Clay Mathematics Institute

Contents

2008

James Carlson

Dear Friends of Mathematics,

I would like to single out four activities of the Clay Mathematics Institute this past year that are of special interest. The first was a small workshop, organized on short notice, on the topic of resolution of singularities of algebraic varieties in characteristic $p > 0$. The characteristic zero case was solved by Heisuke Hironaka in a monumental paper, "Resolution of singularities of an algebraic variety over a field of characteristic zero I, II," *Annals of Mathematics* 1964 (pp 109-203 and 205-326). Without resolution of singularities, modern algebraic geometry would be a far different subject. Since Hironaka's paper, resolution of singularities in characteristic zero has been better and better understood by a series of authors including Bierstone and Millman (1997), Encinas and Villamayor (1998) Encinas and Hauser (2002), Cutkosky (2004), Wlodarczyk (2005), and Kollár (2007). A complete proof, suitable for an advanced graduate course, can now be given in a tenth of the space of the original. Such is the progress of mathematics!

In characteristic $p > 0$, much less is known. The case of curves is elementary (normalize) while the case of surfaces, due to Abhyankar (1956) is difficult. Abhyankar also proved resolution for threefolds in characteristic $p > 5$. Cutkosky recently gave a short proof of this result; Cossart and Piltant have given a proof valid in all characteristics.

The major area of progress in the general case has been the work of de Jong (1996) and Abramovich-de Jong (1998), who proved a weaker result: a singular variety is the image under a dominant finite map of smooth variety. This is sufficient for many, but not all applications. For instance, it is still not known whether the dimension of the space of holomorphic q-forms is a birational invariant in characteristic p. In recent years there has been renewed progress on the problem by Hironaka, Villamayor and his collaborators, Wlodarczyck, Kawanoue-Matsuki, Teissier, and others. A workshop at the Clay Institute brought many of those involved together for four days in September to discuss recent developments. Participants were Dan Abramovich, Dale Cutkosky, Herwig Hauser, Heisuke Hironaka, János Kollár, Tie Luo, James McKernan, Orlando Villamayor, and Jaroslaw Wlodarczyk. A superset of this group met later at RIMS in Kyoto at a workshop organized by Shigefumi Mori.

Second was the CMI workshop organized by Rahul Pandharipande and Davesh Maulik. Workshops are intended to foster communication and hence the creation of new mathematical knowledge. This one had a quick payoff: the solution of the Yau-Zaslow conjecture for rational curves on K3 surfaces (see "Noether-Lefschetz theory and the Yau-Zaslow conjecture," A. Klemm, D. Maulik, R. Pandharipande, and E. Scheidegger; arXiv:0807.2477).

Third was the CMI workshop "Stringy Reflections on the LHC." This meeting, organized by Cumrun Vafa, brought together leading string theorists and particle phenomenologists to discuss the potentially observable data that could emerge from the Large Hadron Collider in Geneva after it begins operation in 2009.

Fourth was the Clay Research Summer School, held in Zürich, Switzerland on the subject of Evolution Equations. The month-long school was organized by David Ellwood, Igor Rodnianski, Gigliola Staffilani, and Jared Wunsch. It was the first such school in analysis per se. Especially noteworthy was the large number of participants, one hundred seventy-eight, and the fact that thirty-eight came with their own funding. As with all Clay Summer Schools, a volume with written versions of the courses and the topical lectures will appear in the CMI-AMS proceeeding series.

Sincerely,

James A. Carlson

Clay Research Conference

The second Clay Research Conference, an event devoted to recent advances in mathematical research, was held at MIT on May 12 and 13 at the MIT media lab (Bartos Theatre). The lectures, listed on the right, covered a wide range of fields: algebraic geometry, symplectic geometry, dynamical systems, geometric analysis, and probability theory.

Conference speakers were Kevin Costello (Northwestern University), Helmut Hofer (Courant Institute, NYU), János Kollár (Princeton University), Tom Mrowka (MIT), Assaf Naor (Courant Institute, NYU), Rahul Pandharipande (Princeton University), Scott Sheffield (Courant Institute, NYU), and Claire Voisin (CNRS, IHÉS, and Inst. Math. Jussieu). Abstracts of their talks are given below. Videos of the talks are available on the Clay Mathematics Institute web site, at **www.claymath.org/publications/videos.**

On the afternoon of May 12, the Clay Research Awards were presented to Claire Voisin and to Cliff Taubes. The citations read:

Cliff Taubes (Harvard University) for his proof of the Weinstein conjecture in dimension three.

Claire Voisin (CNRS, IHÉS, and Inst. Math. Jussieu) for her disproof of the Kodaira conjecture.

The Clay Research Award is presented annually to recognize major breakthroughs in mathematical research. Awardees receive the bronze sculpture "Figureight Knot Complement VII/CMI" by artist-mathematician Helaman Ferguson. They also receive flexible research support for a period of one year.

Tom Mrowka delivering his talk at the conference.

Clay Research Awards

Previous recipients of the award, in reverse chronological order are:

2007	Alex Eskin (University of Chicago) Christopher Hacon (University of Utah) and James McKernan (UC Santa Barbara) Michael Harris (Université de Paris VII) and Richard Taylor (Harvard University)
2005	Manjul Bhargava (Princeton University) Nils Dencker (Lund University, Sweden)
2004	Ben Green (Cambridge University) Gérard Laumon (Université de Paris-Sud, Orsay) Bao-Châu Ngô (Université de Paris-Sud, Orsay)
2003	Richard Hamilton (Columbia University) Terence Tao (University of California, Los Angeles)
2002	Oded Schramm (Theory Group, Microsoft Research) Manindra Agrawal (Indian Institute of Technology, Kanpur)
2001	Edward Witten (Institute for Advanced Study) Stanislav Smirnov (Royal Institute of Technology, Stockholm)
2000	Alain Connes (College de France, IHES, Vanderbilt University) Laurent Lafforgue (Institut des Hautes Études Scientifiques)
1999	Andrew Wiles (Princeton University)

The Clay Mathematics Institute presents the Clay Research Award annually to recognize major breakthroughs in mathematical research. Awardees receive the bronze sculpture "Figureight Knot Complement vii/CMI" by Helaman Ferguson and are named Clay Research Scholars for a period of one year. As such they receive substantial, flexible research support. Awardees have used their research support to organize a conference or workshop, to bring in one or more collaborators, to travel to work with a collaborator, and for other endeavors.

Clay Research Conference

Abstracts of Talks

Kevin Costello (Northwestern University)

A Wilsonian point of view on renormalization of quantum field theories

A conceptual proof of renormalizability of pure Yang-Mills theory in dimension four was given based on an approach to Wilson's effective action and the Batalin-Vilkovisky formalism.

Helmut Hofer (Courant Institute, NYU)

A generalized Fredholm theory and some new ideas in nonlinear analysis and geometry

The usual notion of differentiability in infinite-dimensional Banach spaces is Fréchet differentiability. It can be viewed as a straightforward generalization of the finite-dimensional notion. The important feature of Fréchet differentiability is the validity of the chain rule. However, there are different generalizations, and a new one is sc-differentiability. This notion also has a chain rule. Sc-smoothness requires additional structure on the Banach spaces, so-called sc-structure.

The striking difference between "Fréchet-smooth" and "sc-smooth" can be seen when studying maps $r : U \rightarrow U$, satisfying $r \circ r = r$, i.e., retractions, where U is an open subset of a Banach space. If r is Fréchet-smooth, then $r(U)$ is necessarily a submanifold of U. However, there are sc-smooth examples where $r(U)$ is finite-dimensional, but has locally varying dimension. There are also examples where a connected $r(U)$ has finite-dimensional as well as infinite-dimensional parts. If we consider pairs (O, E), where O is a subset of the sc-Banach space E and is also the image of an sc-smooth retraction, we obtain new local models for smooth spaces. We even can define the tangent of $T(O, E)$ by (TO, TE), where $TO = T\,r(T, U)$. Noting that by the chain rule $T r \circ Tr = Tr$ we see that TO is again an sc-smooth retraction. As it turns out, the definition does not depend on r as long as O is the image of r. We also can define the sc-smooth maps between local sc-models. Evidently, many constructions known from differential geometry can be carried over to a new "sc-retraction based differential geometry". Manifolds become M-polyfolds and orbifolds become polyfolds.

Helmut Hofer delivering his talk at the conference.

A nonlinear elliptic differential operator can usually be interpreted as a Fredholm section of a Banach space bundle and, given enough compactness, can be studied topologically. Many interesting problems in geometry are related to elliptic problems that show a lack of compactness, like bubbling-off. However, these problems usually have fancy compactifications. Two such problems of interest are Gromov-Witten theory and the more general symplectic field theory. Due to serious compactness and transversality issues, it is difficult to study them in a classical Banach manifold set-up. However, it turns out that they are much more easily described in sc-retraction-based differential geometry.

Finally, there is a generalization of the classical nonlinear Fredholm theory to the sc-world, which also has a built-in Sard-Smale-type perturbation and transversality theory. In its applications to symplectic field theory the solution spaces are the compactified moduli spaces.

János Kollár (Princeton University)

Local Integrability of holomorphic functions

Question: Let $f(z_1,\ldots,z_n)$ be a holomorphic function on an open set $U \subset \mathbb{C}^n$. For which $s \in \mathbb{R}$ is $|f|^{-s}$ locally integrable? It is not hard to see that there is a largest value s_0 (depending on f and p) such that $|f|^{-s}$ is integrable in a neighborhood of p for $s < s_0$ but not integrable for $s > s_0$. Our aim is to study this "critical value" s_0. Subtle properties of these critical values are connected with Mori's program (especially the termination of flips), with the existence of Kähler-Einstein metrics in the positive curvature case and many other topics.

Tom Mrowka (MIT)
Monopoles, closed Reeb orbits and spectral flow: Taubes' work on the Weinstein conjecture

We survey Cliff Taubes' recent proof of the Weinstein conjecture in dimension three and related topics. Taubes shows how to construct periodic orbits of Reeb vector fields on contact three manifolds from special cycles in the Seiberg-Witten Monopole Floer homology. The proof follows ideas from Taubes' work relating the Seiberg-Witten and Gromov invariants of four-manifolds but with a new twist. It hinges on new results describing the asymptotic behavior of spectral flow for Dirac type operators.

Assaf Naor (Courant Institute, NYU)
Probabilistic reasoning in quantitative geometry

Many problems of an asymptotic and quantitative nature in geometry have recently been solved using a variety of probabilistic tools. Apart from the classical use of the probabilistic method to prove existence results, it turns out thinking "probabilistically," or interpreting certain geometric invariants in a probabilistic way, is a powerful way to bound a variety of geometric quantities. This talk is devoted to surveying the ways in which probabilistic reasoning plays a sometimes unexpected role in topics such as bi-Lipschitz and uniform embedding theory, extension problems for Lipschitz maps, metric Ramsey problems, harmonic analysis, and theoretical computer science. We will show how random partitions of metric spaces can be used to embed them in normed spaces, find large Euclidean subsets, extend Lipschitz functions, and bound the weak (1,1) norm of maximal functions. We will also discuss the role of random projections, and describe the connections between the behavior of Markov chains in metric spaces and Lipschitz extension, lower bounds for bi-Lipschitz embeddings, and the computation of compression exponents for discrete groups.

Rahul Pandharipande (Princeton University)
Curve counting via stable pairs in the derived category

Let X be a projective 3-fold. We construct a moduli space of stable pairs in the derived category of X with a well-defined enumerative geometry. The enumerative invariants are conjectured to be equivalent to the Gromov-Witten theory of X. The geometry is a very natural place to study recent derived category wall-crossing formulae. Fibrations of K3 surfaces provide computable examples. Connections to work of Kawai-Yoshioka and the Yau-Zaslow formula for enumerating rational curves on K3 surfaces are made.

This is joint work with R. Thomas.

Scott Scheffield (Courant Institute, NYU)
Quantum gravity and the Schramm-Loewner evolution

Many "quantum gravity" models in mathematical physics can be interpreted as probability measures on the space of metrics on a Riemannian manifold. We describe several recently derived connections between these random metrics and certain random fractal curves called "Schramm-Loewner evolutions" (SLE).

Claire Voisin (CNRS, IHÉS and Inst. Math. Jussieu)
Hodge structures, cohomology algebras and the Kodaira problemn

We show that there exist, starting from complex dimension 4, compact Kähler manifolds whose cohomology algebra is not that of a projective complex manifold. In particular their complex structure does not deform to that of a projective manifold, while Kodaira proved that compact Kähler surfaces deform to projective ones. The argument uses the notion of Hodge structure on a cohomology algebra, and exhibits algebraic obstructions for the existence of such Hodge structure admitting a rational polarization. We will also explain further applications of this notion, e.g. the fact that cohomology algebras of compact Kähler manifolds are strongly restricted amongst cohomology algebras of compact symplectic manifolds satisfying the hard Lefschetz property.

