSRI • August 15 – December 16, 2011 • Quantitative Geometry
Tobias Colding • MSRI • August 15 – December 16, 2011 • Quantitative Geometry
William Johnson • MSRI • August 15 – December 16, 2011 • Quantitative Geometry
Joe Harris • PCMI Utah • July 3 – July 23, 2011 • Moduli Spaces of Riemann Surface

### Research programs organized and supported by CMI

January 1 – December 31	. Independent University of Moscow, Moscow, Russia	
January 3 – 7	. DRIP: Density of Rational and Integral Points, Ein Gedi Field School, Israel	
January 4 – July 1	Moduli Spaces, Isaac Newton Institute for Mathematical Sciences, Cambridge, UK	
February 20 – 25	Frontiers in Complex Dynamics (Celebrating John Milnor's 80th Birthday), BIRS and Banff Conference Center, Alberta, Canada	
March 22 – 25	. K-theory and Motives, UCLA, Los Angeles, CA	
May 16 – 17	. Clay Research Conference, Harvard University, Cambridge, MA	
May 16 – 21	Conference in honor of Elias Stein's 80th Birthday, Princeton University, Princeton, NJ	
May 16 – 19	Trends in Complex Dynamics Workshop, CMI	
June 20 – August 12	. Ross Program, Ohio State University, Columbus, OH	
July 4 – August 13	. PROMYS, Boston University, Boston, MA	
July 10 – 22	International Summer School for Students, Jacobs University, Bremen, Germany	
July 11 – 15	Geometry & Topology Down Under (Summer School), University of Melbourne, Australia	
July 18 – 22	Geometry & Topology Down Under (Conference), University of Melbourne, Australia	
August 1 – 5	Latin American School of Algebraic Geometry and Applications, Graduate Courses, Buenos Aires, Argentina	
August 8 – 12	Latin American School of Algebraic Geometry and Applications, Research Workshop, Córdoba, Argentina	
August 15 – 19	Research Scholar Michael Harris Workshop on Automorphic Forms, Galois Representations, and Geometric Representation Theory, Facultad de Matemátic Astronomoica v Fisica, Argentina	
August 22 – 26	Michael Harris (Facultad de Matemátic Astronomoica v Fisica, Argentina) Algebraic Methods in Geometry: Commutative and Homological Algebra in Folliations and Singularities, (Conference), CIMAT Guanajuato, Mexico	
August 25 – 28	A Celebration of Algebraic Geometry: A Conference in Honor of Joe Harris' 60th Birthday, Harvard University, Cambridge, MA	
August 29 – September 1	. Logarithmic Geometry and Moduli Workshop, CMI	
August 29 – September 2	Algebraic Methods in Geometry: Commutative and Homological Algebra in Foliations and Singularities, (School), CIMAT Guanajuato, Mexico	cmi annual
November 19 – December 1	Algebraic vs. Analytic Geometry, ESI, Vienna, Austria	report 2011

#### **Program Allocation**

#### Estimated number of persons supported by CMI in selected scientific programs for calendar year 2011

Research Fellows, Research Awardees, Senior Scholars, Research Scholars: 19

PROMYS / Ross Faculty: 12

CMI Workshops: 46

Participants attending Conferences and Joint Programs: >1000

Independent University of Moscow: 95

#### **Research Expenses for Fiscal Year 2011**



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# profile



Davesh Maulik

#### **Interview with Research Fellow Davesh Maulik**

## What first drew you to mathematics? What are some of your earliest memories of mathematics?

Probably my earliest memories of mathematics are from elementary school. I think I was always interested in science very generally. Mathematics as a subject on its own terms came into focus because of my father, who would give me some small math and logic puzzles to play with in my spare time.

## Could you talk about your mathematical education? What experiences and people were especially influential?

I think the strongest influence by far was outside of school, when I started participating in the Ross program at Ohio State University. This is a summer program for high school students, introducing them to rigorous proofs and some basic number theory. It was my first exposure to thinking about open-ended questions and also just to meeting lots of other kids interested in mathematics. After that first year, I came back repeatedly, later as a counselor. I think it's very hard to overstate the potential impact of programs like this on mathematically minded kids. For instance, many of the people I met there are now colleagues, and many others end up in related sciences. In addition to the Ross program, which dates back to Sputnik, there are now several programs of this type around the country, all with different style and emphasis, and all pretty great.

Columbus, Ohio, also has wonderful record stores, where I spent a lot of time, so it was a pretty classic summer.

#### Did you have a mentor? Who helped you develop your interest in mathematics, and how?

I had a very active and supportive high school teacher and, because of the Ross program, knew many older people who fostered my interest. Frankly, if not for them, it would never have occurred to me that being a research mathematician was something people could do professionally.

## You were educated primarily in the United States of America. Could you comment on the distinctive features of a mathematical education in that country?

I don't have a detailed picture of education in other countries. I think one distinctive feature is that most US schools are by default liberal-arts style. So typically one's math classes are just a fraction of the total course load, while in many other places, one specializes much earlier. In my case, for example, since I wasn't sure what to do after graduating, I was taking a lot of premed classes. While this isn't universal, the result is that it may be later before many American students spend all their intellectual time pursuing mathematics.

On the other hand, my sense is that there are more opportunities in the US for extracurricular math activities, especially in terms of getting exposed to research activity, REU's and other similar programs. I don't think this balance is objectively better or worse than the alternatives, and probably depends on the student.

#### What attracted you to the particular problems you have studied?

What I've always found appealing about algebraic geometry is that it is very broad and only seems to grow broader with time, as it constantly absorbs ideas from other

subjects. What I like about this is that it supports many different styles of mathematics and mathematicians—very combinatorial for some questions, very abstract for others—and it means there always seem to be new corners to learn about.

