Poisson kernel. Given $f$ on $T$, extend $f$ as a harmonic $f(z)$ to $D$.

$$f = e^{i\eta}$$
$$u = z^n \quad n > 0$$
$$e^{-i\eta}$$
$$u = \overline{z}^n \quad n \leq 0$$

$$f(z) = \sum_{n>0} a_n z^n$$
and in $D$

$$2i \text{ Im } f(z) = \sum_{n>1} a_n \overline{z}^n + a_0 \overline{\alpha_0} - \sum_{n<-1} \overline{a_n} \overline{z}^n$$

$$f(z) = \sum_{n>0} a_n z^n$$
and in $D$

$$2 \text{ Re } f(z) = \sum_{n>0} a_n z^n + \overline{a_n} \overline{z}^n$$

$$2 \text{ Re } f(e^{i\eta}) = \sum_{n>0} a_n e^{i\eta} + \overline{a_n} e^{-i\eta}$$

$$\int e^{-i\eta} \frac{f(e^{i\eta})}{2\pi} \frac{d\phi}{2\pi} = \left\{ \begin{array}{ll}
a_n & n > 1 \\
a_0 + \overline{\alpha_0} & n = 0 \\
\overline{a_n} & n < -1 \end{array} \right.$$
\[ f(z) = \sum_{n=0} a_n z^n \quad \text{analytic in } D \]

\[ 2i \, \text{Im} \, f(z) = \frac{1}{2\pi i} \left( f(z) - \overline{f(z)} \right) \]

\[ = \sum_{n>0} a_n z^n - \overline{a_n} \overline{z}^n \]

\[ \int \int_{\Gamma} (2i \, \text{Im} \, f(z)) \frac{d\phi}{2\pi} = a_n \quad n>1 \]

\[ a_0 - \overline{a_0} \quad n=0. \]

\[ f(z) = a_0 + \sum_{n>1} \int \int_{\Gamma} e^{-i\phi} (2i \, \text{Im} \, f(e^{i\phi})) \frac{d\phi}{2\pi} \]

\[ \int (2i \, \text{Im} \, f(e^{i\phi})) \frac{d\phi}{2\pi} = a_0 - \overline{a_0} \]

\[ \frac{a_0 - \overline{a_0}}{2} \]
\[ f(z) = \sum_{n \geq 0} a_n z^n \quad \text{analytic in } D \]

\[ J = \oint f(z) - \overline{f(z)} = \sum_{n \geq 0} a_n z^n - \overline{a_n} \overline{z}^n \]

\[ \int e^{-i\theta} J(e^{i\theta}) \frac{d\theta}{2\pi} = a_n \quad n \geq 1 \]

\[ = a_0 - \overline{a_0} \quad n = 0 \]

\[ f(z) = \sum_{n \geq 0} a_n z^n \]

\[ \text{Im } f(z) = \sum_{n \geq 0} \frac{a_n z^n - \overline{a_n} \overline{z}^n}{2i} \]

\[ \int e^{-i\theta} \text{Im } f(e^{i\theta}) \frac{d\theta}{2\pi} = \left\{ \begin{array}{ll}
\frac{a_0 - \overline{a_0}}{2i} & n = 0 \\
\frac{a_n}{2i} & n \geq 1
\end{array} \right. \]
\( f(z) = \sum_{n>0} a_n z^n \quad \text{and in } \Omega \)

\( f(z) - \overline{f(z)} = \sum_{n>0} a_n z^n - \overline{a_n} \overline{z^n} \)

\( f(e^{i\theta}) - \overline{f(e^{i\theta})} = \sum_{n>0} a_n e^{in\theta} - \overline{a_n} e^{-in\theta} \)

\[
\int e^{-in\theta} (f(e^{i\theta}) - \overline{f(e^{i\theta})}) \frac{d\theta}{2\pi} = \begin{cases} a_0 - \overline{a_0} & n = 0 \\ a_n & n > 1 \end{cases}
\]

\( f(0) = a_0 \)

\[
\int \left( \sum_{n=1}^{\infty} \frac{z^n e^{-in\theta}}{2^n} + \frac{1}{2} \right) (f(e^{i\theta}) - \overline{f(e^{i\theta})}) \frac{d\theta}{2^n} = \frac{1}{2} (a_0 - \overline{a_0}) + \sum_{n=1}^{\infty} z^n a_n = f(z) - \frac{a_0 + \overline{a_0}}{2}
\]

\[
\frac{1}{2} + \frac{ze^{-i\theta}}{1-ze^{-i\theta}} = \frac{1-ze^{-i\theta} + 2ze^{-i\theta}}{2(1-ze^{-i\theta})} = \frac{1}{2} \frac{1+ze^{-i\theta}}{1-ze^{-i\theta}}
\]

\( f(z) = \sum_{n>0} a_n z^n \quad \text{and in } \Omega \)

\[
\int (f(z) - \overline{f(z)}) \frac{d\theta}{2\pi} = \begin{cases} a_n & n > 0 \\ a_0 - \overline{a_0} & n = 0 \end{cases}
\]
Do resistance case first

1. connected
4 vertices
4 edges
1 loop.

\[ \frac{1}{\frac{1}{a+b} + \frac{1}{c+d}} \]

LC 1-port has an impedance \( Z(s) \) with certain properties such that you need to understand properly.