Clay Research Awards

The Weinstein Conjecture

Classical mechanics, as formulated by Hamilton, takes place in the context of a configuration space of positions and momenta. Mathematically, this is a manifold M with a symplectic structure and a distinguished function H, the *Hamiltonian*. The symplectic structure is given by a closed 2-form ω such that $\omega^n(x) \neq 0$, for all x in M, where M is of dimension $2n$. Such a manifold carries a natural vector field X_H defined by the condition

$$\omega(X_H, Y) = dH(Y)$$

for all Y. This, the *Hamitonian vectorfield*, defines a flow $\phi_t(x)$ on the manifold. If $x = (q, p)$ give the position and momentum of a particle, its trajectory as time evolves is given by $\phi_t(x)$ for varying t. The flow itself is defined by Hamilton's equations,

$$\dot{p} = \frac{\partial H}{\partial q}, \qquad \dot{q} = -\frac{\partial H}{\partial p}$$

where q and q are Darboux coordinates, giving conjugate positions and momenta.

The flow just defined determines a *dynamical system*. A fundamental problem is whether or not there exist closed orbits for such a system. For example, we hope that the orbit of the earth is both closed and quite stable. Orbits, of course, lie on the level sets of H, which is commonly taken to be the total energy.

In the late 1970s Rabinowitz and Weinstein proved that for $H : R^{2n} \longrightarrow \mathbb{R}$ which has either star-shaped or convex level sets, the corresponding Hamiltonian flow has a periodic orbit on the level sets. In searching for a common generalization, Weinstein observed that a contact structure could be seen as the engine which makes the arguments work.

A contact manifold is an odd-dimensional manifold with a one-form A such that $A \wedge dA^n$ is everywhere nonzero. The kernel of A is a maximally nonintegrable field of hyperplanes in the tangent bundle; the Reeb vector field generates the kernel of dA and pairs to one with A. For a motivating example, consider the unit sphere in $\mathbb{C}^n$, where A is the a standard form which annihilates the maximal complex subspace of the tangent space. If Z is a coordinate vector for $\mathbb{C}^n$, then $A = \sqrt{-1}(\bar{\partial} - \partial) log||Z||^2$ is such a form. In this case the Reeb vector field is the field tangent to the circles in the fibration $S^{2n-1} \longrightarrow \mathbb{CP}^n$ from the sphere to the associated complex projective space.

Cliff Taubes and Claire Voisin delivering their acceptance speeches.

The Weinstein conjecture, stated some thirty years ago, asks whether the Reeb vector field for a contact manifold always has a a closed orbit. By contrast, there exist arbitrary vector fields on the three-sphere not annihilated by dA with no closed orbits. These are the counterexamples to the Seifert Conjecture of Schweitzer, Harrison and Kuperberg.

Hofer proved the Weinstein Conjecture in many special cases in dimension three, for example, the three-sphere and contact structures on any three-dimensional reducible manifold. Taubes' solution to the general conjecture in dimension three is based on a novel application of the Seiberg-Witten equations to the problem. The orbits come from special cycles in the Seiberg-Witten Monopole Floer Homology.

Cliff Taubes receiving the 2008 Clay Research Award that was presented by Landon and Lavinia Clay and President James Carlson.

The Kodaira Conjecture

Geometric structures on a topological manifold often impose restrictions on what kind of manifolds can arise. For example, a symplectic manifold must have nonzero second Betti number, since the symplectic form ω is non-trivial in cohomology. Indeed, if the manifold has dimension $2n$, then ω^n has nonzero integral. Yet more restrictive is the notion of a Kähler manifold – a symplectic manifold for which the form ω has type $(1,1)$ in a compatible complex structure. In that case many topological conditions are satisfied: the odd Betti numbers are even, the cohomology ring is formal, and there are numerous restrictions on the fundamental group. Kähler manifolds abound: any projective algebraic manifold, that is, any submanifold of complex projective space defined by homogeneous polynomial equations, is a Kähler manifold. In complex dimension one, the converse is true: any Kähler manifold (a Riemann surface) is complex projective. In complex dimension two, the converse is false, but just barely: every complex Kähler manifold is the deformation of a projective algebraic manifold. This fact was proved by Kodaira, using his classification theorem for complex surfaces.

The question then arises: is every compact Kähler manifold deformable to projective algebraic one? Although never explicitly stated by Kodaira, this question has become known as the *Kodaira Conjecture*. Alas, the proof in dimension two gives no clue about what happens in higher dimension. The crux of the problem, however, is to show that on the given complex manifold M, can one deform the complex structure so as to obtain a positive $(1,1)$ class in the rational cohomology. That is, one must show that the Hodge structure is *polarizable*. The fundamental theorem here is due to Kodaira: from a closed, rational, positive, $(1,1)$ form, one may construct an imbedding of the underlying manifold into projective space.

Claire Voisin receiving the 2008 Clay Research Award that was presented by Landon and Lavinia Clay and President James Carlson.

There have been various attempts to prove or disprove the conjecture. Since any deformation of M has the same diffeomorphism type as M, a disproof requires a topological invariant defined for Kähler manifolds that distinguishes the projective algebraic ones from those that are not.

The starting point for Voisin's counterexample is the construction of a complex torus T which is not projective algebraic because of the existence of a "wild" endomorphism Φ. This is an endomorphishm whose eigenvalues are non-real and distinct, and such that the Galois group of the field generated by the eigenvalues is as large as possible. An example is given by the companion matrix of the polynomial $x^4 - x + 1$. The second exterior product of a weight one Hodge structure with a wild endomorphism carries no nonzero rational $(1,1)$ classes, so long as the space of elements of type $(1,0)$ has dimension strictly greater than one. Therefore the complex manifold T is not projective algebraic, though it can, of course, be deformed to an algebraic torus. The actual counterexample is a suitable blowup of $T \times T$. Consider the subvarieties $T \times \{0\}$, $\{0\} \times T$, the graph of the diagonal, and the graph of Φ. Blow up the points of intersection of the diagonals of the identity and of Φ and also the intersection of $T \times \{0\}$ with the graph of Φ. Then blow up the proper transforms of the subvarieties to obtain a Kähler manifold V with $H^2(V) \cong \Lambda^2 H^1(T)$. Any deformation of a blowup of a complex torus is obtained first by deforming the torus and then deforming the blowup. From this one sees that the wild endomorphism is preserved. Therefore the Hodge structure $H^2(V)$ contains no rational $(1,1)$ classes, and so V, and indeed any Kähler manifold with the same cohomology ring as V, is not projective algebraic. The same kind of construction yields a disproof of the Kodaira conjecure in dimension four or greater. Voisin also gives simply connected counterexamples in dimension six and greater.

Summary of 2008 Research Activities

Reseach Fellow Adrian Ioana

The activities of CMI researchers and research programs are sketched below. Researchers and programs are selected by the Scientific Advisory Board (see inside back cover).

Clay Research Fellows

Spyros Alexakis received his Ph.D. from Princeton University under the supervision of Charles Fefferman in 2005. He is currently on leave from his position as Assistant Professor at the University of Toronto. He began his two-year appointment in July 2009 while a visiting researcher at MIT.

Adrian Ioana, a native of Romania, received his B.S. from Unversity of Bucharest and his Ph.D. in 2007 at UCLA under the direction of Professor Sorin Popa. He began his three-year appointment while a post-doc at Caltech.

Xinyi Yuan received his PhD in 2008 from Columbia University under the direction of Shou-Wu Zhang. His three-year appointment began at the Institute for Advanced Study where he is a visitor in the School of Mathematics.

Alexakis, Ioana, and Yuan joined CMI's current group of research fellows Mohammed Abouzaid (MIT), Artur Avila (IMPA Brazil), Maria Chudnovsky (Columbia University), Soren Galatius (Stanford University), Bo'az Klartag (Princeton University), Ciprian Manolescu (Columbia University), Davesh Maulik (Columbia University), Maryam Mirzakhani (Princeton University), Sophie Morel (Institute for Advanced Study), Samuel Payne (Stanford University), and David Speyer (MIT) and Teruyoshi Yoshida (Harvard University)

Research Scholars

Dipendra Prasad (TIFR, Mumai). September 24, 2007–June 13, 2008 at University of California, San Diego.

Senior Scholars

Senior Scholars John Lott and Gang Tian at Institut Henri Poincaré. May 1–June 30.

Senior Scholar Rob Lazarsfeld at PCMI/IAS. July 6–26.

Senior Scholar Henri Gillet at the Fields Institute Program on Arithmetic Geometry, Hyperbolic Geometry and Related Topics. September 1–November 30.

Senior Scholar Fedor Bogomolov at the Centro di Ricerca Matematica Program on Groups in Algebraic Geometry. September 1–November 30.

Senior Scholar Richard Schoen at the Mittag-Leffler Institute Program on Geometry, Analysis and General Relativity. September 1–December 15.

Senior Scholar Werner Mueller at the MSRI Program on Analysis of Singular Spaces. September 10–October 21.

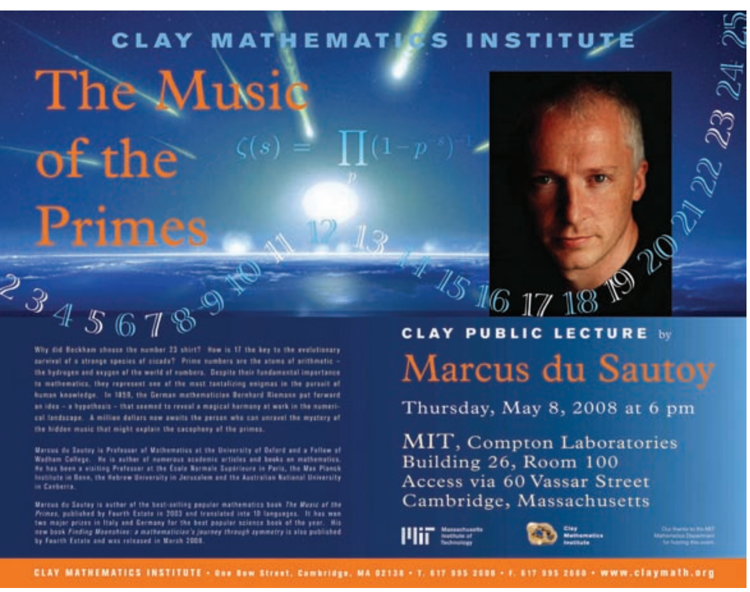
Senior Scholar Gunther Uhlmann at the MSRI Program on Analysis of Singular Spaces. September 16–December 15.

Liftoff Fellows

CMI appointed twenty Liftoff Fellows for the summer of 2008. Clay Liftoff Fellows are recent Ph.D. recipients who receive one month of summer salary and travel funds before their first academic position. See www.claymath.org/liftoff

Mark Braverman	Irine Peng
Dawei Chen	Jonathan (Jay) Pottharst
Michael Eichmair	Yanir Rubinstein
Inessa Epstein	Travis Schedler
David Fithian	David Smyth
Qëndrim Gashi	Robert Waelder
Marketa Havlíčková	Micah Warren
Kai-Wen Lan	Chenyang Xu
Manish Patnaik	Karen Yeats
Ron Peled	David Zywina

Research Fellow Xinyi Yaun



Marcus du Sautoy discussed the mystery of prime numbers, the history behind the Riemann hypothesis and the ongoing quest to solve it in his May 2008 Clay Public Lecture at MIT. A video recording of his talk may be viewed at www.claymath.org/public_lectures/dusautoy.php.

Research Programs organized and supported by CMI

January 3–12. Cycles, Motives, and Shimura Varieties at TIFR, Mumbai, India.

January 31–Feb 3. Shrinking Target Properties at Brandeis University.

March 8. Symposium for Undergraduates in the Mathematical Sciences at Brown University, Providence, Rhode Island.

March 16–19. Algebraic Statistics, Machine Learning, and Lattice Spin Models at Banbury Conference Center, Cold Spring Harbor Laboratory.

March 16–21. Recent Progress on the Moduli Space of Curves at Banff International Research Station, Canada.

March 20–23. CMI Workshop on K3s: Modular Forms, Moduli, and String Theory.

March 25–29. Conference on Algebraic Cycles II at Ohio State University, Columbus, Ohio.

March 28–30. CMI Workshop: Automorphic Forms in Moduli Problems of Schottky and Brill-Noether Type.

April 5–13. Additive Combinatorics, Number Theory, and Harmonic Analysis at the Fields Institute, Toronto, Canada.

May 8. “The Music of the Primes,” Clay Public Lecture by Marcus du Sautoy at MIT.

May 11–18. Workshop on Global Riemannian Geometry, National Autonomous University of Mexico, Cuernavaca (IMATE-UNAM Cuernavaca), Mexico.

May 12–13. Clay Research Conference at MIT.