For the specific questions I have studied, there is a lot of chance involved. Since my subject has a lot of wide-open conjectures, much of my work has started from trying to make some incremental progress there and seeing what happens. I also enjoy problems that seem like they may require learning some new techniques, or bordering on another area of mathematics.

#### Can you describe your research in accessible terms? Does it have applications to other areas?

Most of my research deals with the enumerative geometry of algebraic varieties. These are objects defined by polynomial equations and one is interested in counting objects (curves inside these varieties, vector bundles on them, etc.) satisfying geometric constraints. In this language, some of the original problems are very classical, going back a hundred years or so. In its modern form, there is currently a great deal of insight (usually conjectural) coming from other subjects—mathematical physics, symplectic geometry, and representation theory. Often this leads to the appearance of some structure - for instance, modular forms and other special functions—that is quite surprising from the original formulation and that we would like to understand better. I don't know if it has applications, exactly, but there is a great deal of fruitful interplay with other areas of mathematics, as a source of questions and techniques.

## Can you share with us some of your future plans and ambitions? What research problems and areas are you likely to explore in the future?

This is pretty difficult to predict always, even in the short term. The research questions I mentioned earlier are ongoing, and I continue to think about parts of them. However, I have recently been thinking about some geometric questions about surfaces in finite characteristic. Even though I am far from an expert with this subject (or really because of my in-expertise), I'd like to pursue this arithmetic direction some more.

## Could you comment on collaboration versus solo work as a research style? Are certain kinds of problems better suited to collaboration?

Collaboration seems as much a matter of the temperament of the people as any specific problem. For me, it is a very important part of my work, most of which has been joint with collaborators. It has been crucial in shaping my interests and as a way of learning new material on the fly. Also, I am not a very fast thinker, so it can be an efficient way of weeding out bad ideas! Of course, even in an active collaboration, most of the time is still spent alone getting a handle on the problem.

#### Regarding individual work versus collaboration, what do you find most rewarding or productive?

I certainly find collaboration more productive, although in both approaches, the reward is largely the same, when after fumbling around for a while, you start to intuit how things might come together if you're lucky.

#### How has the Clay Fellowship made a difference for you?

It's been very important in terms of giving me flexibility and freedom to pursue my research interests at a more measured pace. I spent my fellowship at Columbia (where I've since returned) and MIT, and being able to move around like that was wonderful. Also, the

staff at the CMI has been very supportive in terms of travel needs, e.g. when I wanted to spend a semester at MSRI for an algebraic geometry program, or visit a collaborator in Europe.

## What advice would you give to young people starting out in math (i.e., high school students and young researchers)?

Well, I don't feel qualified to offer serious advice to anyone! I think the hardest transition is getting used to thinking about questions that can't be looked up in a book somewhere or solved in a couple of hours. The sooner a student gets accustomed to that ritual frustration, probably the better.

#### What advice would you give lay persons who would like to know more about mathematics—what it is, what its role in our society has been and is, etc.? What should they read? How should they proceed?

This is a tough question, and there are maybe many different approaches here. For instance, if the goal is to learn something about the history of mathematical thought and how it's developed over time, then I think there are many popular-audience books along these lines, using specific results or historical figures as a focus. These are great for getting a sense of interesting mathematical concepts and which problems mathematicians have struggled with over the years. If the goal is to get exposed to toy questions that they can try to reason through, then I think the kind of books I read when younger (authors like Martin Gardner, Raymond Smullyan, etc.) could be fun, although less representative of what modern mathematics is.

I think another possible objective is to get a sense of what mathematical reasoning entails and how it might be applied in an everyday context. Here one wouldn't necessarily learn about higher mathematical ideas. I can think of plenty of short features and magazine columns along these lines, but I don't know offhand of a more systematic treatment.

#### How do you think mathematics benefits culture and society?

Like other sciences, mathematics is a way of understanding the world around us, but at a more abstract level. How closely these questions end up tied to "reality" depends on the area of mathematics in practice. A lot of progress in applied sciences has been based on very deep mathematics. However, I think it is good to keep an open mind about these potential applications, since they can be very difficult to predict in the moment.

#### Please tell us about things you enjoy when not doing mathematics.

I enjoy the usual things: getting together with friends and family, watching and playing basketball, listening to music, and going to concerts when I find the time. These days, however, I spend most of my free time with my wife, dangling shiny objects in front of my newborn son.

"I think the hardest transition [for young math researchers] is getting used to thinking about questions that can't be looked up in a book somewhere or solved in a couple of hours. The sooner a student gets accustomed to that ritual frustration, probably the better."

#### **CMI Workshops**

#### Logarithmic Geometry and Moduli | August 29 – September 1, 2011

#### by Dan Abramovich

The arrival day of the conference on Logarithmic Geometry and Moduli coincided with the arrival day of Hurricane Irene. Thankfully a good number of invited participants were nevertheless able to arrive. We focused the program around the core interests of the participants who were present, namely logarithmic Gromov-Witten theory.

Logarithmic geometry is a branch of algebraic geometry which first made its appearance in Kazuya Kato's paper Logarithmic structures of Fontaine-Illusie of 1989. A logarithmic algebraic variety is a usual algebraic variety with an additional structure imposed on it. (The structure comes in the form of a sheaf of monoids M together with a monoid homomorphism  $M \rightarrow \mathcal{O}_X$  to the structure sheaf). The point is that some *singular* algebraic varieties become *logarithmically smooth* when an appropriate logarithmic structure is imposed on them. Thus logarithmic structures are a way to discover "hidden smoothness" in singular spaces, which is often quite useful.

The prototypical example of a singular variety which is logarithmically smooth is a singular toric variety. Consider for example the surface S in the complex 3-space defined by the equation  $xy = z^k$ . As soon as k > 1 the surface is singular, but there is a canonical toric logarithmic structure which is smooth; a familiar outcome of this logarithmic smoothness is the fact that the sheaf of logarithmic forms is locally free.