Start at the LC side. \( \Gamma \) connected

\[ V = \frac{C'(\Gamma)}{R} \longrightarrow C'(\Gamma) \longrightarrow H'(\Gamma) \]

\[ \downarrow \]

\[ V/W \]

**Examples:**

\[ W = C^0(\Gamma_{ext}) \]

Then \( V/W = \frac{C^0(\Gamma_{ext})}{R} \)

This seems correct.

\[ \text{e.g.} \]
Return to LC circuits + partial units.

Go over steps carefully.

direct current, resistance \( R \) values for edges

a set of external vertices containing \( \Omega \)

Data: external bridge

How are you going to handle independent batteries.

Could this square be related to scattering?

It looks like you want to drop the ground.

\[ a \rightarrow C \xrightarrow{V_1-V_2} C' \rightarrow H' \rightarrow 0 \]

Problem: Find the impedance of a "bridge"
You want to go over the calculation of the quadratic form \( s \| f \|_{+}^{2} + s^{-1} \| f \|_{-}^{2} \) on \( H_+ \oplus H_- \) when restricted to \( V \subset H_+ \oplus H_- \) and afterward when pushed down to \( V/W \).

Let the components of the inclusion \( V \subset H \) be \( j_+ : V \rightarrow H_+ \). Use the inner norm \( \| f \|_{+}^{2} = \| f \|_{+}^{2} + \| f \|_{-}^{2} \) on \( H \) to make \( V \) an Euclidean subspace of \( H \), and to make \( V/W \) an Euclidean quotient space. The variable quadratic form induced on \( V \) from \( s \| f \|_{+}^{2} + s^{-1} \| f \|_{-}^{2} \) is

\[
Q_s(f) = s \| f \|_{+}^{2} + s^{-1} \| f \|_{-}^{2} = s^*(f^* f_+ + s^{-1} f_+ f_-)
\]

Put \( p = f_+ f_+^* \); it's a self-adjoint operator set \( 0 \leq p \leq 1 \) since \( 1 - p = 1 - f_+ f_+^* = f_+ f_- f_+ \geq 0 \).

\[
V = (f_+ f_-^*)(f_+ f_-) = (f_+^*)(f_+ f_-) \text{ is the self-adjoint projection on } H
\]

with image \( (f_+ f_-) V \).

You can split \( V \) into eigenspaces for the operator \( p = f_+ f_+^* \):

\[
V = \bigoplus_{0 \leq \lambda \leq 1} V_{\lambda}, \quad p(f) = \lambda f \text{ for } f \in V_{\lambda}
\]

Try your Grassmannian version. Given \( V \subset (H_+ \oplus H_-) \), let \( F = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \) on \( V \). Suppose \( V = (1) H_+ \).

\[
V_+ = (-T^*) H_-. \text{ Then } \quad F \begin{pmatrix} 1 & -T^* \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -T^* \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} 
\]

\[
F(1+X) = (1+X) \varepsilon, \quad F = (1+X) \varepsilon (1+X)^{-1} = \begin{pmatrix} 1+X \\ 1-X \end{pmatrix} \varepsilon
\]
\( \psi \rightarrow (H_+^*) \quad s f_+ + s^* f_- = 1 \psi \)

\[ f_+ = \sum_{0 \leq \lambda \leq 1} \lambda \pi_\lambda, \quad \sum \pi_\lambda = 1 \quad \text{V} \quad f_- = \sum_{0 \leq \lambda \leq 1} (1-\lambda) \pi_\lambda \]

\[ \sum_{0 \leq \lambda \leq 1} (s \lambda + s^* (1-\lambda)) \pi_\lambda. \quad \text{now you need to link} \]

\( 0 \leq \lambda \leq 1 \) to frequency \( 0 \leq \omega \leq \infty \).

Rate \( \frac{s^2}{s^2 + \omega^2} \)?

\[
\frac{1}{\lambda - 1} = \frac{1-\lambda}{\lambda} = \omega^2 \quad \lambda = \frac{1}{1 + \omega^2}
\]

Transformation \( s \lambda + s^* (1-\lambda) = \frac{s^2 \lambda + (1-\lambda)}{s} \)

\[ s \frac{1}{1 + \omega^2} + s^* \frac{\omega^2}{1 + \omega^2} = \frac{1}{s} \frac{s^2 + \omega^2}{1 + \omega^2} \]

Let \( \sum_{0 \leq \omega \leq \infty} \frac{s^2 + \omega^2}{1 + \omega^2} \pi_\omega \)

You notice \( s = 1 \) gives \( 1 \) on \( \psi \).