May 18–23. HIRZ80 at Emmy Noether Research Institute for Mathematics at Bar Ilan University, Ramat Gan, Israel.

Summary of 2008 Research Activities

May 19–25. Lie Theory and Geometry: The Mathematical Legacy of Bertram Kostant at PIMS in Vancouver, BC.

June 2–13. Conference on Motives, Quantum Field Theory and Pseudodifferential Operators at Boston University.

June 9–13. A Celebration of Raoul Bott's Legacy in Mathematics at CRM, Montreal.

June 15–28. School and Workshop on Aspects of Moduli Theory at the De Giorgi Center, Scuola Normale Superior di Pisa.

June 16–20. Analysis and Topology in Interaction at the Il Pallazone in Cortona, Italy.

June 22–28. Symmetries in Mathematics and Physics at the Palazzone della Scuola Normale Superiore, Cortona, Italy.

June 23–July 18. CMI Summer School: Evolution Equations at the ETH, Eidgenossische Technische Hochschule, Swiss Federal Institute of Technology, Zurich, Switzerland.

June 28–July 2. Conference on Modular Forms and Arithmetic in honor of Ken Ribet's 60th birthday at MSRI, Berkeley, CA.

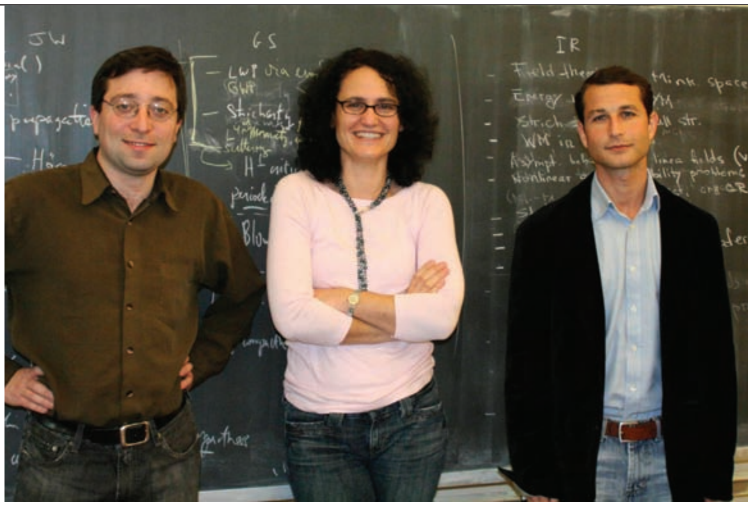
July 14–26. Algebraic Geometry, D-modules, Foliations and their Interactions, Buenos Aires, Argentina.

July 16–18. 60 Miles: A conference in Honor of Miles Reid's 60th Birthday, LMS, London.

August 4–15. CMI Workshop: The Foundations of Algebraic Geometry: Grothendieck's EGA Unbound.

October 13–16. CMI Workshop: Stringy Reflections on LHC.

October 14. "A Tribute to Euler," Clay Public Lecture by William Dunham at Harvard University.



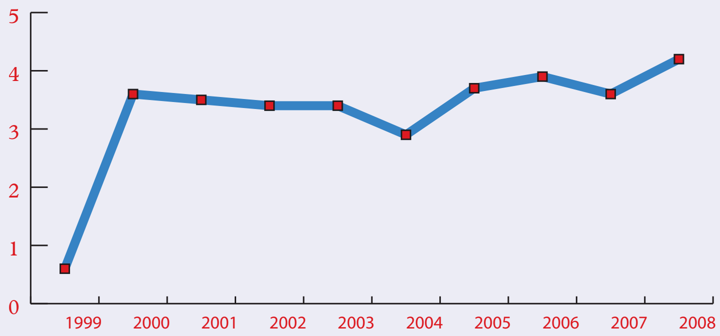
CMI Summer School Scientific Committee members Jared Wunsch, Gigliola Staffilani and Igor Rodnianski, (and David Ellwood, not depicted).

Program Allocation

Estimated number of persons supported by CMI in selected scientific programs for calendar year 2008:

Research Fellows, Research Awardees, Senior Scholars, Research Scholars, Book Fellows, Liftoff Fellows	30
Summer School participants and faculty	134
PROMYS/Ross, participants and faculty	28
CMI Workshops	76
Participants attending Conferences and Joint Programs	> 1000

IRS Qualifying Charitable Expenses of CMI Since Inception



Total through September 30, 2008: Over $32 million

Interview with Research Fellow Maryam Mirzakhani

Maryam Mirzakhani, a native of Iran, is currently a professor of mathematics at Stanford. She completed her Ph.D. at Harvard in 2004 under the direction of Curtis T. McMullen. In her thesis she showed how to compute the Weil-Petersson volume of the moduli space of bordered Riemann surfaces. Her research interests include Teichmüller theory, hyperbolic geometry, ergodic theory, and symplectic geometry.

What first drew you to mathematics? What are some of your earliest memories of mathematics?

As a kid, I dreamt of becoming a writer. My most exciting pastime was reading novels; in fact, I would read anything I could find. I never thought I would pursue mathematics before my last year in high school. I grew up in a family with three siblings. My parents were always very supportive and encouraging. It was important for them that we have meaningful and satisfying professions, but they didn't care as much about success and achievement. In many ways, it was a great environment for me, though these were hard times during the Iran-Iraq war. My older brother was the person who got me interested in science in general. He used to tell me what he learned in school. My first memory of mathematics is probably the time that he told me about the problem of adding numbers from 1 to 100. I think he had read in a popular science journal how Gauss solved this problem. The solution was quite fascinating for me. That was the first time I enjoyed a beautiful solution, though I couldn't find it myself.

Could you talk about your mathematical education? What experiences and people were especially influential?

I was very lucky in many ways. The war ended when I finished elementary school; I couldn't have had the great opportunities that I had if I had been born ten years earlier. I went to a great high school in Tehran, Farzanegan, and had very good teachers. I met my friend Roya Beheshti the first week after entering middle school. It is invaluable to have a friend who shares your interests, and helps you stay motivated. Our school was close to a street full of bookstores in Tehran. I remember how walking along this crowded street, and going to the bookstores, was so exciting for us. We couldn't skim through the books like people usually do here in a bookstore, so we would end up buying a lot of random books.

Also, our school principal was a strong-willed woman who was willing to go a long way to provide us with the same opportunities as the boys' school. Later, I got involved in Math Olympiads that made me think about harder problems. As a teenager, I enjoyed the challenge. But most importantly, I met many inspiring mathematicians and friends at Sharif University. The more I spent time on mathematics, the more excited I became.

At Sharif University, we had problem-solving sessions and informal reading groups with my classmates. The friendship and support of all the people I met there and later at Harvard helped me a lot in many different ways. I am grateful to all of them.

Did you have a mentor? Who helped you develop your interest in mathematics, and how?

Many people have had a great influence on my math education, from my family and teachers in high school to professors at Sharif University, and later at Harvard.

You were educated in Iran. Could you comment on the differences between mathematical education there and in the US?

It is hard for me to comment on this question since my experience here in the U.S. is limited to a few universities, and I know very little about the high school education here.

However, I should say that the education system in Iran is not the way people might imagine here. As a graduate student at Harvard, I had to explain quite a few times that I was allowed to attend a university as a woman in Iran. While it is true that boys and girls go to separate schools up to high school, this does not prevent them from participating say in the Olympiads or the summer camps.

... the education system in Iran is not the way people might imagine here. As a graduate student at Harvard, I had to explain quite a few times that I was allowed to attend a university as a woman in Iran.

But there are many differences: in Iran you choose your major before going to college, and there is a national entrance exam for universities. Also, at least in my class in college, we were more focused on problem solving rather than taking advanced courses.

What attracted you to the particular problems you have studied?

When I entered Harvard, my background was mostly combinatorics and algebra. I had always enjoyed complex analysis, but I didn't know much about it. In retrospect, I see that I was completely clueless. I needed to learn many subjects which most undergraduate students from good universities here know. I started attending the informal seminar organized by Curt McMullen. Well, most of the time I couldn't understand a word of what the speaker was saying. But I could appreciate some of the comments by Curt. I was fascinated by how he could make things simple, and elegant. So I started asking him questions regularly, and thinking about problems that came out of these illuminating discussions. His encouragement was invaluable. Working with Curt had a great influence on me, though now I wish I had learned more from him! By the time I graduated I had a long list of raw ideas that I wanted to explore.

Most problems I work on are related to geometric structures on surfaces and their deformations. In particular, I am interested in understanding hyperbolic surfaces.

Can you describe your research in accessible terms? Does it have applications to other areas?

Most problems I work on are related to geometric structures on surfaces and their deformations. In particular, I am interested in understanding hyperbolic surfaces. Sometimes properties of a fixed hyperbolic surface can be better understood by studying the moduli space that parametrizes all hyperbolic structures on a given topological surface.

These moduli spaces have rich geometries themselves, and arise in natural and important ways in differential, hyperbolic, and algebraic geometry. There are also connections with theoretical physics, topology, and combinatorics. I find it fascinating that you can look at the same problem from different perspectives, and approach it using different methods.

What research problems and areas are you likely to explore in the future?

It's hard to predict. But I would prefer to follow the problems I start with wherever they lead me.

Could you comment on collaboration versus solo work as a research style? Are certain kinds of problems better suited to collaboration?

I find collaboration quite exciting. I am grateful to my collaborators for all I have learned from them. But in some ways I would prefer to do both; I usually have some problems to think about on my own.

What do you find most rewarding or productive?

Of course, the most rewarding part is the "Aha" moment, the excitement of discovery and enjoyment of understanding something new, the feeling of being on top of a hill, and having a clear view. But most of the time, doing mathematics for me is like being on a long hike with no trail and no end in sight!

I find discussing mathematics with colleagues of different backgrounds one of the most productive ways of making progress.

How has the Clay Fellowship made a difference for you?

It was a great opportunity for me; I spent most of my time at Princeton which was a great experience. The Clay Fellowship gave me the freedom to think about harder problems, travel freely, and talk to other mathematicians. I am a slow thinker, and have to spend a lot of time before I can clean up my ideas and make progress. So I really appreciate that I didn't have to write up my work in a rush.

What advice would you give to young people starting out in math (i.e., high school students and young researchers)?

I am really not in a position to give advice; I usually use the career advice on Terry Tao's web page for myself! Also, everyone has a different style, and something that works for one person might not be so great for others.

What advice would you give lay persons who would like to know more about mathematics—what it is, what its role in our society has been and so on? What should they read? How should they proceed?

This is a difficult question. I don't think that everyone should become a mathematician, but I do believe that many students don't give mathematics a real chance. I did poorly in math for a couple of years in middle school; I was just not interested in thinking about it. I can see that without being excited mathematics can look pointless and cold. The beauty of mathematics only shows itself to more patient followers.

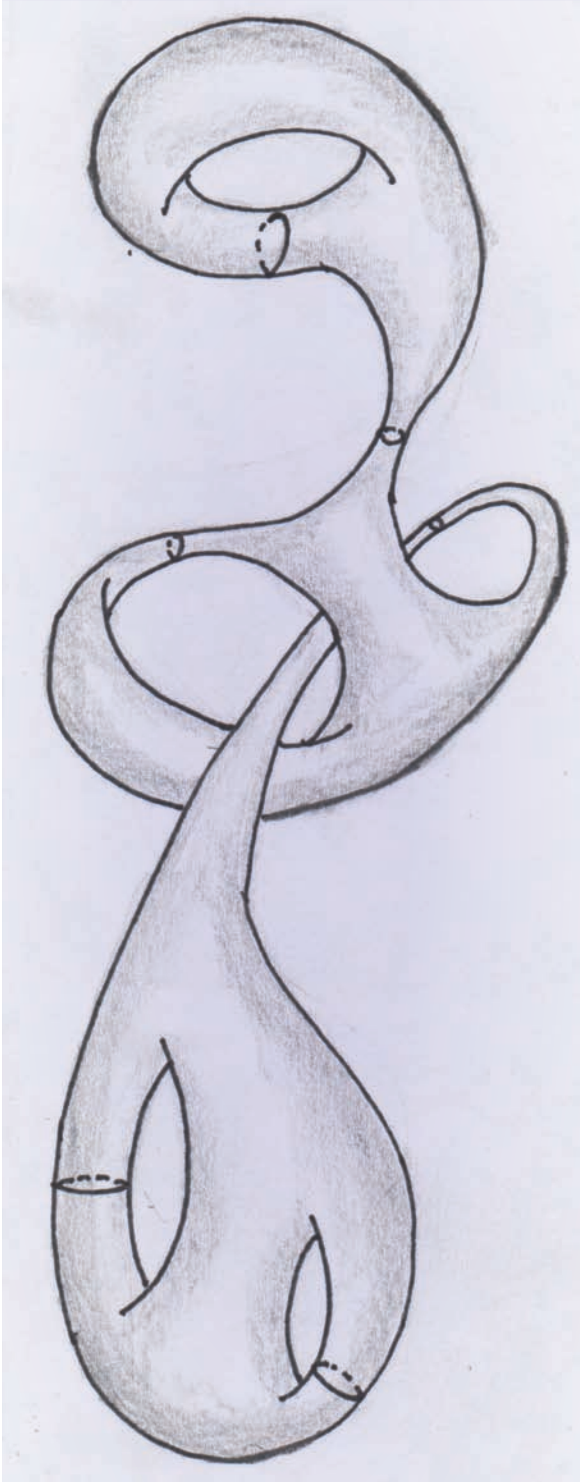
Please tell us about things you enjoy when not doing mathematics.