An example of a morphism which is logarithmically smooth but not smooth is the map  $\pi : A^2 \rightarrow A^1$ , where  $A^2$  has coordinates x, y and  $A^1$  has coordinate t, given

by the equation t = xy. The map is clearly singular at the origin, but again the map of toric logarithmic structures is smooth. Again a familiar outcome is the fact that the sheaf of relative logarithmic forms is locally free. It can be said in general that logarithmic smoothness is the notion which combines both usual smooth maps and toric maps, and allows for arbitrary base change.

In the example  $\pi: A^2 \rightarrow A^1$  of a logarithmically smooth (but singular) map, the fibers are nodal curves, which are prominent in compactifications of moduli spaces. This suggests that logarithmic structures should be a good way to treat families of varieties degenerating to normal crossings singularities, and in particular should be well adapted for the compactification of moduli spaces.

The first examples of logarithmic geometry in moduli spaces was given by Fumiharu Kato's paper Log smooth deformation and moduli of log smooth curves (2000). Here he showed that the famous Deligne-Mumford-Knudsen moduli space of stable pointed curves  $M_{a,n}$  is a moduli space of logarithmically smooth curves. Other applications to moduli spaces were given by Olsson and Alexeev, where it is shown that logarithmic geometry can isolate the desired components of moduli spaces which otherwise contain some "undesirable elements".

Our workshop centered around constructions of spaces of stable logarithmic maps, namely stable maps  $f: C \rightarrow X$ where C is a logarithmic curve and X is a logarithmic variety. This is an extension of Kontsevich's spaces of stable maps;



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**Bumsig Kim** 

Paul Hacking

Mark Gross

#### Organizers

Dan Abramovich (Brown University) Mark Gross (Univeristy of California San Diego) Bernd Siebert (Universität Hamburg) James Carlson (CMI) – *ex officio* 

#### Speakers

Dan Abramovich (Brown University) Brett Parker (MPI, ANU) Paul Hacking (University of Massachusetts Amherst) Mark Gross (University of California San Diego) Bernd Siebert (Universität Hamburg) William Gillam (Brown University) Qile Chen (Columbia University) Bumsig Kim (Korea Institute for Advance Study) Jonathan Wise (Stanford University)

it has the advantage that it allows for the construction of Gromov-Witten invariants of a singular variety *X*, as long as it has a logarithmically smooth structure. Lectures were given on constructions of these moduli spaces (Gross, Siebert, Kim), on setting up Gromov-Witten theory (Gillam), gluing formulas (Chen), comparison with other approaches (Wise) and emerging applications (Hacking).

Another focus of the workshop was on another theory of Gromov-Witten invariants of singular varieties, namely Brett Parker's theory of exploded manifolds. A series of lectures by Parker described his theory and made bridges with the logarithmic approach.

#### **Trends in Complex Dynamics** | May 16 – 19, 2011

#### by Bob Devaney and Sarah Koch

In the middle to late 1990's, the field of complex dynamics expanded its horizons considerably. Now rational maps, entire transcendental maps, and meromorphic functions are major topics of research. The behavior of these maps is often very different from that of polynomial maps. Furthermore, people have been investigating higher dimensional holomorphic maps like the planar Henon map. These types of maps were investigated at length back in the 1970's, but the lack of complex analytic tools hindered these efforts. Now this area is once again back in focus.

At the Trends in Complex Dynamics Workshop a very diverse, talented group of mathematicians convened at the Clay Math Institute for four days of mathematical lectures, productive discussions, and fruitful collaborations. Many different areas of Complex Dynamics were featured in the talks; Hubbard, Kahn and Tan Lei talked about current work involving the Mandelbrot Set and quadratic polynomials, Fagella, Jarque and Moreno Rocha gave presentations on transcendental dynamics, while Meyer, Pilgrim and Nekrashevych presented work which was more topological in nature. Epstein and DeMarco talked about the developing link between complex dynamics and number theory, Shishikura presented work involving Teichmueller spaces, and Cantat discussed complex dynamics in several variables and automorphism groups.

In addition to the lectures from the 13 speakers, we had a number of very successful contributed talks in our "breakout sessions." The contributed talks included presentations from a number of local faculty, postdocs and graduate students, and Paul Blanchard, Elizabeth Russell,



Jeremy Kahn

Adam Epstein

Tan Lei

#### program overview

#### Organizers

Jim Carlson (CMI) Robert Devaney (Boston University) Sarah Koch (Harvard University)

#### Speakers

Dan Look, Russell Lodge and Antonio Garijo gave lectures during our breakout sessions.

There were a number of collaborations that were created, and a number which were enhanced during out workshop; when our participants were not engaged in listening to one of the scheduled lectures, they were often actively participating in a smaller discussion amongst themselves. They dispersed throughout the Clay Math Institute and even the Harvard Math Department for these discussions; it was clear that the lectures stimulated valuable interaction among the participants. Nuria Fagella (Universitat de Barcelona) Xavier Jarque (Universitat de Barcelona) Monica Moreno Rocha (CIMAT) Mitsuhiro Shishikura (Kyoto University) Laura DeMarco (University of Illinois, Chicago) Adam Epstein (University of Warwick) John H. Hubbard (Cornell University) Kevin Pilgrim (Indiana University) Jeremy Kahn (Stony Brook University, SUNY) Volodymyr Nekrashevych (Texas A&M University) Daniel Meyer (Jacobs University) Serge Cantat (Université de Rennes 1) Tan Lei (Université de Angers)

#### **CMI Supported Conferences**

#### **The International Mathematical Summer School for Students** | July 10 – 22, 2011

#### by Etienne Ghys, Dierk Schleicher and Sergei Tabachnikov

More than 100 participants from 29 countries attended the inaugural International Mathematical Summer School for Students at Jacobs University, Bremen, Germany. Inspired by the series of Russian language summer schools "Contemporary Mathematics" that have taken place in Dubna near Moscow for more than ten years, the goal of the school was to bring together top-notch mathematicians of today and tomorrow by inviting a number of the brightest high school and university students and having them actively interact with some of the world's finest mathematics researchers and communicators.