The quadratic form \( Q_5 \) on \( (H_+^*) \) does not yield an operator unless there is already a quadratic form on \( H \). What structure is there on \( C^1 = C^1_\perp \oplus C^1_c \).

Energy \( \sum \frac{V_\perp^2}{L_5} \sum V_c^2 C_5 \)
What you've learned. LC circuit described by the real vector space $H = C^1 = H_+ \oplus H_-$. Equipped with the family of quadratic forms

$$Q_\delta \left( \begin{pmatrix} x_+ \\ x_- \end{pmatrix} \right) = 8 \| x_+ \|^2 + 8^{-1} \| x_- \|^2$$

$x_+ = \{ \varphi_\sigma, \sigma \in \text{C-type} \}$

$x_- = \{ \varphi_\sigma, \sigma \in \text{L-type} \}$

$$\| x_+ \|^2 = \sum \varphi_\sigma^2$$

$$\| x_- \|^2 = \sum \frac{1}{\gamma} \varphi_\sigma^2$$

Energy calculation

$$CV = \dot{I}$$

$$CVI = IT = \dot{I} \left( \frac{I^2}{2} \right)$$

Energy in the capacitor at time $t$ is

$$\int_{-\infty}^{t} VI \, dt = \int_{-\infty}^{t} \dot{I} \left( \frac{I^2}{2c} \right) \, dt = \frac{I^2}{2C}$$

Charge of capacitor $Q = CV$

$I = C \partial_t V$

Power $VI = CV \partial_t V = \dot{I} \left( \frac{1}{2} CV^2 \right)$

Energy stored in capacitor at time $t$ is

$$\int_{-\infty}^{t} VI \, dt = \left[ \frac{1}{2} CV^2 \right]_{-\infty}^{t} = \frac{1}{2} CV(t)^2$$
Try for some progress on LC circuits
connected
\( \Gamma \), a graph, whose edges are either L or C type
\( V = \mathbb{C}^\infty(\Gamma)/\mathbb{C} \overset{\delta}{\rightarrow} \mathbb{C}(\Gamma) \)

\[ \begin{align*}
V/W & \quad \text{e.g. } W = \mathbb{C}(\Gamma_0, \text{int}) \\
V/W & = \mathbb{C}(\Gamma_0, \text{ext})/\mathbb{C}
\end{align*} \]

Here you have set \( \Gamma_0 \) of vertices partitioned
into \( \Gamma_0, \text{ext} \) and \( \Gamma_0, \text{int} \). You want the internal voltages to be free.

It seems that you should work over \( \mathbb{R} \) when
defining an "abstract" LC circuit. Such a thing is equivalent to
subquotient of a polarized real
Euclidean space.

\[ V \overset{\phi}{\rightarrow} H_+ \oplus H_- = H \]

\[ \begin{align*}
V/W & \quad \text{On } H \text{ you the quadratic form}
& \quad s||\phi_+||^2 + s^{-1}||\phi_-||^2 = Q_s
\end{align*} \]

which induces a \( \mathbb{R} \) family of quadratic forms on
\( V/W \) depending rational on \( s \). Your first task
will be to understand this well. Need to understand
\( \phi \) dynamics: \( s \) is the "frequency" parameter
occurring in the L.T. You are studying
a module over the (times translation) group, \( \mathbb{R} \). The L.T.
turns this into a module over functions of \( s \).
Now you have a new idea — to work with the DE’s hopefully to understand variational principles — to handle circuit constraints. Where to start? A specific example!

You need a way to handle the constraints. What is a solution? Prove a solution exists and is unique.
Aim: To obtain a real functions of time picture of some simple LC circuits. Try

\[
\frac{V_a}{I_a} = Ls + \frac{1}{Cs}
\]

But, simpler is the closed circuit:

\[
\frac{V_a}{I_a} = \frac{C^{-1}s}{s^2 + \omega^2}
\]

What exactly is the system of equations you want to solve? In terms of the 4 variables \( V_L, V_C, I_L, I_C \) you have

\[
V_L = L \frac{d}{dt} I_L \quad I_C = CV_C
\]

circuit equations \( V_L = V_C \), \( I_L + I_C = 0 \)

4 equations in 4 variables

\[
V_L = L \frac{d}{dt} I_L \quad -I_L = CV_L = CL \frac{d}{dt} I_L
\]

\[
(CL \frac{d^2}{dt^2} + 1)I_L = 0
\]

\[
CLs^2 + 1 = 0 \quad s = \pm i \sqrt{\frac{1}{LC}}
\]

What comes next should be one of the two cases above with a "forcing" term - current or voltage source applied to the series or parallel LC circuit.
As an example, consider a simple LC circuit. This is a 1-port. Two simple cases:

**Series case:**
- Variables are $V_C, I_C, V_L, I_L, V_a, I_a$
- $I_a = I_C = I_L$
- $V_a = V_C + V_L$
- $I_C = CV_C$
- $V_L = LI_L$

  Take L.I.T. $I_a = IC = ITL$, $V_a = V_C + V_L$
  - $I_a = CSV_C$
  - $V_L = LSIA$
  - $V_a = \left(\frac{1}{CS} + LS\right)IA$

**Parallel case:**
- Variables are $V_C = V_L$, $I_a = I_C + I_L$
- $I_C = CV_C$
- $V_L = LI_L$

  Take L.I.T. $IC = CSVA$
  - $IL = \frac{1}{LS}VA \Rightarrow IA = (CS + \frac{1}{LS})VA$

Go back to your notes. Something seems strange if you are using real functions of time. Start where?

Start with resistance, a network, direct current sources (batteries). Review the existence of the linear equations.
Consider a resistance network, a graph where each edge has a resistance of $R > 0$ ohms. Assume connected. Picture

\[
\tilde{\mathcal{C}}^0 \xrightarrow{\delta} \mathcal{C}^1 \rightarrow \mathcal{H}^0
\]

\[
\tilde{\mathcal{C}}_0 \xleftarrow{\partial} \mathcal{C}_1 \leftrightarrow \mathcal{H}_1
\]

cycles

quality pairing, diagonal

type pairing \[ V_{\sigma} \cdot I_{\sigma} = \begin{cases} 0 & \text{if } \sigma \neq \rho \\ V_{\sigma}I_{\sigma} & \text{if } \sigma = \rho \end{cases} \]

This pairing \[ V \cdot I = \sum_{\sigma} V_{\sigma}I_{\sigma} \]
is the power dissipated by a configuration of voltage drops + currents.

At some point you must list the circuit equations:

1) Voltage drops given by a potential, a function on the nodes (modulo constants).

This means restricting $\mathcal{C}^1$ to $\delta \tilde{\mathcal{C}}^0$.

Do there some statement about 1-cycle currents?

\[ \exists \text{ take a } f \]
Try again to understand the circuit to get a clear picture of the circuit equations.

Data. $\Gamma$ graph connected, "metric" on edges. at least 2 nodes. Consider 1-port, specify 2 nodes: +, - nodes.

State of the network: for each $\sigma$ $V_\sigma, I_\sigma$ related by Ohm's Law.

$\{V_\sigma\}$ conservative - same as $\{V_x\}$ mod const.

$\{I_{x}\} = \partial \{I_\sigma\}$ supported at +, - nodes.

\[ \text{What's the difficulty? Start again: } \Gamma \text{ connected graph. } \Gamma \text{ edge with "metric" given by pos, nos. } R > 0 \text{ for each edge. Given also a dist. pair of nodes } +, - \text{ called external nodes for attaching a battery or current source.}

Then you be able to calculate the state of the network with the source attached. This state consists of $V_\sigma, I_\sigma$ for each edge $\sigma$.

\[ \text{better to say } 1\text{-cochain } \{V_\sigma\}, \text{ 1-chain } \{I_\sigma\}\]

satisfying Ohm: $V_\sigma = R \cdot I_\sigma$. $V$ is 1-coboundary.

$\partial I$ vanishes at each of node.

Claim. There's a space of such states of dim 1.
this kinematics seems completely clear, since interpretation via symplectic quotients. Next comes dynamics. Wait, what is the conclusion? Answer might be that

Geometric situation (kinematics)
\[ \mathbb{R} \left[ \begin{array}{c} 0 \\ \mathbb{R} \end{array} \right] \xrightarrow{\mathbb{R}[+1,-1]} \mathcal{C}_0 \xrightarrow{\mathcal{C}_1} \mathbb{C} \]

Symplectic quotient situation

Puzzle about energy a geometric pairing between voltage and current. How is this related to the energy going into a resistance?

Given an LC 1-port, do there an associated partial unitary \( U \) of rank 1? Reason: The LC 1-port is described by a rational function, pos. res.
\( V_L = V_e - V_C \)
\( V_L = LsI \)
\( I = CsV_C = V_e \frac{Cs}{1 + LCS^2} \)

\[ Q_s \text{ should be } \frac{1}{Ls} V_L^2 + CsV_C^2 \]

\[ \frac{1}{Ls} (V_e - V_C)^2 + CsV_C^2 \]

\[ \frac{1}{Ls} (V_e - V_C)(-1) + CsV_C = 0 \]

\[ \frac{1}{Ls} V_e = \left( \frac{1}{Ls} + Cs \right) V_C \]

\[ V_C = V_e \frac{1}{1 + LCs^2} \]

\[ V_L = V_e - V_C = V_e \left( 1 - \frac{1}{1 + LCs^2} \right) \]

\[ V_L = V_e \frac{LCS^2}{1 + LCS^2} \]

\[ I = \frac{Cs}{1 + LCS^2} \]

\[ \frac{V_e}{I} = \frac{V_e Cs}{V_e 1 + LCS^2} = \frac{1 + LCS^2}{Cs} = Ls + \frac{1}{Cs} \]
Start with a connected R network with basepoint.