Mostly, I spend time with my family and husband. But for myself, I prefer solo activities; I enjoy reading and exercising in my free time.

Recent Research Articles

"Ergodic Theory of the Earthquake Flow." *Int Math Res Notices* (2008) Vol. 2008.

"Ergodic Theory of the Space of Measured Laminations," with Elon Lindenstrauss. *Int Math Res Notices* (2008) Vol. 2008.



Riemann Surface and Geodestics. Pencil sketch by Jim Carlson.

A Tribute to Euler by William Dunham

Koehler Professor of Mathematics, Muhlenberg College

Visiting Professor of Mathematics, Harvard University

With acclaim normally reserved for matinee idols, Leonhard Euler has recently been basking in the mathematical limelight. The cause of this spike in publicity was his tercentenary. Euler was born in Switzerland in 1707, and thus 2007 provided the perfect opportunity for mathematicians to celebrate his life and work. Discussions, conferences, and special events were held in his honor. The Mathematical Association of America published not one, not two, but *five* books about his remarkable career. And, in October of 2008 ("tercentenary plus one"), Euler was the subject of my Clay Public Lecture at Harvard University. His story, in both its personal and scholarly dimensions, is one of the great tales from the history of mathematics.

Leonhard Euler (1707—1783)

As a youth, Euler showed such signs of genius that he was mentored by Johann Bernoulli, who was then on the faculty at the University of Basel. Euler would later recall his sessions with the illustrious Bernoulli as an exciting, if daunting, experience. For his part, Johann recognized the special talent of his young student. Indeed, Bernoulli—a person not naturally self-effacing—would later write these laudatory words to Euler: "I present higher analysis as it was in its childhood, but you are bringing it to man's estate."

When he was 15, Euler graduated from the University of Basel, and by the age of 20 he had won a prize from the Paris Academy. In those days, the Academy would challenge the mathematicians of Europe with specific, and often quite difficult, problems. In this instance, the problem required a mathematical analysis of the placement of masts on a sailing ship. Euler's submission received what amounted to a second prize, an achievement all the more remarkable because of his Swiss— i.e., landlocked— upbringing. (He would win the Academy's first prize a dozen times over the course of his career.)

On the heels of this success, Euler applied for a faculty position at his alma mater. To his dismay, the job went to Benedict Staehelin, an individual who thereby earned the distinction of being perhaps the worst hiring choice in history. But Euler's fortunes improved with an offer from the St. Petersburg Academy in Russia. His appointment came through the influence of Daniel Bernoulli, son of Johann, who had himself secured a position at St. Petersburg a few years before.

And so Euler bade farewell to Switzerland and moved to St. Petersburg in 1727. He stayed until 1741 when he accepted a call to the rival Berlin Academy. There he worked under Frederick the Great until friction between them proved too much. Euler returned to St. Petersburg in 1766, where he remained until his death in 1783.

It was during his first Russian stint that he married Katharina Gsell, and the Eulers would eventually have 13 children. However, child mortality took a dreadful toll in those days, and only five of their children would survive to adolescence. The accompanying sorrow defies comprehension.

Meanwhile, Euler faced a physical challenge of his own. By his early 30s, he had lost vision in his right eye. A modern diagnosis, insofar as such a thing is possible, attributes this to an ocular infection that was untreatable at the time. Visual limitations aside, Euler continued his research unabated and maintained his productivity up to the year 1771, when he lost sight in his other eye. This was due to a cataract. Such a malady, easily corrected today, was a most serious matter back then. Euler's doctors tried eye surgery to save his vision (and no one wants to contemplate the horrors of eye surgery in the 18th century), but the procedure was unsuccessful. By 1771, Euler was essentially blind.

It is tempting to conclude that this marked the end of his career, but Euler would not be stopped. He instructed his assistants to read aloud the newly arrived books and journals, and he in turn dictated his ideas to a tableful of scribes working furiously to keep up. It is said that Euler could create mathematics faster than most people can write it, and he daily put his assistants to the test. A case in point: in 1775, when he was blind, he produced a paper a week! Like Beethoven, who wrote music that he never heard, Euler created mathematics that he never saw. This triumph in the face of adversity makes Euler's the most inspirational story in the history of mathematics.

Such a biography, although compelling, might be forgotten had the results he produced been of minor interest. But nothing could be further from the truth. If one measures a mathematician's impact along the three "axes" of quantity, diversity, and significance, then Euler is pretty much off the charts on all three. Let me address each in turn.

In terms of *quantity*, Euler has no peer. Indeed, a major challenge for those who sought to publish his collected works was the "simple" task of locating them all. This was complicated by the fact that Euler published 228 papers *after he died*, making the deceased Euler one of history's most prolific mathematicians.

In any case, by the dawn of the 20th century, the scholar Gustav Eneström had identified a total of 866 books and papers that Euler produced over his long career. Eneström briefly described each of these in a catalogue that itself ran to 388 pages. With this massive document as its guide, the Swiss Academy of Sciences began publishing Euler's collected works —his *Opera Omnia*—in 1911, when the first hefty volume appeared. Thereafter, the books kept coming and coming … and coming. At the moment, there are 75 volumes in print, totaling over 25,000 pages, but the project is not yet complete. By the time all of the papers and letters and notebooks are in print, Euler will have kept his publishers busy for more than a century. There is nothing else like this in all of mathematics.

In terms of its *diversity*, Euler's work covers a range of subject matter that can only be described as "universal." Consider the following dichotomies:

Pure/Applied: Euler, of course, made innumerable contributions to pure mathematics, but he was also the leading applied mathematician of his day. In fact, a good half of those 75 volumes of the *Opera Omnia* treat subjects like mechanics, acoustics, and optics —subjects that are today classified under physics or applied math.

Continuous/Discrete: Euler was as comfortable working in the continuous realm (e.g., calculus and differential equations) as he was working in the discrete one (e.g., number theory and combinatorics). Such breadth has become a rarity in our age of specialization.

Advanced/Elementary: Euler certainly contributed to the advanced mathematics of his time, but he was also successful writing about elementary topics. For instance, in 1738 he published his *Rechenkunst*, an arithmetic text for the schools, and his best-selling work of all was *Letters to a German Princess* of 1768, a survey of popular science written for the layperson.

Old/New: Euler made some remarkable discoveries in the venerable subject of plane geometry, discoveries that would have been accessible to old Euclid himself. Yet Euler also worked in fields so new that he was making them up as he went along.

But quantity and diversity do not fully account for Euler's mathematical reputation. There is one additional dimension of excellence that is surely the greatest of all: the *significance* of his work. It is remarkable how many seminal ideas in our discipline can be traced back to him. Consider, for instance:

The concept of function. It was Euler who elevated the "function" into its starring role in analysis. Prior to that, people had applied calculus to the "curve," a quasi-precise idea rooted in, and limited by, geometrical understanding. In his classic 1748 text, *Introductio in Analysin Infinitorum*, Euler emphasized functions and introduced the special types—polynomial, exponential, logarithmic, trigonometric, and inverse trigonometric—that still occupy center stage in analysis.

A Tribute to Euler by William Dunham

$$e^{+v\sqrt{-1}} = cof.\,v + \sqrt{-1}.\,fin.\,v$$

Euler Identity from his 1748 classic *Introductio in Analysin Infinitorum*

The Euler Identity. It was Euler who gave us the formula $e^{ix} = \cos x + i\sin x$. Those who encounter this for the first time are apt to regard it as a typo, so peculiar is its fusion of the exponential and the trigonometric, the real and complex. It was from this that Euler deduced such strange consequences as $i^i = \frac{1}{\sqrt{e^{\pi}}}$, about which the Harvard mathematician Benjamin Peirce is reported to have said, "Gentlemen, we have not the slightest idea of what this equation means, but we may be certain that it means something very important."

The Euler Polyhedral Formula. In a 1752 study of polyhedra, Euler observed that $V + F = E + 2$, where V is the number of vertices, F is the number of faces, and E is the number of edges of a solid figure. Because of the utter simplicity of this relationship, Euler confessed that, "I find it surprising that these general results in solid geometry have not previously been noticed by anyone, so far as I am aware." Of course, no previous mathematician had had Euler's penetrating insight.

The Basel Problem. In the late 17th century, Jakob Bernoulli had challenged the mathematical community to find the *exact* sum of the infinite series $\sum_{k=1}^{\infty} 1/k^2$. This remained an open question for a generation until Euler, then a young and still relatively unknown mathematician, stunned the world by finding the sum to be $\pi^2/6$. As much as anything, this discovery made Euler famous.

The Euclid-Euler Theorem. Four volumes of the *Opera Omnia* address the theory of numbers, and Euler made untold contributions to this ancient and challenging field. One of these harks back to Book IX of the *Elements*, where Euclid had demonstrated that a whole number will be perfect (i.e., the sum of its proper divisors) if it is of the form $N = 2^{k-1}(2^k - 1)$, where the rightmost factor is prime. There matters stood for two millennia until Euler proved that this sufficient condition is also necessary for an *even* number to be perfect. Taken together, these results characterize even perfect numbers in the so-called "Euclid-Euler theorem," surely one of the most illustrious hyphenations in the history of mathematics. (By the way, Euler suggested that the matter of odd perfect numbers was likely to be "most difficult"—an indisputably accurate assessment.)

The Euler Product-Sum Formula. In 1737, Euler proved that $\sum_{k=1}^{\infty} 1/k = \prod_{p} \frac{1}{1-1/p}$, where the product on the right is taken over all the primes. Of course, he was here equating a divergent series (the harmonic) with a divergent product, so one might dismiss it as so much drivel. But Euler saw how to exploit his formula to establish the divergence of $\sum_{p} 1/p$.

This non-trivial theorem, which employed techniques of analysis to attack questions of number theory, prompted the 20th century mathematician André Weil to comment, "One may well regard these investigations as marking the birth of analytic number theory."

Such spectacular results notwithstanding, we have barely scratched the surface. Consider this partial list of other Eulerian "hits": The Bridges of Königsberg (1736); the original partitioning theorem for whole numbers (1740); the Euler line of a triangle (1767); the first textbook on the calculus of variations (1744); the analysis of Greco-Latin squares (1782); the landmark study of continued fractions (1744); the gamma function (1729); and the influential mechanics text of 1736 that cast Newton's physics in the language of Leibniz's calculus.

$$P = \frac{1}{(1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{5})(1-\frac{1}{7})(1-\frac{1}{11})(1-\frac{1}{13})\ \&c.,}$$

fiet

$$P = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \&c.,$$

Euler product-sum formula from his 1748 classic *Introductio in Analysin Infinitorum*

And, as if these achievements were not enough, there are some downright quirky results scattered among his papers. For instance, he sought four different whole numbers, the sum of any pair of which is a perfect square. (If you think this is easy, try it.) Euler came up with this fearsome foursome: 18530, 38114, 45986, and 65570.

From all of this, it should be clear why I chose to focus my Clay Public Lecture on Euler and his triumphs (those interested can find a video of the talk at www.claymath.org/video). It should be equally clear why the mathematical community so enthusiastically celebrated Euler's 300th birthday. For, if anyone stands as the mathematical counterpart of Shakespeare or Rembrandt or Bach, it is the incomparable master, Leonhard Euler.

Other pertinent texts (in English):

Emil Fellmann, *Leonhard Euler*, (trans., E. and W. Gautschi), Birkhauser, 2007.

Andreas and Alice Heyne, *Leonhard Euler: A Man to be Reckoned With*, Birkhauser, 2007. [This is the "Euler Comic Book" I mentioned in the lecture. It's actually pretty good!]

Euler in translation: *Leonhard Euler, Introduction to Analysis of the Infinite* (2 vols.), (trans. John Blanton), Springer-Verlag, 1988.

Leonhard Euler, *Foundations of Differential Calculus*, (trans. John Blanton) Springer-Verlag, 2000.

Leonhard Euler, *Elements of Algebra*, (trans. John Hewlett), Springer-Verlag, 1840 (reprint).

Surveys:

Edward Sandifer, *The Early Mathematics of Leonhard Euler*, MAA, 2007.

Edward Sandifer, *How Euler Did It*, MAA, 2007.

William Dunham, *Euler: The Master of Us All*, MAA, 1999.