The summer school lasted 12 days and offered daily lectures and parallel talks. Each speaker presented at least one plenary talk in the beginning, allowing students to get to know the speakers before choosing between parallel talks. On many evenings, there were special sessions on various topics, including a session on the mathematics of tides the evening before a full-day excursion to an island in the Wadden Sea. A particular highlight was an evening devoted to a discussion on "Free Will" by a mathematician, a physicist, and a neurobiologist and philosopher. The contributors were John Conway (Princeton University), codiscoverer of the Free Will Theorem, Peter Schupp (Jacobs University) on quantum mechanics and determinism and Gerhard Roth (Universität Bremen, President of the German "Studienstiftung"), with research interests on neurobiological, philosophical, and legal aspects of free will. After presentations by the three experts there was a lively discussion with participants, led by Etienne Ghys.

Throughout the week the presentations covered a large area of mathematics, including:

- Probability: Wendelin Werner's four-lecture course started with discrete random walks and harmonic functions on graphs and ended with the non-recurrence of 3D Brownian motion and an implication for real life: the big gradient of the electric potential at the sharp end of an electrode.
- Number theory: Don Zagier gave a breathtaking walk of constant discovery, moving from the very origins of algebraic number theory to the Birch and Swinnerton-Dyer conjecture (one of the seven Millennium Problems).
- "Physical Mathematics": Tadashi Tokieda's outstanding lectures were on mathematics where it touches the real world—applied mathematics for real physics processes. Water waves, tsunamis and tides, estimating the power of a blast and stabilization of the upper equilibrium for a rotating pendulum—all of this was blended into some magic of discovery, of constant brainwork and constant



O COURTESY OF CHRIS NICHOLLS

Don Zagier

simplicity, simple arguments breaking through complicated questions.

- Complex Dynamical Systems: John Hubbard impressed the students with illustrating and explaining the theory of holomorphic dynamics and the Mandelbrot set that he and Adrien Douady had discovered.
- Coding Theory, Games and Surreal Numbers, The Free Will Theorem, and More: John Conway always manages to impress his audience with his broad interests and unexpected discoveries on many different areas of mathematics.
- Geometry in different flavours was the topic of the contributions by Etienne Ghys, Sergei Tabachnikov, and Dmitry Fuchs.
- Polygons and combinatorics was the topic of lectures by Günter Ziegler, a leading researcher and communicator of mathematics, and many others.

The summer school was organized by Jacobs University. It was supported by the Volkswagen foundation (VolkswagenStiftung, Hannover) and the Clay Mathematics Institute. We hope that this series of summer schools will develop an equally lasting tradition as our Russian partner.

#### **Project Leaders**

Etienne Ghys (École Normale Supérieure de Lyon) Dierk Schleicher (Jacobs University) Sergei Tabachnikov (Pennsylvania State University)

#### **Scientific Committee**

Etienne Ghys (École Normale Supérieure de Lyon) Frances Kirwan (University of Oxford) Dierk Schleicher (Jacobs University) Alexei Sossinsky (University of Moscow) Sergei Tabachnikov (Pennsylvania State University) Anatoliy Vershik (University of St. Petersburg) Wendelin Werner (Université Paris-Sud, Orsay) Jean-Christophe Yoccoz (Université Paris-Sud, Orsay) Don Zagier (Max Planck Institute Bonn and Collège de France) Günter M. Ziegler (Freie Universität Berlin)

#### Speakers

Martin Andler (Université Versailles-Saint-Quentin) Mario Bonk (University of California, Los Angeles) John Conway (Princeton University) Dmitry Fuchs (University of California, Davis) Étienne Ghys (École Normale Supérieure de Lyon) John Hubbard (Cornell University and Université de Marseille) Victor Kleptsyn (Université de Rennes) Rostislav Matveev (Universität Leipzig) Gerhard Roth (Universität Bremen) Dierk Schleicher (Jacobs University) Peter Schupp (Jacobs University) Sergei Tabachnikov (Pennsylvania State University) Tadashi Tokieda (University of Cambridge) Wendelin Werner (Université Paris-Sud, Orsay) Don Zagier (Max Planck Institute Bonn and Collège de France) Günter Ziegler (Freie Universität Berlin)



John Conway

#### program overview



Buenos Aires group PHOTO COURTESY OF ALICIA DICKENSTEIN

#### Latin American School on Algebraic Geometry and Applications Buenos Aires and Cordoba, Argentina | August 1 – 12, 2011

#### by Alicia Dickenstein

More than 199 participants, including 59 invited speakers from all over the world, came to Argentina for this summer program consisting of graduate courses and a research workshop. Algebraic geometry has long been a central subject, with deep and substantial connections to almost every aspect of modern mathematics, including recent applications to different sciences. Even though there are several researchers in the field in Latin America, this is not an area with a tradition and a visible community. The aim of this event was to establish a framework for long-term fruitful connections and synergies between these researchers and students from over 30 different countries.