\[
\tilde{C}_\text{int} \leftarrow \tilde{C}_0 \longrightarrow C' \longrightarrow H'
\]

you get a positive def g.f. on \( C' \) induces one on \( \tilde{C}_\text{int} \). Dual spaces

\[
\tilde{C}_\text{int} \leftarrow \tilde{C}_0 \overset{\delta}{\longrightarrow} C'
\]

\[
\tilde{C}_0 \longrightarrow C_0 \overset{\partial}{\longrightarrow} C'
\]

\[
\tilde{C}_0 = C_0(X_r;*)
\]

You need to identify the induced positive quad. forms with the direct solution of the circuit equations. The main part of the story is that \( \text{Voltage and Current spaces are naturally dual via the power pairing (this is kinematics), and then you have the quadratic form induced by energy, also the power).? NO somehow different -1}
Anyway what?

\[ V, I \]

\[ V = IR \]

\[ P = VI = \frac{V^2}{R} = I^2R \]

Now

Maxwell equations.

\[ \mathbf{E} \quad \mathbf{B} \]

\[ \nabla \cdot \mathbf{B} = 0 \quad \nabla \times \mathbf{E} + \partial_t \mathbf{B} = 0 \]

\[ \nabla \cdot \mathbf{E} = \mathbf{j} \quad \nabla \times \mathbf{B} = \mathbf{j} + \partial_t \mathbf{E} \]
Discuss what? Ans: Clean picture about LC ports. Subsequent of polarized Euclidean space. You have to review this carefully in order to avoid a mistake. Also to learn whether there are any complex hermitian aspects. You can look at a subquotient of a polarized complex Hilbert space.

\[
\begin{pmatrix}
\bar{d}^* \\
\bar{d}
\end{pmatrix} \rightarrow \begin{pmatrix} H_+ \\ H_- \end{pmatrix}
\]

\[
\begin{pmatrix} d_+ \\ d_- \end{pmatrix} \rightarrow \begin{pmatrix} d_+^* \\
-d_-^* \end{pmatrix}
\]

\[
\begin{pmatrix} d_+ \end{pmatrix}^* = \begin{pmatrix} d_+^* \\
-d_-^* \end{pmatrix}
\]

J axiom means \[d_+ d_-^* = d_+^* d_- + d_-^* d_+ = 1 \]

get proj. \[
\begin{pmatrix} d_+ \end{pmatrix} \begin{pmatrix} d_+^* \\
-d_-^* \end{pmatrix} \text{ on } H \text{ with inner product}
\]

\[
\begin{pmatrix} 1 & d_+^* \\ d_-^* & 1 \\
-d_+^* & 1 \\
d_-^* & -d_+^* \end{pmatrix}
\]

What information is there? Just two involutions \(F, \varepsilon\)

\[
1 + X = \begin{pmatrix} 1 & -t \\ t & 1 \end{pmatrix}
\]

\[
q^{1/2} = \frac{1 + X}{\sqrt{1 - x^2}} = \begin{pmatrix} 1 & -t \\ t & 1 \end{pmatrix} \frac{1}{\sqrt{1 + t^2}} \frac{1}{1 + t^2}
\]

\[
F = q^{1/2} \otimes q^{-1/2} = \begin{pmatrix} 1 - t \\ t \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 + t \\ t \end{pmatrix} = \frac{1}{1 + t^2} \frac{1}{1 + t^2} \begin{pmatrix} x - t^2 & 2t \\ 2t & 1 - t^2 \end{pmatrix}
\]

\[
\begin{pmatrix} x - t^2 & 2t \\ 2t & 1 - t^2 \end{pmatrix}
\]
the inclusion. What is the structure behind this? Simply a Hilbert space reps of the infinite dihedral group $\mathbb{Z}_2 \times \mathbb{Z}/2$. i.e. two involutions $F, e$ on a Hilbert space 

\[ V = J^*J = (d^+ d^-)(d^+ + d^- f^-) \]

projection on \( V = J J^* = (d^+ d^-) = (d^+ d^* d^* d^-) \)

\[ F = 2e - 1 = \begin{pmatrix} 2d^+ d^* - 1 & 2d^+ d^- \\ 2d^- d^* & 2d^- d^- - 1 \end{pmatrix} \]

not useful.

