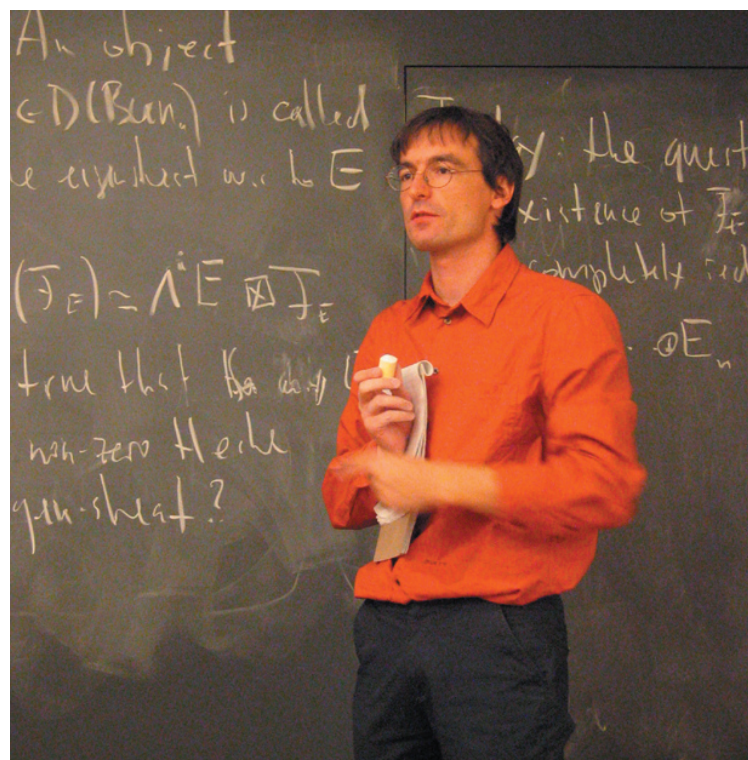# Interview with Research Fellow Dennis Gaitsgory

Biography

During his time at Tel-Aviv University, Dennis Gaitsgory studied algebra, algebraic geometry and representation theory under the guidance of Joseph Bernstein, receiving his PhD in 1998. His thesis, entitled "Automorphic sheaves and Eisenstein series," develops ideas of A. Beilinson, V. Drinfeld and G. Laumon that deal with the geometrization of the theory of automorphic forms.

Since the fall of 1996, he has spent most of the academic year in the United States. He was twice a member of the Institute for Advanced Study in Princeton, New Jersey (1996–1997 and 1998–1999), and in 1997 he was admitted as a Junior Fellow of the Harvard Society of Fellows. He held a Clay Research Fellowship from 2000–2004.

Dennis Gaitsgory joined the Department of Mathematics at the University of Chicago in the Fall of 2001 as an Associate Professor.

Dennis Gaitsgory lecturing on "Two Faces of Geometric Eisenstein Series" at the Clay Research Fellow Lecture Series in November 2004.

Interview

What first drew you to mathematics? What are some of your earliest memories of mathematics?

My father, who is an applied mathematician, was making diligent efforts from my early childhood to attract me to mathematics. In the beginning he would give me math puzzles to solve, but this never quite worked: as I understand now, I've never had a knack for "isolated difficult problems." (For the same reason, I invariably did poorly at math Olympiads). Later, when I was about twelve, we started to read some math books together, so by the time I entered the university I knew the basics of analysis, linear algebra and differential equations. However, as soon as I enrolled in my first several classes at the university (Tel Aviv, summer 1990), my attitude to mathematics underwent a sudden phase transition: something "clicked," and I started

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to derive great pleasure from carefully thinking through the mathematical theories that I was being taught. I think that it was then that I began to appreciate the real aesthetics of the subject, i.e., what makes it pleasant to do mathematics, apart from the thrill of "to know a bit more of higher math."

Did you have a mentor? Who helped you develop your interest in mathematics, and how?

It would be fair to say that after my father my next mentor was Sasha Braverman. Having come from a math high school in Moscow, he had a much clearer idea than I about what was going on in modern mathematics. I recall a scene in the library, where Sasha (sixteen at that time) was leafing through a book on algebraic topology. When I asked reverently what homology was, he somberly replied that I would be better off not knowing it (which, of course, resulted in my starting to read the same book that very afternoon). It went on in this fashion for about a year until Sasha convinced me that we should go together to Joseph Bernstein (who was

spending his sabbatical at Tel Aviv University that year) and ask him to "give us a problem"—which effectively was asking him to become our adviser.

In 1998 you received your PhD from Tel-Aviv University, where you studied under Joseph Bernstein. Could you recount some of your mathematical experiences at Tel-Aviv University and with Professor Bernstein?

The first several months after meeting Bernstein were exceptionally hard. I was exposed to so much new mathematics, which I felt I should have learned long ago. Besides, Bernstein was running a seminar where even more fascinating things were discussed, and which I was not able to follow at all. But after a while I got over my frustration, mostly by acquiring enough impertinence to ask a question every time I didn't understand something.