The first week in Buenos Aires was devoted to a graduate school with four courses taught by well-known specialists: Peter Beelen (Technical University of Denmark) introduced algebraic geometry codes, discussing their construction and their decoding; Joe Harris (Harvard University) covered the basics of intersection theory and its applications; Robin Hartshorne (University of California, Berkeley) offered an introductory course in deformation theory; and Diane Maclagan (University of Warwick) provided an elementary introduction to the emerging field of tropical algebraic geometry where a variety is replaced by

a combinatorial shadow, known as the tropical variety. The second week in Cordoba offered plenary lectures

by leading experts as well as research sessions with talks on a wide range of topics in algebraic geometry, including algebraic curves and their moduli, vector bundles and their moduli, surfaces and higher dimensional varieties, classical projective geometry, intersection theory and enumerative geometry, D-modules and differential algebra, Kähler geometry and generalizations, Hodge theory, complex dynamics and holomorphic foliations, automorphic forms, theta functions and modular forms, algebraic groups, arithmetic algebraic geometry, elliptic curves, abelian varieties, singularity theory, computational methods in algebraic geometry and commutative algebra, connections with combinatorics, toric varieties, connections with theoretical physics, mirror symmetry, and applications of algebraic geometry to other sciences.

The program was a joint venture of CIMPA (Centre International de Mathématiques Pures et Appliquées) and ICTP (International Center for Theoretical Physics), combined with the School of Exact and Natural Sciences of the University of **Buenos Aires**.

All the classes were recorded in HD video and are freely available at: http://mate.dm.uba.ar/~visita16/ELGA-2011/ index.shtml following the link to photos and videos: http://mate.dm.uba.ar/~visita16/ELGA-2011/version/v1/ images-en.shtml

#### Organizers

María Laura Barberis (Universidad Nacional de Córdoba) Fernando Cukierman (Universidad de Buenos Aires) Alicia Dickenstein (Universidad de Buenos Aires) Javier Fernández (Instituto Balseiro) Roberto Miatello (Universidad Nacional de Córdoba)

#### **Graduate Course Speakers**

Peter Beelen (Technical University of Denmark) Joe Harris (Harvard University) Robin Hartshorne (University of California, Berkeley) Diane Maclagan (University of Warwick)

#### **Workshop Speakers**

Carolina Araujo (IMPA) Nicolás Botbol (Universidad de Buenos Aires) Leticia Brambila Paz (CIMAT) Roberto Callejas-Bedregal (Universidade Federal de Paraíba) Cicero Carvalho (Universidade Federal de Uberlândia) François Charles (IRMAR) Juliana Coelho (Universidade Federal Fluminense) Maria Angelica Cueto (Columbia University) Luis Dieulefait (Universitat de Barcelona) Sandra Di Rocco (Kungliga Tekniska Högskolan) Joan Elias (Universitat de Barcelona) Eduardo Esteves (IMPA) Renato Fehlberg Junior (Universidade de São Paulo) Javier Fernandez de Bobadilla (Instituto de Ciencias Mathemáticas) Eduardo Friedman (Universidad de Chile) Lothar Goettsche (International Centre for Theoretical Physics) Dorian Goldfeld (Columbia University) Lucio Guerberoff (University of California, Los Angeles) Joe Harris (Harvard University) Michael Harris (Institut de Mathématiques de Jussieu) Robin Hartshorne (University of California, Berkeley) Reimundo Heluani (IMPA) Marcos Jardim (IMECC) Gabriela Jeronimo (Universidad de Buenos Aires) Abhinav Kumar (Massachusetts Institute of Technology) Shrawan Kumar (University of North Carolina at Chapel Hill) Laurentiu Maxim (University of Wisconsin, Madison)

James McKernan (Massachusetts Institute of Technology) Margarida Melo (University of Coimbra) Rosa María Miró Roig (University of Barcelona) Ruxandra Moraru (University of Waterloo) Daniela Moura Prata dos Santos (UNICAMP) Juan Carlos Naranjo (University de Barcelona) Ariel Pacetti (Universidad de Buenos Aires) Ivan Pan (Universidade Federal do Rio Grande do Sul) Sam Payne (Yale University) Gregory Pearlstein (Michigan State University) Jorge Vitorio Pereira (IMPA) Ragni Piene (University of Oslo) Gian Pietro Pirola (Università di Pavia) Enrique Reyes (Cinvestav) Claudia Reynoso (CIMAT) Anita Rojas (Universidad de Chile) Osvaldo Santillán (Universidad de Buenos Aires) Florent Schaffhauser (Universidad del los Andes) Laura Schaposnik (University of Oxford) José Seade (Universidad Nacional Autónoma de Mexico) Anne Shiu (University of Chicago) Aron Simis (Universidade Federal de Pernambuco) Michael Singer (North Carolina State University) Marcio Soares (Universidade Federal de Minas Gerais) Gonzalo Tornaría (Centro de Matemática) Armando Treibich Kohn (Universidad de la Republica Regional Norte) Bernardo Uribe (Universidad de los Andes) Giancarlo Urzua (Pontifica Universidad Católica de Chile) Israel Vainsencher (Universidade Federal de Minas Gerais) Marie France Vigneras (Institut de Mathématiques de Jussieu)

Filippo Viviani (Università Roma Tre)



Cordoba group

PHOTO COURTESY OF ALICIA DICKENSTEIN

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2011

## publications

#### **Selected Articles by Research Fellows**

#### **Tim Austin**

Equidistribution of joinings under off-diagonal polynomial flows of nilpotent Lie groups, to appear in *Ergodic Theory and Dynamical Systems*. arXiv:1105.5612

On discontinuities of cocycles in cohomology theories for topological groups, submitted. arXiv:1112.1465

#### **Mohammed Abouzaid**

Exact Lagrangians in plumbings, with Ivan Smith, to appear in *Geometric and Functional Analysis*. arXiv:1107.0129

Lagrangian fibrations on blowups of toric varieties and mirror symmetry for hypersurfaces, with Denis Auroux, and Ludmil Katzarkov, submitted. arXiv:1205.0053

#### **Davesh Maulik**

Supersingular K<sub>3</sub> surfaces for large primes, submitted. arXiv:1203.2889

Macdonald formula for curves with planar singularities, with Zhiwei Yun, to appear in *Crelle*. arXiv:1107.2175