William Dunham (ed.), *The Genius of Euler*, MAA, 2007.

The Mathematical Association of America named William Dunham as the recipient of the 2008 Beckenbach Book Prize for *Euler: The Master of Us All*, MAA, 1999.

"Mathematician William Dunham has written a superb book about the life and amazing achievements of one of the greatest mathematicians of all time. Unlike earlier writings about Euler, Professor Dunham gives crystal clear accounts of how Euler ingeniously proved his most significant results, and how later experts have stood on Euler's broad shoulders. Such a book has long been overdue. It will not need to be done again for a long long time." *—Martin Gardner*

"William Dunham has done it again! In *Euler: The Master of Us All*, he has produced a masterful portrait of one of the most fertile mathematicians of all time. With Dunham's beautiful clarity and wit, we can follow with amazement Euler's strokes of genius which laid the groundwork for most of the mathematics we have today." *—Ron Graham, Chief Scientist, AT&T*

Euler the Master of Us All is available through the MAA at the following website: www.maa.org.

The BBC Series *The Story of Math* by Marcus du Sautoy

In October 2008, Marcus du Sautoy presented a landmark series for the BBC on the history of mathematics. Called *The Story of Math*, the four one-hour programs took viewers from the pyramids of Cairo to the deserts of Arizona, from the backwaters of Kerela to the suburbs of St. Petersburg, in pursuit of where and how mathematics evolved over the last seven millennia.

Program One covers the mathematics of the ancient world: Egypt, Babylon, and Greece, including how the Egyptians used early ideas of the calculus to calculate the volumes of pyramids. Program Two takes viewers on a journey through the mathematics of the east from China through to India where we discover that the Kerelean school of mathematicians already knew Leibniz's infinite series for pi some centuries before its discovery in the West. Program Three presents the mathematics of Europe from Descartes, via Euler through to Riemann. Program Four encompasses the mathematics of the modern era, from Hilbert and Cantor through to Perelman's proof of the Poincaré Conjecture.

The reaction to the programs has been fantastic. Over half a million people viewed the first program. It was the seventh most downloaded program on i-player, beaten by two episodes of Eastenders and Little Britain in the USA. The Program received a four-star review from the Times—despite the reviewer saying he didn't understand a word: "where was du Sautoy when the dumbing-down debate was had."

Marcus du Sautoy is the Simonyi Professor for the Public Understanding of Science and Professor of Mathematics at the University of Oxford and a Fellow of New College. He is author of the best-selling popular mathematics book *The Music of the Primes* published in 2003 and translated into 10 languages. It has won two major prizes in Italy and Germany for the best popular science book of the year. His book *Symmetry* was released in March 2008.

In May of 2008 **Marcus du Sautoy** discussed the mystery of prime numbers, the history behind the Riemann hypothesis and the ongoing quest to solve it in his Clay Public Lecture at MIT. A video recording of his talk may be viewed at www.claymath.org/public_lectures/dusautoy.php.

Marcus du Sautoy outside the modern library in Alexandria talking about Euclid.

The series is partly funded by the Open University and there is an accompanying course for those interested in discovering more (www.openuniversity.co.uk/storyofmaths). The series forms part of Marcus du Sautoy's work as a Senior Media Fellow for the EPSRC. Combining stunning graphics with colorful locations, *The Story of Math* hopes to bring alive the intellectual journey that has taken mathematicians from fractions to fractals, from the circle to the hypersphere.

The Open University and the BBC have been in partnership for more than 30 years, providing educational programming to a mass audience. In recent times this partnership has evolved from late-night programming for delivering courses to peak-time programs with a broad appeal to encourage wider participation in learning.

Du Sautoy describes his experience hosting the series with enthusiasm, "I didn't really know a lot about the history of my subject. I always believed that what matters most is the mathematics. If you know the theorems and the proofs, is it really important who created them or in what circumstances? Certainly the way we are taught mathematics both in school and at university reinforces this a-historical message. So you might think that with such a mentality, I wouldn't be the ideal candidate to present a landmark series on the history of math for the BBC."

"But in some ways I think that it's worked in my favor. The series has become a real journey of discovery for me. Uncovering quite how much the ancient

A DVD of all four episodes is now available at www.ouw.co.uk/products/XM004_DVD01.shtm.

Egyptians and Babylonians knew about mathematics before the ancient Greeks has been a revelation for someone brought up on the myth that it all started with Pythagoras. I was amazed to discover quite how much the Indian mathematicians of the medieval period knew about infinite series and pre-calculus. And visiting the places where Descartes, Fermat, Euler, and Cantor grew up brought these characters alive for me in a way that I hope will come over on the screen."

"The programs pick up on this intellectual journey and mirror it with a real physical journey. The hope was to make something that looked like a cross between Michael Palin meets the *Ascent of Man*. The programs open with the story of the mathematics of ancient Eygpt and Babylon. Cairo and the pyramids provide an exotic location for the former. But unfortunately health and safety at the BBC stopped us from braving war-torn Iraq for the sake of mathematics. So Damascus, an outpost of the Babylonian empire, became our backdrop to talk about the mathematics hidden inside the clay tablets that have survived."

The film crew for program One at the pyramids in Egypt.

Horse riding in the Atlas Mountains of Morocco discussing Omar Khayyam's contributions to mathematics.

"The second program took us to the East and an exploration of Chinese and Indian mathematics. One of the highlights for me was the pilgrimage to Gwalior to see a tiny little temple hanging off the side of a mountain fort. Big enough to fit the presenter and a cameraman inside, we scoured the inscriptions on the walls for the first known example of the number zero, one of the greatest and revolutionary inventions made in India."

"The mathematics of India found its way to Europe via the spice routes through central Asia. Again health and safety denied us a trip to Iran to recreate the adventures of Omar Khayam (the British sailors had not long before been released from captivity). So Morocco became our central Asian backdrop where we found some fantastic horses to ride across the Atlas mountains in my reincarnation of the great Persian poet and mathematician. (My director informed me afterwards that he had decided to leave that reckless afternoon out of the health and safety report.)"

"Programs Three and Four took us to the colder climes of Europe and then on to the US to a town called Descartes; Fermat's home town Beaumont-de-Lomagne for Fermat Day; St Petersburg for the mighty Euler and the elusive Perelman; Göttingen for Gauss, Riemann, and Hilbert; the Nervenklinik in Halle for the unsettled Cantor; the Paris café where Bourbaki began (now a fast food burger joint); and the Arizona desert to look for Julia Robinson's childhood haunts. But if I had to pick out one location that excited me more than any other, it has to be our one day trip from St. Petersburg to the gray city of Kaliningrad. This is the modern name for Königsberg, the home of the seven bridges that some see as the beginning of modern topology. The city was bombed heavily during the second world war and today only three of the original bridges are left standing. Two of the others have been rebuilt—they now take a huge dual carriageway through the center of the town."

"Despite the ugly nature of this modern city, I felt I was in a mathematical Disneyland. To be able to make the journey over the bridges to see if there is a path filled me with a childish excitement that my crew just couldn't understand. Of course with just the five existing bridges it is in fact possible to make the journey today, unlike the seven bridges that the inhabitants of Königsberg were faced with."

"My crew was only too pleased to leave behind the grim skyline of Kaliningrad but for me it was one of the days out of the months of filming that I will always treasure. For me it encapsulated what this whole series is about—bringing alive the stories behind the amazing intellectual journey that mathematicians have made over the last seven millennia."

The Story of Math is an Open University/BBC co-production that aired on BBC FOUR in October, 2008. The series was produced by BBC Executive Producer David Okuefuna, BBC Series Producer Kim Duke, and Open University Executive Producer Catherine McCarthy. Academic Advisors from The Open University were Professors Robin Wilson, Jeremy Gray and Dr. June Barrow-Green.

CMI Supported Conferences

A Celebration of Raoul Bott's Legacy in Mathematics
June 9–13, 2008, Centre de Recherche Mathématiques, Montreal

by Robert Kotiuga, Associate Professor of Electrical and Computer Engineering, Boston University

The background for the conference, its rationale, as well as abstracts and titles of talks are archived on the conference website: www.crm.math.ca/Bott08. The conference was cosponsored at the level of $13,000 by the Clay Mathematics Institute, and was awarded a $25,000 grant from the NSF (USA). Currently, an extended conference proceedings is being planned.

"A Celebration of Raoul Bott's Legacy in Mathematics" was a forward-looking mathematical conference that was not organized around a mathematical topic, but a mathematical personality. Most of the speakers were either students or coauthors of Raoul Bott, or feel that their work clearly reflects the influence Bott had on them. Minimal effort was given to a systematic covering of the topics covered in Bott's collected works published over a decade ago. Rather, Bott's colleagues from six consecutive decades were given a free hand to rework and understand past work in terms of current developments. The abstracts posted on the website summarize the mathematical aspects of the conference and document where the organizational approach leads. One talk out of the mathematical mainstream was the talk by Jim Lambek, who reminisced about Raoul Bott as an engineering student at McGill University in the 1940s. Numerous other anecdotes about Bott were given in the first panel session entitled "Raoul Bott as Teacher, Mentor, and Colleague," and in the banquet speeches.

In addition to being a profound and influential researcher, it is well known that Bott was a wonderful lecturer. This has been documented in many places, and the conference produced some posthumous testimony of this. At the end of the second panel session, “Examining Raoul Bott’s Legacy in Mathematics,” the conference organizer emphasized that the conference was not organized around a mathematical topic but a mathematician, and asked the younger attendees what they thought of the concept. A student who identified himself as a graduate student working in an unrelated field made what was considered a remarkable comment. He said he learned more in his area of expertise than he did at other mathematics conferences because speakers at this conference seemed to make an extraordinary effort to communicate their ideas in the simplest, and most visual terms possible. What was more remarkable was that the instant consensus in the room was that this was a manifestation of all of the speakers being influenced by Bott’s lecturing style and his insistence on understanding deep mathematical concepts in the simplest terms possible.

Another unique aspect of the conference was the visual memory of Bott—from the “picture gallery” on the website, to pictures of him from six distinct decades on the conference poster, to the screening of Vanessa Scott’s film—*A Peek into the Book*. The unique combination of forward-looking mathematics and intimate connection to the Bott family would not have been possible without the effort of Bott’s daughter, Candace Bott, who spoke at the banquet, introduced her niece’s film, and was indispensable in helping with all visual aspects of the conference. In addition to producing an extended conference proceedings, popular demand initiated an attempt to distribute Vanessa Scott’s film more widely. The rough cut documentary portrait, as it was screened in its unfinished state, is now available upon written request to the Clay Institute.

Organized by Robert Kotiuga

Scientific Advisory Committee:

Sir Michael Atiyah	David Mumford
Octavian Cornea	Graeme Segal
David Ellwood	Stephen Smale
Jacques Hurtubise	Jim Stasheff
Francois Lalonde	Edward Witten

Speakers :

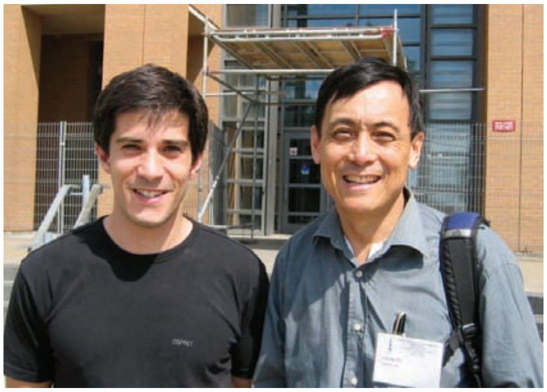
Michael Atiyah	Joseph Kohn
Paul Baum	Robert Kotiuga
James Bernhard	Peter D. Lax
Ralph Cohen	John Morgan
Octav Cornea	Stephen Smale
Marco Gualtieri	András Szenes
James Heitsch	Constantin Teleman
Nancy Hingston	Susan Tolman
Morris Hirsch	Loring Tu
John Hubbard	Cumrun Vafa
Lisa Jeffrey	Jonathan Weitsman
Nitya Kitchloo	Edward Witten

Panelists:

Michael Atiyah	Nitya Kitchloo
Paul Baum	James Stasheff
Nancy Hingston	Susan Tolman
Jacques Hurtubise	Loring Tu

Banquet Speakers:

Michael Atiyah
Candace Bott
Stephen Smale

Conference speakers Cumrun Vafa, Marco Gualtieri, and Loring Tu.

Banquet speakers Candace Bott and John Hubbard.