For the next two years Bernstein invested a lot of time talking to Sasha and me, as well as teaching courses more or less at our request, primarily in algebraic geometry and representation theory. All this was done while emphasizing his quite unique way of attaining the clearest and simplest, even if sophisticated, understanding of mathematical structures.

I should say that after such an experience I found it difficult to attend courses taught by other people, be it in Tel Aviv or elsewhere: I always expected a lecturer to explain things at the same level of clarity, and to reduce them to the same level of beautiful simplicity as did Bernstein, which is quite rare.

How would you compare your mathematical experiences in Israel, Russia, and the US?

It so happened that in my research I have mostly interacted with the same circle of disciples of I. M. Gelfand (this group of people is also widely known as "Russian mafia"). This international organization transcends the borders of states, so sitting in a seminar room I often couldn't quite determine its venue: Chicago, Tel Aviv, or Moscow (where I have never been as an adult).

What attracted you to the areas of mathematics in which you work and the problems you have studied?

At one point (in 1994, I think), Beilinson came to Tel Aviv for the Schur lectures (as at many other universities, there is an annual lecture series named after a renowned mathematician. In Tel Aviv, distinguished people are invited to speak about recent developments in their fields at the Issai Schur Memorial Lectures). Beilinson spoke about Geometric Langlands. Although I understood less than thirty percent, it was enough for me to be completely overcome by the sense of beauty of what he talked about. In my eyes it combined perfectly the aspects of algebraic geometry and representation theory that I liked most.

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As it turned out, Bernstein, while still at Harvard, had run a seminar on a different approach to the same problem. Obviously, I was most eager to take it up. To this day, manifestations of Langlands duality in geometric representation theory serve as a source of motivation for most problems that I study.

Dennis Gaitsgory and Joseph Bernstein during the School on Automorphic froms in Jerusalem, April 2001.

How did your Clay Research Fellowship benefit your work?

CMI has benefited my work in a very simple way: by allowing me to concentrate on my research for three years. What more can a mathematician ask for?

How do you think mathematics benefits our culture?

First, we should define what "culture" means. To be brief, let us say that culture is the collection of intellectual assets of the society that are not connected with needs of physical survival, or, more broadly, material prosperity. In this definition, it ranges from what is known as "popular culture," i.e., those aspects that can be appreciated by the vast majority of people, to — mathematics. The latter is the purest intellectual pursuit that can be adequately understood only by a tiny minority.

Curiously enough, mathematics, which (as I guess) is perceived, either consciously or subconsciously, as a form of art by most people who practice it professionally, borders with applied sciences. And throughout history, applied problems were often a source of motivation for developments that later took a purely theoretical turn.

Please tell us about things you enjoy when not doing mathematics.

With regret I must admit that I agree with the maxim that (in most cases) a mathematician denies himself (most of the) simple pleasures of life. He (or she) is just thinking all the time, while life passes by. In spite of that, there are things not connected to mathematics without which I would feel that I am missing out on too much.

One of these things is travel. Having spent my childhood and early adolescence in Communist-era Russia, I grew up with the conviction that I would be a life-time prisoner, bound to that country. Therefore, as soon as I found myself in the West, I simply couldn't satisfy the thirst for the sensation that I could go to (almost) any country that I chose, and that those abstract colored shapes on the globe

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were actual destinations. However, as I grow older, I am starting to feel that being a mere tourist does not answer all my needs, and I would like to have a deeper involvement in the countries that I visit.

Another thing that I greatly enjoy is being passionate, but not too consistent or serious about any of my hobbies. Acting in this spirit, in the course of the past year I was doing yoga, studying Arabic, and taking tango lessons. If I were a really devoted student in any of the above three endeavors, it would take up all of my free time. But outside the sphere of my professional life I prefer to try more things, even at the expense of not making sufficient progress.

Recent Research Articles:

On a vanishing conjecture appearing in the geometric Langlands correspondence, AG/0204081. Annals of Math., 160 (2004), no. 2, 617—682.

D-modules on the affine Grassmannian and representations of affine Kac-moody algebras, joint with E. Frenkel, AG/0303173, Duke Math. J. 125 (2004), no. 2, 279—327.

Representations of algebraic groups over a 2-dimensional local field, joint with D. Kazhdan, RT/0302174, GAFA 14 (2004), no. 3, 535—574.

Construction of central elements in the affine Hecke algebra via nearby cycles, alg-geom/9912074, Invent. Math. 144 (2001), no. 2, 253—280.

Geometric Eisenstein series, joint with A. Braverman, alg-geom/9912097, Invent. Math. 150 (2002), no. 2, 287—384.

Differential operators on the loop group via chiral algebras, joint with S. Arkhipov, AG/0009007, Int. Math. Res. Not. 2002, no. 4, 165—210.