#### Sucharit Sarkar

A Khovanov homotopy type, with Robert Lipshitz, submitted. arXiv:1112.3932

A Steenrod square on Khovanov homology, with Robert Lipshitz, submitted. arXiv:1204.5776

#### **Books**

#### **Grassmannians, Moduli Spaces and Vector Bundles**

Editors: David A. Ellwood, Emma Previato

CMI/AMS, 2011, 180 pp., softcover, ISBN: 0-8218-5205-1. List Price: \$55. AMS Members: \$44. Order Code: CMIP/14.. www.claymath.org/ publications/VectorBundles/

This collection of cutting-edge articles on vector bundles and related topics originated from a CMI workshop, held in October

2006, that brought together a community indebted to the pioneering work of P. E. Newstead, visiting the United States for the first time since the 1960s. Moduli spaces of vector bundles were then in their infancy, but are now, as demonstrated by this volume, a powerful tool in symplectic geometry, number theory, mathematical physics, and algebraic geometry. This volume offers a sample of the vital convergence of techniques and fundamental progress taking place in moduli spaces at the outset of the twenty-first century.

#### Peter Scholze

On the cohomology of compact unitary group Shimura varieties at ramified split places, with Sug Woo Shin, submitted to *Journal of the American Mathematical Society*. arXiv:1110.0232

Perfectoid Spaces, submitted to *Publications Mathematiques de l'IHES*. arXiv:1111.4914

#### Adrian loana

Uniqueness of the group measure space decomposition for Popa's HT factors. arXiv:1104.2913

Compact actions and uniqueness of the group measure space decomposition of II\_1 factors. arXiv:1112.0831

#### Xinyi Yuan

Volumes of Arithmetic Okounkov Bodies, preprint. arXiv:1201.2221

Effective Bound of Linear Series on Arithmetic Surfaces, with Tong Zhang, preprint. arXiv:1201.2216

#### **On Certain L-Functions**

Editors: James Arthur, James W. Cogdell, Steve Gelbart, David Goldberg, Dinakar Ramakrishnan, Jiu-Kang Yu

CMI/AMS, 2011, 647 pp., softcover, ISBN: 0-8218-5204-3. List Price: \$129. AMS Members: \$103.20. Order Code: CMIP/13. www.claymath.org/publications/Shahidi/



This volume constitutes the proceedings of the conference organized in honor of the 6oth birthday of Freydoon Shahidi, who is widely recognized as having made groundbreaking contributions to the Langlands program. The articles in this volume represent a snapshot of the state of the field from several viewpoints. Contributions illuminate various areas of the study of geometric, analytic, and number theoretic aspects of automorphic forms and their L-functions, and both local and global theory are addressed.

### Motives, Quantum Field Theory, and Pseudodifferential Operators

Editors: Alan Carey, David Ellwood, Sylvie Paycha, Steven Rosenberg

CMI/AMS, 2010, 349 pp., softcover. ISBN: 0-8218-5199-3. List price: \$89. AMS Members: \$71.20. Order Code: CMIP/12. www.claymath.org/publications/Motives Quantum/



This volume contains articles related to the conference "Motives, Quantum Field Theory, and Pseudodifferential Operators" held at Boston University in June 2008, with partial support from the Clay Mathematics Institute, Boston University, and the National Science Foundation. There are deep but only partially understood connections between the three conference fields, so this book is intended both to explain the known connections and to offer directions for further research.

#### Quanta of Maths; Proceedings of the Conference in honor of Alain Connes

Editors: Etienne Blanchard, David Ellwood, Masoud Khalkhali, Matilde Marcolli, Henri Moscovici, Sorin Popa



CMI/AMS, 2010, 675 pp., softcover, ISBN: 0-8218-5203-5. List price:

\$129. AMS Members: \$103.20. Order Code: CMIP/11. www. claymath.org/publications/Quanta\_Maths/

The work of Alain Connes has cut a wide swath across several areas of mathematics and physics. Reflecting its broad spectrum and profound impact on the contemporary mathematical landscape, this collection of articles covers a wealth of topics at the forefront of research in operator algebras, analysis, noncommutative geometry, topology, number theory and physics.

## Homogeneous Flows, Moduli Spaces and Arithmetic

Editors: Manfred Einsiedler, David Ellwood, Alex Eskin, Dmitry Klein, Elon Lindenstrauss, Gregory Margulis, Stefano Marmi, Jean-Christophe Yoccoz



CMI/AMS, 2010, 438 pp., softcover, ISBN: 0-8218-4742-2. List price:

\$99. AMS Members: \$79.20. Order Code: CMIP/10. www. claymath.org/publications/Homogeneous\_Flows/

This book contains a wealth of material concerning two very active and interconnected directions of current research at the interface of dynamics, number theory and geometry. Examples of the dynamics considered are the action of subgroups of SL(n,R) on the space of unit volume lattices in Rn and the action of SL (2, R) or its subgroups on moduli spaces of flat structures with prescribed singularities on a surface of genus >= 2.

#### The Geometry of Algebraic Cycles

Editors: Reza Akhtar, Patrick Brosnan, Roy Joshua

CMI/AMS, 2010, 187 pp., softcover, ISBN: 0-8218-5191-8. List Price: \$52. AMS Members: \$41.60. Order Code: CMIP/9. www.claymath.org/ publications/Algebraic\_Cycles/

The subject of algebraic cycles has



its roots in the study of divisors, extending as far back as the nineteenth century. Since then, and in particular in recent years, algebraic cycles have made a significant impact on many fields of mathematics, among them number theory, algebraic geometry, and mathematical physics. The present volume contains articles on all of the above aspects of algebraic cycles.

#### Arithmetic Geometry

Editors: Henri Darmon, David Ellwood, Brendan Hassett, Yuri Tschinkel.