Photos courtesy of Loring Tu

CMI Supported Conferences

A Celebration of Raoul Bott's Legacy in Mathematics, continued

Although this is not the place to summarize the individual talks, it is worth commenting on the unanticipated outcomes of the conference. The masterful presentations of Michael Atiyah were not surprising, in light of his lifetime of outstanding achievements and collaborations. So it was with many of the other talks; the distinguished speakers lived up to their reputations. The speakers who talked about localization and singularity theory clearly built on the last two decades of Bott's research. The talks by Vafa and Witten dwelled on a stream of dualities that quantum field theory has been offering mathematics in recent decades, and the tantalizing new connections to number theory. Many other talks rounded out the conference in other ways. However, there were several unexpected developments where a big picture seemed to evolve magically out of smaller parts, and it is useful to focus on one that was not obvious before the conference. Loosely speaking, it pertains to Chas-Sullivan string topology and its relation to Floer homology via Morse theory. Here, on one hand, the work of Hingston and Goresky recast string topology in terms of Morse theory as applied to loop spaces by Bott in the 1950s. On the other hand, the work of Kitchloo and Cohen build on Morse theory in the context of quantum topology, and refine the use of Morse theory in low dimensional topology. In the talks given by these speakers, as well as those of Cornea and Teleman, one could sense where manifold topology was headed in the next few years, and while many of the new results would be unknown to Bott, the connection to his mathematical perspective and legacy is inescapable!

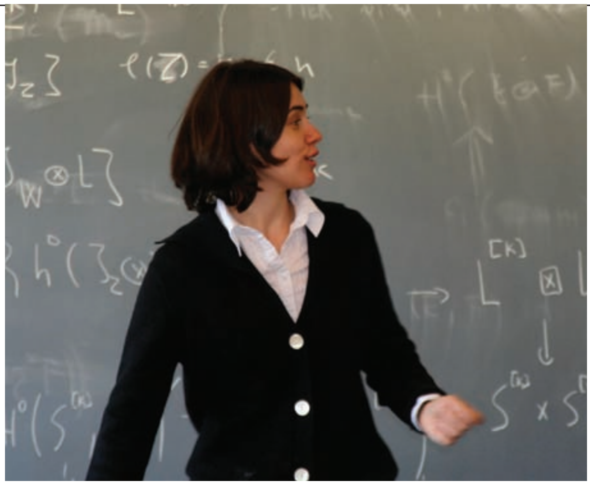
Finally, the organizer would like to express his gratitude to all involved. What a wonderful bunch of people to work with! The close connections to Raoul Bott clearly had something to do with making this a wonderful event.

A Conference in Algebraic Geometry Honoring F. Hirzebruch 80th Birthday, May 18–23, 2008, ENI, Bar Ilan University, Israel

CMI had the privilege of cosponsoring HIrz80, a Conference on Algebraic Geometry, held at Bar Ilan University, Israel, on the occasion of Professor Fritz Hirzebruch's eightieth birthday. Organized by Professor Mina Teicher of Bar Ilan with funding from the Israel Science Foundation (ISF), the six-day conference brought together forty mathematicians from Israel, the US, England, Canada, Italy, Germany, France, Russia, and Korea. Professor Hirzebruch is well known to us for his many contributions to mathematics, including his Riemann-Roch theorem, proved in 1954. This result is one of the great leaps forward that made algebraic geometry in dimension greater than two possible. Nonetheless, his influence was much wider than his own work in mathematics. He played a major role in the rebuilding of German mathematics after the war, including the founding and operation of the Max Planck Institute in Bonn, the Arbeitstagung, and mentoring many mathematicians just starting their careers.

The range of talks at the conference was broad, e.g., Faltings on p-adic period domains, Lubotzky on counting arithmetic groups and surfaces, and Griffiths on singularity and enumerative properties of families of Calabi-Yau varieties. Please see http://u.cs.biu.ac.il/~eni/Hirz80.html for a full list of titles and abstracts. The hospitality, meals, and excursions (Sea of Galilee and the Old City in Jerusalem) were beautifully organized by Professor Teicher with the help of Miryam Shabtay. Despite the very full lecture schedule, there was ample time between talks and in the evening to discuss mathematics. This is one of the joys of a good conference, and was notably so at Hirz80.

CMI Workshops



K3s: Modular Forms, Moduli, and String Theory

March 20 - 23, 2008

by Rahul Pandharipande (Princeton University) and Davesh Maulik (CMI)

The main goal of this workshop was to bring together mathematicians from various areas related to the study of K3 surfaces and their moduli. Research over the last twelve months had produced results and conjectures in the form of identities involving modular forms, hypergeometric series, and K3 moduli on the one hand, and the geometry of classical Noether-Lefschetz loci in the moduli of K3s on the other. A narrower goal was to make progress on a circle of rather concrete conjectures arising from this interaction.

The workshop consisted of fourteen research talks centered on the geometry of K3 surfaces. The group of participants included a successful mix of string theorists, complex geometers, and arithmetic geometers. Although the differences in language and backgrounds covered were large, the workshop fostered productive dialogue. The narrow goal of making progress on concrete conjectures was also met. Organizers R. Pandharipande and D. Maulik, together with A. Klemm and E. Scheidegger, succeeded to prove the Yau-Zaslow conjecture about rational curves on K3 surfaces for all (not necessarily primitive) curve classes using a combination of techniques motivated by the workshop. (See "Noether-Lefschetz theory and the Yau-Zaslow conjecture," A. Klemm, D. Maulik, R. Pandharipande, and E. Scheidegger; arXiv: 0807.2477v1 [math.AG].)

The organization and infrastructure of the CMI were instrumental in the progress made at the workshop. Through the effort of its staff, the focus of the participants was entirely mathematical; moreover, the environment was extremely conducive for collaboration.

A list of abstracts for all the talks and other information can be found at:

www.claymath.org/workshops.

Organizers: David Ellwood, Rahul Pandharipande, and Davesh Maulik

Participants:
Kai Behrend (University of British Columbia)
Jim Bryan (University of British Columbia)
Ron Donagi (University of Pennsylvania)
Lothar Gottsche (ICTP)
Joe Harris (Harvard University)
Brendan Hassett (Rice University)
Albrecht Klemm (University of Wisconsin)
Conan Leung (The Institute of Mathematical Sciences)
Eduard Looijenga (Utrecht University)
Davesh Maulik (CMI)
Alina Marian (Institute of Advanced Study)
Greg Moore (Rutgers University)
Rahul Pandharipande (Princeton University)
Tony Pantev (University of Pennsylvania)
Emanuel Scheidegger (University of Eastern Piedmont)
Domingo Toledo (University of Utah)
Yuri Tschinkel (New York University)
Wei Zhang (Columbia University)
Aleksey Zinger (State University of New York)

CMI Workshops

Automorphic Forms in Moduli Problems of Schottky and Brill-Noether Type

March 28-30, 2008

by Emma Previato, Professor of Mathematics, Boston University

Organized by David Ellwood and Emma Previato

Participants

Hershel Farkas (The Hebrew University of Jerusalem and Stony Brook)
Eleanor Farrington (Boston University)
Samuel Grushevsky (Princeton University)
Jay Jorgenson (The City College of New York)
Yaacov Kopeliovich (MEAG/Munich Reinsurance, New York, NY)
Irwin Kra (Stony Brook University)
Igor Krichever (Columbia University)
Alan Mayer (Brandeis University)
Emma Previato (Boston University)

This workshop addressed the dual nature of certain special functions, in their dependence on moduli as automorphic forms, and their dependence on a Fourier-Mukai dual variable, which describes a moduli space of bundles, theta functions being the prime example. A distinctive feature of the workshop was that of bringing together experts of the analytic and of the algebraic techniques, to discuss open problems in these two somewhat separate areas, rooted in classical (nineteenth-century) mathematics and bearing on cutting-edge issues such as renormalization for superstring amplitudes in mathematics physics.

The authors of the foundational “Theta constants, Riemann surfaces and the modular group” (American Mathematical Society, 2001), H.M. Farkas and I. Kra, were both present. Farkas and Kopeliovich talked about recent work on Thomae’s formulas for non-hyperelliptic curves, generalizing the classical expression for the Weierstrass points of the curve in terms of thetanulls: Thomae’s formulas are crucial in the study of the KZB (Knizhnik–Zamolodchikov–Bernard) connection. Kra led the “open questions” session, proposing the theme of a stratification of Teichmüller spaces by hyperbolic geodesics. Prymian theta functions and varieties also figured prominently. Krichever, whose work recently settled the Schottky problem for Jacobians and certain Prymians using linear differential equations in two variables, a startling discovery, proposed new equations for Prym theta functions, and Farrington talked about the Klein curve and her thesis project of applying to it the Prymian construction of an AgM (Arithmetic-geometric-Mean) in genus three, due to R. Donagi and R. Livné (*Ann. Scuola Norm. Sup. Pisa Cl. Sci.* (4) 28 (1999), 323-339).

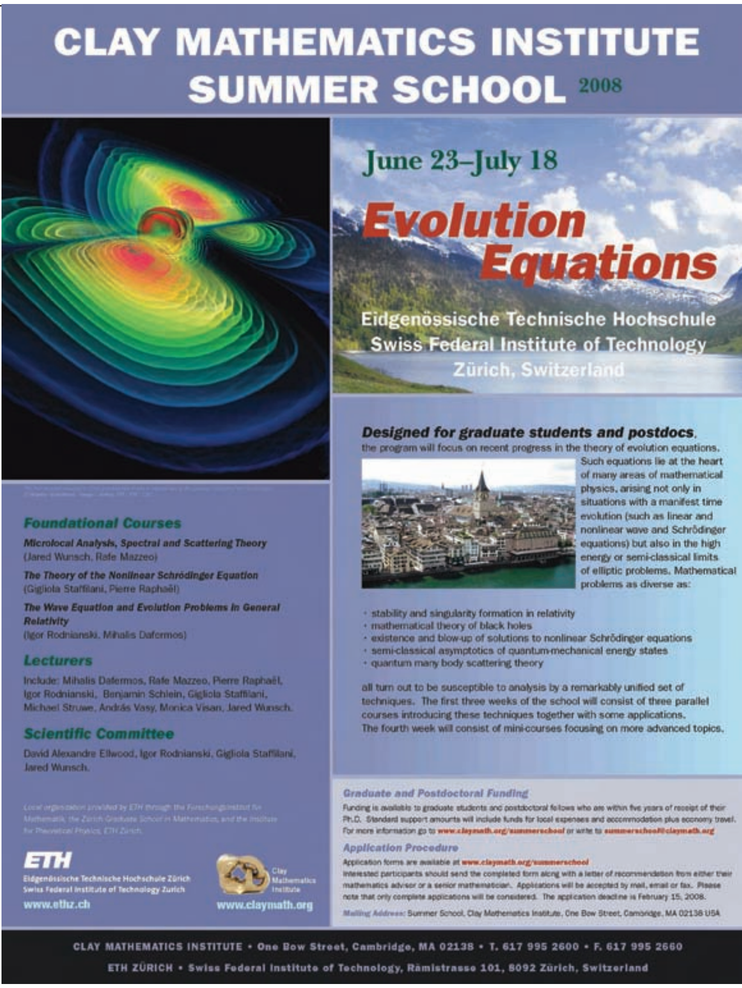
As regards subloci of moduli space defined by thetanulls, Grushevsky gave an exposition of his recent characterization of genus-4 Jacobians with a vanishing thetanull (joint with R. Salvati Manni). Mayer presented a highly innovative and provocative set of ideas that go back to his work with A. Andreotti in the 1960s, for example, a conjectural stratification of the moduli space of Riemann matrices by rank of quadrics viewed as modular forms; ideas for a proof include degeneration and Heisenberg-group actions. Jorgenson rightly concluded the workshop celebrating the analytic origin of the theta function through kernel functions for the Laplacian on a Riemann surface, and computing modern invariants (e.g. Arakelov metric, Falting’s delta function) by a hyperbolic kernel for certain modular curves.

The synergy that filled the Institute’s hospitable headquarters never flagged, conjectures and proofs were reached by the interplay of geometry and analysis, and Jorgenson gave an informal presentation of his work in progress, “Zeta functions, heat kernels and spectral asymptotics on degenerating families of discrete tori” (arXiv:0806.2014 joint with G. Chinta and A. Karlsson). R.D.M. Accola, B. Harris, R.C. Gunning, I. Dolgachev, and L. Takhtajan had accepted invitations but were unable to attend.

For more information, please see:

www.claymath.org/workshops/

Evolution Equations at the Swiss Federal Institute of Technology, Zürich, Switzerland



by Dean Baskin (Stanford), Jacques Smulevici (Cambridge), and Vedran Sohinger (MIT)

The Clay Mathematics Institute 2008 Summer School took place in the wonderful setting provided by the Eidgenössische Technische Hochschule Zürich and focused on recent progress in the theory of evolution equations. These equations lie at the heart of many areas of mathematical physics, arising not only in situations with a manifest time evolution but also in the high energy or semi-classical limits of elliptic problems.

The study of evolution equations has a long history but its rich and varied landscape make it an ever renewed field with many interesting open questions and conjectures waiting to be solved.