Approach 1: Start with $H = \left( \begin{array}{c} H^+ \\ H^- \end{array} \right)$ and $e$ Let $F$ be another involution in $H$, ask what structure arises on $H$. Decomposition of $H$ into irreducible reps of the dihedral group.

\[ g \frac{g - 1}{2} \] is in the center of the dihedral group.

\[ \frac{F - eF}{2} \begin{pmatrix} d^+ d^* - \frac{1}{2} & -d^+ d^- \\ -d^- d^* & d^- d^- + \frac{1}{2} \end{pmatrix} \]

\[ + \begin{pmatrix} d^+ d^* - \frac{1}{2} & d^+ d^- \\ -d^- d^* + \frac{1}{2} & -d^- d^- \end{pmatrix} \]

\[ = \begin{pmatrix} 2d^+ d^* - 1 & 0 \\ 0 & -2d^- d^- + 1 \end{pmatrix} \]
\[ g = \text{unitary} \quad \mathbb{L}_2 = \{ f \mid g f = z f \} \]

\[
\Rightarrow \quad \varepsilon (g^{-1} f) = \varepsilon (g^{-1} \cdot z) = z^{-1}(\varepsilon f).
\]

\[ \varepsilon : \mathbb{L}_2 \sim \mathbb{L}_{2^{-1}} \quad \text{Assume } H = \mathbb{L}_2 + \mathbb{L}_{2^{-1}} \]

\[ H = \mathbb{L}_2 + \mathbb{L}_{2^{-1}} \quad \text{Let } f \in \mathbb{L}_2 \text{ then } \varepsilon f \in \mathbb{L}_{2^{-1}} \]

Suppose \( H = C f + C \varepsilon f \quad g f = z f \quad g \varepsilon f = z^{-1} \varepsilon f \)

\[
(g + g^{-1})(\varepsilon f + \varepsilon f) = (z + z^{-1}) \varepsilon f + (z^{-1} + z) \varepsilon f
\]

\[
= \quad (z + z^{-1})(\varepsilon f + \varepsilon f)
\]

\[ \varepsilon : \mathbb{L}_2 \sim \mathbb{L}_{2^{-1}} \quad \text{Assume } H = \mathbb{L}_2 + \mathbb{L}_{2^{-1}} \]

where \( \mathbb{L}_2 = \{ f \in H \mid g f = z f \} \quad \text{such for } g^{-1} \)

\[ \varepsilon : \mathbb{L}_2 \sim \mathbb{L}_{2^{-1}} \quad \text{Now } H = H_+ \oplus H_- \]

So

\[
\begin{bmatrix}
\mathbb{L}_2 \\
\mathbb{L}_{2^{-1}}
\end{bmatrix}
\sim
\begin{bmatrix}
H_+ \\
H_-
\end{bmatrix}
\]

Better way to proceed is:

\[ F \begin{pmatrix} 1 & -T^* \\ T & 1 \end{pmatrix} \]

\[ F(1 + X) = (1 + X) \varepsilon (1 + X)^{-1} = \left( \frac{1 + X}{1 - X} \right) \]
\[ x = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix} \quad x^2 = \begin{pmatrix} -|\omega|^2 & \omega \\ -\omega & 1 \end{pmatrix} \]

\[ 1 + x = \begin{pmatrix} 1 & -\omega \\ \omega & 1 \end{pmatrix} \]

\[ \frac{1 + x}{1 - x^2} = \frac{1}{1 + |\omega|^2} \begin{pmatrix} 1 - |\omega|^2 & -2\bar{\omega} \\ 2\omega & 1 - |\omega|^2 \end{pmatrix} \]

It's sort of clear that the reality question occurs here with \( a \) being real.

\[ \varepsilon = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{on} \quad (H^+ + H^-) \quad \text{and} \quad V \subseteq (H^+ + H^-) \]

Given \( F, \varepsilon \) in \( H \), let \( \xi_1 \) be an eigenvector for \( F \) with eigenvalue \( \varepsilon \): \( F \xi_1 = \varepsilon \xi_1 \), let \( \xi_2 = \varepsilon \xi_1 \), then \( F \xi_2 = F \varepsilon \xi_1 = F \xi_1 \).

Thus, \[ g = F \xi_1 = \varepsilon \xi_1 \Rightarrow \varepsilon (\xi_1) = g (\xi_1) = g^{-1} (\varepsilon \xi_1). \] Put \( \varepsilon \xi_1 = \xi_2 \).
Put \( \mathbf{W} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \) 
\( \varepsilon = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \) 
\( g = \begin{pmatrix} 1 & 0 \\ 0 & 2^{-1} \end{pmatrix} \)

\[
\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

\[
\begin{pmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 2^{-1} \\ 1 & -1 \end{pmatrix}^\frac{1}{2} = \begin{pmatrix} 1 & 2 \\ 1 & -1 \\ -2 & 1 \\ 2 & -1 \end{pmatrix}^\frac{1}{2} = \begin{pmatrix} \frac{2 + \sqrt{2}}{2} & \frac{2 - \sqrt{2}}{2} \\ \frac{2 - \sqrt{2}}{2} & \frac{2 + \sqrt{2}}{2} \end{pmatrix}
\]

in new basis

\[
\varepsilon = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad g = \begin{pmatrix} \cos & i \sin \\ i \sin & \cos \end{pmatrix} \quad F = \begin{pmatrix} \cos & -i \sin \\ i \sin & \cos \end{pmatrix}
\]

\[
\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad g = \begin{pmatrix} 2 & 0 \\ 0 & 2^{-1} \end{pmatrix}
\]

\[
g^\varepsilon = \begin{pmatrix} 2 & 0 \\ 0 & 2^{-1} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 2^{-1} & 0 \end{pmatrix}
\]