CMI/AMS 2009, 562 pp., softcover. ISBN:0-8218-4476-8. List price: \$119. AMS Members: \$95.20. Order Code: CMIP/8. www.claymath.org/publications/Arithmetic\_Geometry/

This book is based on survey lectures given at the 2006 CMI Summer School at the Mathematics Institute of the University of Gottingen. It introduces readers to modern techniques and outstanding conjectures at the interface of number theory and algebraic geometry.

#### Dirichlet Branes and Mirror Symmetry

Editors: Michael Douglas, Mark Gross.

CMI/AMS 2009, 681 pp., hardcover. ISBN: 0-8218-3848-2. List price: \$109. AMS Members: \$87.20. Order Code: CMIM/4. www.claymath.org/ publications/Dirichlet\_Branes/



The book first introduces the

notion of Dirichlet brane in the context of topological quantum field theories, and then reviews the basics of string theory. After showing how notions of branes arose in string theory, it turns to an introduction to the algebraic geometry, sheaf theory, and homological algebra needed to define and work with derived categories. The physical existence conditions for branes are then discussed, culminating in Bridgeland's definition of stability structures. The book continues with detailed treatments of the



### publications

Strominger-Yau-Zaslow conjecture, Calabi-Yau metrics and homological mirror symmetry, and discusses more recent physical developments.

#### Analytic Number Theory: A Tribute to Gauss and Dirichlet

Editors: William Duke, Yuri Tschinkel.

CMI/AMS, 2007, 265 pp., softcover. ISBN: 0-8218-4307-9. List Price: \$50. AMS Members: \$40. Order Code: CMIP/7. www.claymath.org/ publications/Gauss\_Dirichlet/



This volume contains the proceedings of the Gauss-Dirichlet Conference held in Göttingen from June 20–24 in 2005, commemorating the 150th anniversary of the death of Gauss and the 200th anniversary of Dirichlet's birth. It begins with a definitive summary of the life and work of Dirichlet by J. Elstrodt and continues with thirteen papers by leading experts on research topics of current interest within number theory that were directly influenced by Gauss and Dirichlet.

#### **Ricci Flow and the Poincaré Conjecture**

Authors: John Morgan, Gang Tian.

CMI/AMS, 2007, 521 pp., hardcover. ISBN: 0-8218-4328-1. List price: \$71. AMS Members: \$56.80. Order Code: CMIM/3. www.claymath.org/ publications/ricciflow/



This book presents a complete

and detailed proof of the Poincaré conjecture. This conjecture was formulated by Henri Poincaré in 1904 and has remained open until the recent work of Grigory Perelman. The arguments given in the book are a detailed version of those that appear in Perelman's three preprints.

#### The Millennium Prize Problems

Editors: James Carlson, Arthur Jaffe, Andrew Wiles.

CMI/AMS, 2006, 165 pp., hardcover. ISBN: 0-8218-3679-X. List Price: \$30. AMS Members: \$24. Order Code: MPRIZE. www.claymath.org/publications/ Millennium\_Problems/



This volume gives the official

description of each of the seven problems as well as the rules governing the prizes. It also contains an essay by Jeremy Gray on the history of prize problems in mathematics.

#### Surveys in Noncommutative Geometry

Editors: Nigel Higson, John Roe.

CMI/AMS, 2006, 189 pp., softcover. ISBN: 0-8218-3846-6. List Price: \$50. AMS Members: \$40. Order Code: CMIP/6. www.claymath. org/publications/Noncommutative\_Geometry/



In June of 2000, a summer school on noncommutative geometry, organized jointly by the American Mathematical Society and the Clay Mathematics Institute, was held at Mount Holyoke College in Massachusetts. The meeting centered around several series of expository lectures that were intended to introduce key topics in noncommutative geometry to mathematicians unfamiliar with the subject. Those expository lectures have been edited and are reproduced in this volume.

#### Floer Homology, Gauge Theory, and Low-Dimensional Topology

Editors: David Ellwood, Peter Ozsváth, András Stipsicz, Zoltán Szábo.

CMI/AMS, 2006, 297 pp., softcover. ISBN: 0-8218-3845-8. List price: \$66. AMS Members: \$52.80. Order Code: CMIP/5. www.claymath.org/ publications/Floer\_Homology/



This volume grew out of the summer school that took place in June of 2004 at the Alfréd Rényi Institute of Mathematics in Budapest, Hungary. It provides a state-of-the-art introduction to current research, covering material from Heegaard Floer homology, contact geometry, smooth fourmanifold topology, and symplectic four-manifolds.

#### Lecture Notes on Motivic Cohomology

Authors: Carlo Mazza, Vladimir Voevodsky, Charles Weibel.

CMI/AMS, 2006, 216 pp., softcover. ISBN: 0-8218-5321-X. List Price: \$47. AMS Members: \$37.60. Order Code: CMIM/2.S. www.claymath.org/publications/Motivic\_Cohomology/

This book provides an account



of the triangulated theory of motives. Its purpose is to introduce the reader to motivic cohomology, to develop its main properties, and finally to relate it to other known

its main properties, and finally to relate it to other known invariants of algebraic varieties and rings such as Milnor K-theory, étale cohomology, and Chow groups.

#### Harmonic Analysis, the Trace Formula and Shimura Varieties

Editors: James Arthur, David Ellwood, Robert Kottwitz.

CMI/AMS, 2005, 689 pp., softcover. ISBN: 0-8218-3844-X. List Price: \$131. AMS Members: \$104.80. Order Code: CMIP/4. www.claymath.org/ publications/Harmonic\_Analysis/

The subject of this volume is the trace formula and Shimura varieties. These areas have been especially difficult to learn because of a



lack of expository material. This volume aims to rectify that problem. It is based on the courses given at the 2003 Clay Mathematics Institute Summer School at Fields Institute, Toronto. Many of the articles have been expanded into comprehensive introductions, either to the trace formula or to the theory of Shimura varieties, or to some aspect of the interplay and application of the two areas.