The program was built around three foundational courses:

- Microlocal Analysis, Spectral and Scattering Theory by Jared Wunsch and Rafe Mazzeo
- The Theory of the Nonlinear Schrödinger Equation by Gigliola Staffilani and Pierre Raphaël
- Wave Equation and Evolution Problems in General Relativity by Igor Rodnianski and Mihalis Dafermos

These courses were supplemented by several mini-courses:

- Derivation of Effective Evolution Equations from Microscopic Quantum Dynamics by Benjamin Schlein
- Nonlinear Schrödinger Equations at Critical Regularity by Monica Visan
- Wave Maps With and Without Symmetries by Michael Struwe
- Quantum N-body Scattering, Diffraction of Waves, and Symmetric Spaces by András Vasy

One of the fundamental bricks of evolution equations is the homogeneous scalar wave equation on Minkowski spacetime $\Box\phi = 0$. Generalizations include inhomogeneous terms, non flat-geometries, non-linearity, higher dimensions, and so forth. For the scalar homogeneous wave equation in Minkowski space, one may easily obtain an explicit representation of the solution using either spherical means or in terms of a Fourier decomposition.

It was therefore natural that one of the foundational courses addressed in particular a generalization of Fourier analysis using microlocal tools and their applications to evolution equations and scattering theory. Jared Wunsch began with an axiomatic treatment of pseudodifferential operators and wavefront sets, which generalize differential operators and singular sets, respectively. As an example of their applications, he provided a proof of the Duistermaat-Hörmander propagation of singularities theorem for operators of real principal type. He then constructed the solution operator for the wave equation and the wave trace via a geometric optics construction. This construction provided a motivation for an axiomatic treatment of the calculus of Fourier integral operators, and the construction of the wave operator within this calculus.

Evolution Equations at the Swiss Federal Institute of Technology, Zürich, Switzerland

Rafe Mazzeo continued the microlocal lectures with a discussion of scattering theory, both time-independent and time-dependent. The former is broadly a study of the continuous spectrum of the Laplacian on noncompact spaces, i.e. the study of solutions of $(-\Delta - \lambda^2)\, u = 0$. Taking the Fourier transform in λ changes the problem into time-dependent scattering, i.e. into the study of the asymptotic behavior of wave evolution.

Building on the constructions in the foundational course, András Vasy's lectures outlined the construction of the resolvent $(-\Delta + V - \lambda)^{-1}$ for N-body potentials V via a geometric resolution. He then illustrated the geometric similarity between this construction and the construction of the resolvent $(-\Delta - \lambda)^{-1}$ on symmetric spaces of noncompact type.

Another foundational course was devoted to the study of hyperbolic wave motion and its applications to general relativity. Igor Rodnianski started with a derivation of wave motions from the equations of different theories of physics such as electromagnetism, acoustics, and general relativity. The representations of the solution of the wave equation in Minkowski spacetime easily provide quantitative estimates, e.g., L^∞ to L^∞ estimates and energy estimates. However, they rely on many features of the Minkowski space and the linear property of the equation.

The method of compatible currents was thus introduced to study more complex systems of hyperbolic equations arising, such as the Euler-Lagrange equations of given Lagrangian. For instance, for the scalar wave equation on a Lorentzian manifold $\Box_g \phi = 0$, one can contract the energy-momentum tensor $T_{\mu\nu}(\phi)$ arising from Noether's theorem with any timelike vector field X^μ. The resulting vector field $J^\mu = T_{\mu\nu} X^\nu$ enjoys several remarkable properties. First, J^μ and its divergence only depend on the 1-jet of ϕ. Moreover, the integral of $J^\mu n_\mu$ over a spacelike hypersurface with normal n_μ controls $\nabla\phi$ in L^2. If we choose X^μ to a be Killing vector, Stokes's theorem gives us a conservation law and other choices of vector fields give us different energy estimates. These methods were applied to obtain global existence results for several non-linear wave equations such as the Yang-Mills equations.

Views of Zürich (above) and the ETH Chemistry building, where courses took place.

Mihalis Dafermos continued the course with an analysis of the wave equation on different black hole spacetimes. His lectures introduced the geometry of one of the simplest family of explicit solutions of the Einstein equations, the so-called Schwarzschild family. This one-parameter family of solutions lives as a subfamily of a two-parameter family known as the Kerr family. Both cases are models of black hole spacetimes. Black hole spacetimes are characterized by the presence of an event horizon, a global geometric property. One may try to use a compatible current with a timelike Killing vector field in order to obtain decay for the wave equation on Schwarzschild. In contrast with Minkowski space, this Killing field becomes null on the event horizon and so the weights in the energy estimates degenerate on the horizon. To bypass this difficulty, one needs to capture the celebrated *red-shift effect* with the introduction of a new vector field. To prove decay, it is also necessary to understand the geometry of the so-called *photonsphere* where trapped null geodesics accumulate. Due to the more complex geometry of the Kerr solution, controlling trapping here requires delicate analysis. Mihalis Dafermos presented the recent results concerning boundedness and decay for Kerr spacetimes sufficiently close to Schwarzchild.

In his series of lectures, Michael Struwe presented a study of the wave map problem. He focused in particular on the application of Strichartz estimates and geometric arguments in physical space for the Cauchy problem. Using these methods, he provided a proof of global existence for two symmetric cases, either when the target manifold is a surface of revolution of dimension 2 satisfying appropriate conditions or when it is a smooth compact Riemannian manifold without boundary and the initial data has radial symmetry.

Benjamin Schlein proved in his mini course that these non-linear dispersive equations, such as the Hartree equation with bounded or Coulomb potentials, arise naturally from microscopic quantum dynamics. The proofs involve in particular the study of the time evolution of the marginal density associated with the wave function describing the quantum system. He then presented the recent results concerning the evolution of Bose-Einstein condensates and the derivation of the Gross-Pitaevskii equation.

The third foundational course was devoted to the nonlinear Schrödinger equation. In this class, we introduced the power-nonlinearity semilinear Schrödinger equation $i \cdot u_t + \frac{1}{2}\Delta u = \lambda \cdot |u|^{p-1} \cdot u$. In her lectures, Gigliola Staffilani studied the *defocusing equation* given by $\lambda = 1$. We first observed the fundamental conservation laws of mass, energy, and momentum for our equation. From the scaling heuristic, we defined the concepts of subcritical, critical, and supercritical nonlinearities that led into the well-posedness theory for our equation. We then proved local well-posedness of the H^1-subcritical nonlinear Schrödinger equation, and, in the case of the defocusing equation, we deduced that the well-posedness was global.

Continuing with our study of the defocusing equation, we arrived at the concept of *scattering*. Our approach to this problem was based on the notion of Morawetz estimates. After recapitulating the standard Morawetz estimate, we turned to the Interaction Morawetz estimate from which we derived scattering for the cubic, defocusing, nonlinear Schrödinger equation in $\mathbb{R}^3$. Having presented the global well-posedness and scattering in the H^1-subcritical case, we set out to learn the analogue of the result in the H^1-critical case.

Pierre Raphaël's class emphasized the focusing equation, where $\lambda = -1$. The goal was to describe the long-time behavior of solutions, and in particular, the singularity formation in the space H^1. In studying this problem, we also considered the related problem of Soliton Stability. We began by proving H^1-global well-posedness in the L^2-subcritical case and turned to the orbital stability of solitons. We used *concentration compactness* as a remedy for the failure of compactness of $L^q \hookrightarrow H^1$. The main topic of the class was the L^2-critical equation. From the soliton characterization

The Summer School barbecue held on the ETH Hönggerberg Campus.

Evolution Equations at the Swiss Federal Institute of Technology, Zürich, Switzerland

ETH Summer School participants joined by organizers CMI Research Director David Ellwood and ETH Professor Gian Michele Graf (center, front row).

in this case, one can obtain the sharp constant in the *Gagliardo-Nirenberg inequality*. From this fact, we deduced global well-posedness for the L^2-critical problem as long as the mass of the initial data is less than that of the ground state soliton.

At this point, we proved that this theorem was sharp by applying the mass-preserving *Pseudo-conformal transformation*. In particular, this gave us a concrete example of a blow-up solution and led to the first *Liouville (Rigidity) Theorem* of the class. We then turned to the study of the blow-up phenomenon and which blow-up rates occur. We considered a solution by writing it in terms of *modulational parameters*, which we substituted into the equation to obtain the evolution equations for the parameters. For the remainder of the course, we studied explicit blow-up regimes for solutions with small super-critical data.

During the conference, Monica Visan talked about the NLS in the case of critical regularity. She discussed global existence and scattering for the mass-critical and for the energy-critical equations. In particular, we proved an extension of a result of Carlos Kenig and Frank Merle, removing the assumption of radial initial data in dimension ≥ 5.

All lecturers included topics of interest to advanced students, but also took care to provide concrete examples that were accessible to non-experts.

View from Zürich's Lindenhof across the Limmat river to the ETH campus.

Selected Articles by Research Fellows

MODHAMMED ABOUZAID

"Morse homology, tropical geometry, and mirror symmetry for toric varieties." To appear in *Selecta Mathematica*.

"An open string analogue of Viterbo functoriality," with P. Seidel. Preprint, arXiv:0712.3177.

SPYROS ALEXAKIS

"Renormalized area and properly embedded minimal surfaces in hyperbolic 3-manifolds," with Rafe Mazzeo.

"A unique continuation theorem for the vacuum Einsein equations."

ARTUR AVILA

"Hausdorff dimension and conformal measures of Feigenbaum Julia sets," with M. Lyubich. *Journal of the American Mathematical Society* 21 (2008), 305-363.

"Absolute continuity of the integrated density of states for the almost Mathieu operator with non-critical coupling," with D. Damanik. *Inventiones Mathematicae* 172 (2008), 439-453.

MARIA CHUDNOVSKY

"The Erodos Hajnal Conjecture for bull-free graphs," with S. Safra. *Journal of Combinatorial Theory,* Ser., B, 98 (2008) 1301-1310.

"Cycles in dense digraphs," with P. Seymour and B. Sullivan. *Combinatorica* 28 (2008), 1-18.

ADRIAN IOANA

"Cocycle superrigidity for profinite actions of property (T) groups."

"Relative property (T) for the subequivalence relations induced by the action of SL(2, Z) on T^2.

SOREN GALATIUS

"Stable homology of automorphism groups of free groups." Submitted to *Ann. of Math*.

"Universal moduli spaces of surfaces with flat connections and cobordism theory," with R. Cohen and N. Kitchloo. Submitted to *Adv. Math*.

BO'AZ KLARTAG

"On nearly radial marginals of high-dimensional probability measures." Submitted.

"Economical toric spines via Cheeger's Inequality," with N. Alon. Submitted.

CIPRIAN MANOLESCU

"On the Khovanov and knot Floer homologies of quasi-alternating links," with P. Ozsvath. *Proceedings of the 14th Gokova Geometry-Topology Conference* (2007), 60-81.

CIPRIAN MANOLESCU continued

"Combinatorial cobordism maps in hat Heegaard Floer theory," with R. Lipshitz and J. Wang. *Duke Mathematical Journal* 145 (2008), 207-247.

DAVESH MAULIK

"Noether-Lefschetz theory and the Yau-Zaslow conjecture," with A. Klemm, R. Pandharipande, and E. Scheidegger. arxiv: 0802.2739.

"Gromov-Witten/Donaldson-Thomas theory correspondence for toric threefolds," with A. Oblomkov, A. Okounkov, and R. Pandharipande. arxiv: 0809.3976.

MARYAM MIRZAKHANI

"Ergodic theory of the earthquake flow." *Int Math Res Notices* (2008) Vol. 2008.

"Ergodic theory of the space of measured laminations," with Elon Lindenstrauss. *Int Math Res Notices* (2008).

SOPHIE MOREL

"Complexes pondérés sur les compactifications de Baily-Borel: Le cas des variétés de Siegel." *J. Amer. Math. Soc*. 21 (2008), 23-61 ext.

"On the cohomology of certain non-compact Shimura varieties," (with an appendix by R. Kottwitz). To appear in *Annals of Mathematics Studies*.

SAMUEL PAYNE

"Analytification is the limit of all tropicalizations." Submitted. arXiv:0805.4035.

"Positivity for toric vector bundles," with M. Hering and M. Mustata. To appear in *Ann. Inst. Fourier.* arXiv:0805.1916.

DAVID SPEYER

"Powers of Coxeter elements in infinite groups are reduced." To appear in *Proceedings of the AMS*.

"Matching polytopes, toric geometry, and the non-negative part of the Grassmannian," with Alex Postnikov and Lauren Williams. To appear in *Journal of Algebriac Combinatorics*.

TERUYOSHI YOSHIDA

"Local class field theory via Lubin-Tate theory." To appear in *Annales de la Faculté des Sciences de Toulouse*, 17-2 (2008).