You learn what? You've started with an irreducible repn of \((\mathbb{Z}/2) \times (\mathbb{Z}/2)\), used the abelian normal subgroup \(g\), to get the product \( \varepsilon = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \) 
\( g = \begin{pmatrix} 2 & 0 \\ 0 & 2^{-1} \end{pmatrix} \) in \( \mathbb{C}^2 \)

\[
F = \begin{pmatrix} 2 & 0 \\ 0 & 2^{-1} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 2^{-1} & 0 \end{pmatrix}
\]

This is an irreducible repn over \( \mathbb{C} \) for \( z \neq \pm 1 \), but you want one over \( \mathbb{R} \).

\[
\varepsilon = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

\[
1 + X \cdot \begin{pmatrix} 0 & -1 \\ 1 & t \end{pmatrix}
\]
An LC network is connected with ground. 

\[ C_0 \rightarrow C' \rightarrow H' \]

\[ C_0 \leftarrow C_1 \leftarrow H_1 \]

What is your aim? To link the LC response to pure unitaries satisfying a reality condition. It seems you can do this for a 1-port using the characterization of response functions of LC 1-ports. Such a \( Z(s) \) is a real rational function of \( s \) with simple poles on the imaginary axis having poles at \( \pm j \omega \).

**Example**

\[ Z = \frac{Ls + \frac{1}{Cs}}{C} \]

Simple pole, pos. res. at \( \pm j \omega \).

\[ \frac{V}{I_{C} + I_{L}} = \frac{V}{C_{s}V + \frac{V}{L_{s}}} = \frac{1}{C_{s} + \frac{1}{L_{s}}} = \frac{L_{s}}{L_{C}s^{2} + 1} \]

\[ = \frac{C^{-1}s}{s^{2} + \frac{1}{L_{C}}} = \frac{C^{-1} \frac{5}{s^{2} + \omega^{2}}}{s^{2} + \omega^{2}} \]
For discrete transmission line:

\[
\begin{align*}
V_0 - V_1 &= L_1 s I_1 \\
I_1 - I_2 &= C_1 s V_1 \\
V_1 - V_2 &= L_2 s I_2 \\
V_0 / I_0 &= L_1 s + V_1 / I_1 = L_1 s + \frac{1}{C_1 s +} \\
I_1 / V_1 &= C_1 s + \frac{I_2}{V_1} \\
V_1 / I_2 &= L_2 s + \frac{V_2}{I_2} \\
\end{align*}
\]

For continuous:

\[
\begin{align*}
V_x - V_{x+dx} &= \lambda_x dx s I_x \\
I_x - I_{x+dx} &= \theta_x dx s V_x \\
2'V + s\delta(x) I &= 0 \\
2' I + s\delta(x) V &= 0
\end{align*}
\]

Need to adjust \( \alpha \) so that signal speed = 1.
Is it true that $\mathbb{L}nW^0 + W$ is Lagrangian in $W^0/W$?

\[ (\mathbb{L}nW^0 + W)^0 = (\mathbb{L} + W) \cap W^0 = (\mathbb{L}nW^0) + W. \]

Suppose we have $T \backslash W^0 = W^0$, and we choose $\mathbb{L}$ fixed. Then it seems to have a retraction of Lagrangian subspaces of $T$ onto the subspace of $\mathbb{L}$ between $W$ and $W^0$, i.e. $W^0 \subset \mathbb{L}$, namely

\[ \mathbb{L} \mapsto \mathbb{L}nW^0 + W = (\mathbb{L} + W) \cap W^0. \]

This is a retraction of the symplectic Grass of $T$ into the symplectic Grass of $W^0/W$. For $L_0 \subset W_0 \subset \mathbb{L}_0 \cap W^0$

inclusion $\text{Hom}(L_0, T/L_0) \hookrightarrow \text{Hom}(L_0/W, W^0/L_0)$.
First thing you need to do is to check that the reattachment takes place somewhere where $l = \frac{\lambda}{e A_x}$.

So we will assume $\lambda = \frac{\lambda}{e A_x}$ and $l = \frac{\lambda}{e A_x}$.