#### **Global Theory of Minimal Surfaces**

Editor: David Hoffman.

CMI/AMS, 2005, 800 pp., softcover. ISBN: 0-8218-3587-4. List Price: \$131. AMS Members: \$104.80. Order Code: CMIP/2. www.claymath.org/ publications/Minimal\_Surfaces/



2001 CMI Summer School held at MSRI. The subjects covered include minimal and constant-mean-curvature submanifolds, geometric measure theory and the doublebubble conjecture, Lagrangian geometry, numerical simulation of geometric phenomena, applications of mean curvature to general relativity and Riemannian geometry, the isoperimetric problem, the geometry of fully nonlinear elliptic equations, and applications to the topology of three-manifolds.

#### **Strings and Geometry**

Editors: Michael Douglas, Jerome Gauntlett, Mark Gross.

CMI/AMS, 2004, 376 pp., softcover. ISBN: 0-8218-3715-X. List Price: \$76. AMS Members: \$60.80. Order Code: CMIP/3. www.claymath.org/ publications/Strings\_Geometry/



This volume is the proceedings of the 2002 Clay Mathematics Institute Summer School held at the Isaac Newton Institute for Mathematical Sciences in Cambridge, UK. It contains a selection of expository and research articles by lecturers at the school and highlights some of the current interests of researchers working at the interface between string theory and algebraic geometry. The topics covered include manifolds of special holonomy, supergravity, supersymmetry, D-branes, the McKay correspondence and the Fourier-Mukai transform.

#### **Mirror Symmetry**

Editors: Cumrun Vafa, Eric Zaslow.

CMI/AMS, 2003, 929 pp., hardcover. ISBN: 0-8218-2955-6. List Price: \$137. AMS Members: \$109.60. Order Code: CMIM/1. www.claymath.org/ publications/Mirror\_Symmetry/

This thorough and detailed exposi-



tion develops mirror symmetry from both mathematical and physical perspectives and will be particularly useful for those wishing to advance their understanding by exploring mirror symmetry at the interface of mathematics and physics. This one-of-a-kind volume offers the first comprehensive exposition on this increasingly active area of study. It is carefully written by leading experts who explain the main concepts without assuming too much prerequisite knowledge.

#### Strings 2001

Editors: Atish Dabholkar, Sunil Mukhi, Spenta R. Wadia.

CMI/AMS, 2002, 489 pp., softcover. ISBN: 0-8218-2981-5. List Price: \$86. ASM Members: \$68.80. Order Code: CMIP/1. www.claymath.org/publications/Strings\_2001/

This multi-authored book summarizes the latest results across all areas of string theory from the perspective of world-renowned experts, including Michael Green, David Gross, Stephen Hawking, John Schwarz, Edward Witten and others. The book comes out of the "Strings 2001" conference,



organized by the Tata Institute of Fundamental Research (Mumbai, India), the Abdus Salam ICTP (Trieste, Italy), and the Clay Mathematics Institute (Cambridge, MA, USA). Individual articles discuss the study of D-branes, black holes, string dualities, compactifications, Calabi-Yau manifolds, conformal field theory, noncommutative field theory, string field theory, and string phenomenology. Numerous references provide a path to previous findings and results.

To order books please visit www.ams.org/bookstore.

### publications

CMI's library holdings include two collections of mathematical books that were acquired as gifts by the Institute:

#### Raoul Bott Library, gift received from the Bott family in 2005.

701 volumes consisting of books, journals, and preprints on topology, geometry, and theoretical physics.

#### *George Mackey Library, gift received from the Mackey family in 2007.*

1,310 volumes consisting of books and periodicals related to quantum mechanics, group representations, and physics, in addition to titles on a wide range of historical, philosophical, and scientific topics. CMI's Digital Library includes the following facsimiles of significant historical mathematical books and manuscripts that are accessible online at: www.claymath.org/library/historical

## *Euclid's Elements, Constantinople, 888 AD (Greek). MS at the Bodleian Library.*

The oldest extant manuscript and printed editions of Euclid's Elements, in Greek (888 AD) and Latin (1482 AD), respectively. High resolution copies of the manuscript are available for study at the Bodleian Library, Oxford University. Full online edition available at rarebookroom.org.

#### Riemann's 1859 Manuscript

The manuscript in which Riemann formulated his famous conjecture about the zeroes of the zeta function.

#### Felix Klein Protokolle

The Klein Protokolle, comprising 8,600 pages in twentynine volumes, records the activity of Felix Klein's seminar in Goettingen for the years 1872-1912

### 2012 CMI calendar

Date	Event	Location
January 1 – December 31	PRIMES	MIT, Cambridge, MA
January 1 – December 31	Independent University of Moscow	Moscow, Russia
January 9 – May 20	Senior Scholars Richard Kenyon and Greg Lawler, "Random Spatial Processes"	MSRI
Spring 2012	Senior Scholar Xavier Buff, "ICERM's Complex and Arithmetic Dynamics"	ICERM
June 3 – 30	CMI Summer School on the Resolution of Singular Algebraic Varieties	Obergurgl, Austria
June 18 – 19	Clay Research Conference	University of Oxford, Oxford, UK
June 18 – July 27	Ross Program	Ohio State University, Columbus, OH
July 1 – 21	Senior Scholars Bill Thurston, Martin Bridson and Alex Lubotzky, "Geometric Group Theory"	PCMI, Park City, UT
July 1 – August 11	PROMYS	Boston University, Boston, MA
August 20 – 30	International Summer School of Mathematics for Young Students	Lyon, France
August 20 – September 20	Senior Scholars Karen Smith and Claudio Procesi, "Commutative Algebra"	MSRI
September 3 – 7	SAVANT: Synergies and Vistas in Analytic Number Theory	University of Oxford, Oxford, UK

www.claymath.org