XINYI YUAN

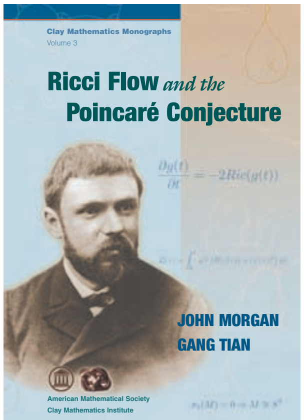
"On volumes of arithmetic line bundles."

"Heights of CM points I: Gross-Zagier formula," with Shou-wu Zhang and Wei Zhang.

Books & Videos

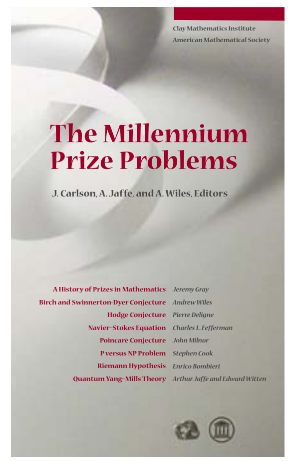
Publications

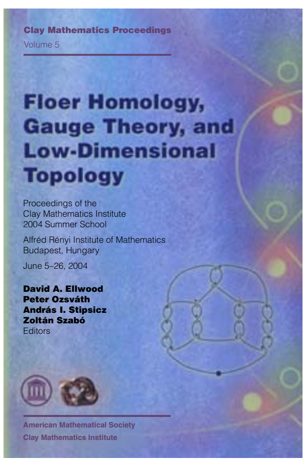
Analytic Number Theory: A Tribute to Gauss and Dirichlet; Editors: William Duke, Yuri Tschinkel. CMI/AMS, 2007, 265 pp. www.claymath.org/publications/Gauss_Dirichlet. This volume contains the proceedings of the Gauss–Dirichlet Conference held in Göttingen from June 20–24 in 2005, commemorating the 150th anniversary of the death of Gauss and the 200th anniversary of Dirichlet's birth. It begins with a definitive summary of the life and work of Dirichlet by J. Elstrodt and continues with thirteen papers by leading experts on research topics of current interest within number theory that were directly influenced by Gauss and Dirichlet.



Ricci Flow and the Poincaré Conjecture; Authors: John Morgan, Gang Tian. CMI/AMS, 2007, 521 pp., www.claymath.org/publications/ricciflow. This book presents a complete and detailed proof of the Poincaré Conjecture. This conjecture was formulated by Henri Poincaré in 1904 and has remained open until the recent work of Grigory Perelman. The arguments given in the book are a detailed version of those that appear in Perelman's three preprints.

The Millennium Prize Problems; Editors: James Carlson, Arthur Jaffe, Andrew Wiles. CMI/AMS, 2006, 165 pp., www.claymath.org/publications/Millennium_Problems. This volume gives the official description of each of the seven problems as well as the rules governing the prizes. It also contains an essay by Jeremy Gray on the history of prize problems in mathematics.

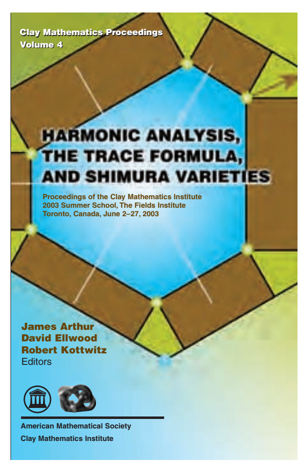




Floer Homology, Gauge Theory, and Low-Dimensional Topology; Proceedings of the CMI 2004 Summer School at Rényi Institute of Mathematics, Budapest. Editors: David Ellwood, Peter Ozsváth, András Stipsicz, Zoltán Szábo. CMI/AMS, 2006, 297 pp., www.claymath.org/publications/Floer_Homology. This volume grew out of the summer school that took place in June of 2004 at the Alfréd Rényi Institute of Mathematics in Budapest, Hungary. It provides a state-of-the-art introduction to current research, covering material from Heegaard Floer homology, contact geometry, smooth four-manifold topology, and symplectic four-manifolds.

Lecture Notes on Motivic Cohomology; Authors: Carlo Mazza, Vladimir Voevodsky, Charles Weibel. CMI/AMS, 2006, 210 pp., www.claymath.org/publications/Motivic_Cohomology. This book provides an account of the triangulated theory of motives. Its purpose is to introduce the reader to Motivic Cohomology, to develop its main properties, and finally to relate it to other known invariants of algebraic varieties and rings such as Milnor K-theory, étale cohomology and Chow groups.

Surveys in Noncommutative Geometry; Editors: Nigel Higson, John Roe. CMI/AMS, 2006, 189 pp., www.claymath.org/publications/Noncommutative_Geometry. In June of 2000, a summer school on Noncommutative Geometry, organized jointly by the American Mathematical Society and the Clay Mathematics Institute, was held at Mount Holyoke College in Massachusetts. The meeting centered around several series of expository lectures that were intended to introduce key topics in noncommutative geometry to mathematicians unfamiliar with the subject. Those expository lectures have been edited and are reproduced in this volume.



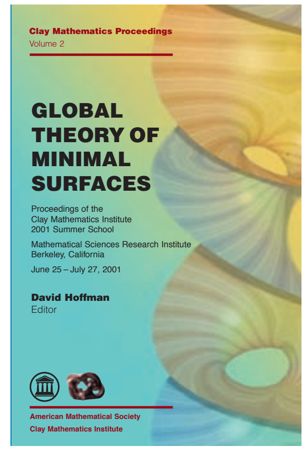
Harmonic Analysis, the Trace Formula and Shimura Varieties; Proceedings of the 2003 CMI Summer School at Fields Institute, Toronto. Editors: James Arthur, David Ellwood, Robert Kottwitz. CMI/AMS, 2005, 689 pp., www.claymath.org/publications/Harmonic_Analysis. The subject of this volume is the trace formula and Shimura varieties. These areas have been especially difficult to learn because of a lack of expository material. This volume aims to rectify that problem. It is based on the courses given at the 2003 Clay Mathematics

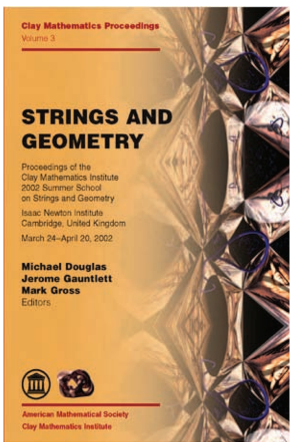
The AMS will provide a discount of 20% to students purchasing Clay publications. To receive the discount, students should provide the reference code CLAY MATH.

Online Orders: *enter "CLAY MATH" in the comments field for each Clay publication ordered.* **Phone Orders:** *give Customer Service the reference code and they will extend a 20% discount on each Clay publication ordered.*

Institute Summer School. Many of the articles have been expanded into comprehensive introductions, either to the trace formula or to the theory of Shimura varieties, or to some aspect of the interplay and application of the two areas.

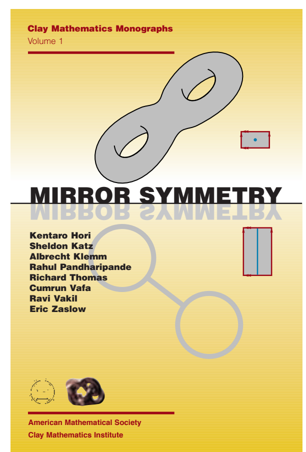
Global Theory of Minimal Surfaces; Proceedings of the 2001 CMI Summer School at MSRI. Editor: David Hoffman. CMI/AMS, 2005, 800 pp., www.claymath.org/publications/Minimal_Surfaces. This book is the product of the 2001 CMI Summer School held at MSRI. The subjects covered include minimal and constant-mean-curvature submanifolds, geometric measure theory and the double-bubble conjecture, Lagrangian geometry, numerical simulation of geometric phenomena, applications of mean curvature to general relativity and Riemannian geometry, the isoperimetric problem, the geometry of fully nonlinear elliptic equations, and applications to the topology of three-manifolds.





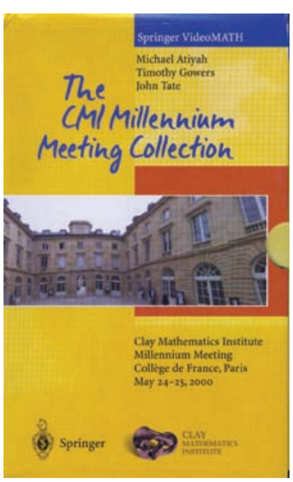
Strings and Geometry; Proceedings of the 2002 CMI Summer School held at the Isaac Newton Institute for Mathematical Sciences, UK. Editors: Michael Douglas, Jerome Gauntlett, Mark Gross. CMI/AMS publication, 376 pp., paperback, ISBN 0-8218-3715-X. List: $69. AMS Members: $55. Order code: CMIP/3. To order, visit www.ams.org/bookstore.

Mirror Symmetry; Authors: Kentaro Hori, Sheldon Katz, Albrecht Klemm, Rahul Pandharipande, Richard Thomas, Ravi Vakil. Editors: Cumrun Vafa, Eric Zaslow. CMI/AMS, 929 pp., hardcover, ISBN 0-8218-2955-6. List: $124. AMS Members: $99. Order code: CMIM/1. To order, visit www.ams.org/bookstore.



Strings 2001; Authors: Atish Dabholkar, Sunil Mukhi, Spenta R. Wadia. Tata Institute of Fundamental Research. Editor: American Mathematical Society (AMS), 2002, 489 pp., paperback, ISBN 0-8218-2981-5. List $74. AMS Members: $59. Order code: CMIP/1. To order, visit www.ams.org/bookstore

Video Cassettes



The CMI Millennium Meeting Collection; Authors: Michael Atiyah, Timothy Gowers, John Tate, François Tisseyre. Editors: Tom Apostol, Jean-Pierre Bourguignon, Michele Emmer, Hans-Christian Hege, Konrad Polthier. Springer VideoMATH, Clay Mathematics Institute, 2002. Box set consists of four video cassettes: the CMI Millennium Meeting, a film by François Tisseyre; The Importance of Mathematics, a lecture by Timothy Gowers; The Millennium Prize Problems, a lecture by Michael Atiyah; and The Millennium Prize Problems, a lecture by John Tate. VHS/NTSC or PAL. ISBN 3-540-92657-7. List: $119, EUR 104.95. To order, visit www.springer-ny.com (in the United States) or www.springer.de (in Europe).

These videos document the Paris meeting at the Collège de France where CMI announced the Millennium Prize Problems. The videos are for anyone who wants to learn more about these seven grand challenges in mathematics.

Videos of the 2000 Millennium event are available online and in VHS format from Springer-Verlag. To order the box set or individual tapes, visit www.springer.com.

2009 Institute Calendar

JANUARY FEBRAURY MARCH	Research Scholar Bryna Kra at Northwestern. January 1–June 4 Research Scholar Fernando Rodriguez-Villegas at University Texas at Austin. January 1–June 30 CMI Workshop on Geometry and Physics of the Landau Ginzburg Model. January 12–16 Research Scholar Richard Schwartz at Caltech. February 1–March 30 Clay Lectures in Mathematics at RIMS, Kyoto: Roman Bezrukavnikov, Dennis Gaitsgory, and Hiraku Nakajiima. March 2–6 Conference on "IV International Symposium on Non-Linear Equations and Free Boundary Problems," at Gran Hotel Dora, Buenos Aires. Argentina. March 17–20
APRIL MAY	Senior Scholar Calire Voisin at MSRI. April 1–30 Conference on "Singularities at MIT." April 5 Conference on "Geometry and Physics: Atiyah80" at International Centre for Mathematical Sciences, Edinburgh. April 20 –22 Senior Scholars Christopher Hacon and Rahul Pandharipande at MSRI. April 9 – May 21 Clay Research Conference at MIT. May 4–5 Conference on the Power of Analysis at Princeton, University. May 5–8 CMI Workshop on Macdonald Polynomials and Geometry. May 9–12
JUNE JULY AUGUST	Conference on Geometry and Functional Analysis, at University of Tel-Aviv. June Conference on Topology of Algebraic Varieties at Jaca, Huesca, Spain. June 22–26 Senior Scholars Benedict Gross and John Tate at IAS/PCMI. June 28 – July 13 Conference on Rennes Arithmetic Geometry Days (Journees de Geometrie Arithmetique de Rennes) at Rennes University. July 6–10 Conference on Dynamical Numbers: Interplay Between Dynamical Systems and Number Theory at Max-Planck-Institut fur Mathematik, Bonn. July 20–24
SEPTEMBER OCTOBER NOVEMBER	Clay Lectures Australia (featuring Clay-Mahler lecturer Terry Tao). Heisuke Hironaka, Soundararajan, Dinakar Ramakrishnan, Langlands program. Etienne Ghys public lecture at MIT. October or November Senior Scholar Clifford Taubes at MSRI. Program on Symplectic and Contact Geometry and Topology. 2009-2010