Toward the other way you have an attraction. Suppose a beam (a lighting camp) to go. You have a reaction $SS(w_0)$ toward $SS(W)$. You seem to have a reaction $SS(w_0)$ toward $SS(W)$.
\[ T = \mathbb{L} \oplus \mathbb{L}^* \quad W = \mathbb{V} \oplus \mathbb{V}^0 \quad W^0 = \mathbb{L} \oplus \mathbb{V}^0 \]

\[ \Gamma = \Gamma_A = (1) \mathbb{L} \subset T. \]

Now want \[ \Gamma' = (\Gamma + W) \cap W^0 = (\Gamma \cap W^0) + W. \]

This \( \Gamma' \) contains \( W \) and is contained in \( W^0 \). \( W \subseteq \Gamma' \subseteq W^0 \). Assume \( \Gamma' = \Gamma_B \), does this make sense? \( W/W = \mathbb{L}/V \oplus \mathbb{L}/V \).

It would seem that if we take \( A : L \rightarrow \mathbb{L}^* \) then cut it down to \( A^{-1}(V^0) \) and extend. You have a quadratic form on \( L \).

Curious. You have \( L = L_0 \). \( T = L \oplus L^* \).

\[ W = \mathbb{V} \oplus \mathbb{V}^0 \quad W^0 = \mathbb{L} \oplus \mathbb{V}^0 \quad W/W = \mathbb{V}/V \oplus \mathbb{L}/V^0 \quad \mathbb{V}/V = (L/V)^* \]

Now take \( \Gamma_A = \{ (x, Ax) \mid x \in \mathbb{L} \} \). Look at \( \Gamma_A \cap W^0 + W \)

\[ \Gamma_A \cap W^0 = \{ (x, Ax) \mid x \in A^{-1}V^0. \text{ e.} \} = \{ (A^t y) \mid y \in V^0 \} \]

The conjecture is that this \( (\Gamma_A \cap W^0 + W)/W \) is the graph of the induced quadratic form on \( L/V \).

What next?? Back to Lagrangian subbundles. You have \( T = V \oplus V^* \) symplectic.

Electrical situation have \( 0 \rightarrow L \rightarrow \mathbb{O} \otimes T \rightarrow \mathbb{Z} \rightarrow 0 \) what to hope for? Electrical. An LC network gives monad gives a quad.

form which is combined with the canon. skew form on \( \mathbb{H}^0(\mathcal{O}(1)) \) to get a symm form.
Basic idea is that the map $L \mapsto L_\mid W^0 \oplus W$ is rational, e.g. regular at $L_\mid N W = 0$. Yes.

closed subset of $\mathbb{P}^1 : W : L \sim W^0$ is of dim 1.

So you have the variety $\text{SG}(T)$ of dim 3 and the variety $\text{SG}(W^0/W)$ of dim 1, and a natural map $\text{SG}(T) \to \text{SG}(W^0/W)$.

$$\text{dim } \text{SG}(T) = \frac{n^2 - n(n-1)}{2} = \frac{n(n+1)}{2}$$

Consider $\{L \in \text{SG}(T) \mid L \cap W = 0\}$ = 4

For $L \in L \cap W = T$ so $L \cap W^0 = 0$ has dim $n-2$

$$\tilde{T} = \{ (z, l, v) \mid l, v \in \text{SG}(T), V \cap W = 0 \}$$

$$\text{dim } \tilde{T} = \frac{(n-n)^2 + (n-n)2 + n^2(n+1)}{2}$$

$$= \frac{(n-n)^2 + 2(n-n)n + n^2}{2} + \frac{n(n+1)^2}{2} = \frac{n^2 + n}{2}$$

$L \cap W = L_\mid N W$

$W^0 \cap W = V$
You should be dealing with a real form of \( \mathbb{P}(\mathbb{C}^2) \) with \( SL(2, \mathbb{C}) \) symmetry. Thus you could look at \( \mathbb{P}(\mathbb{R}^2) \) with \( SL(2, \mathbb{R}) \) action.

You remember something about spinors. Yes, \( \Omega(-1) \otimes \mathbb{R}^2 = \Omega(-2) = \Omega' \).

\[ g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad g^* (f \delta z) = f \left( \frac{a \bar{z} + b}{c \bar{z} + d} \right) \delta \left( \frac{a \bar{z} + b}{c \bar{z} + d} \right) = \frac{ad - bc}{(c \bar{z} + d)^2} \delta z \]

Consider a rational section of \( \Omega(-1) : f(z) \delta \). Then

\[ g^* f(z) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} f \left( \frac{a z + b}{c z + d} \right) \begin{pmatrix} a & b \\ c & d \end{pmatrix} (1) = \frac{1}{c \bar{z} + d} f \left( \frac{a z + b}{c z + d} \right) (1) \]

\[ \begin{array}{c}
 \circ \rightarrow \Omega(-1) \rightarrow \Omega \otimes \mathbb{C}^2 \rightarrow \Omega(1) \rightarrow \circ \\
 \text{Cannon, ion.} \quad \Omega(-1) \otimes \Omega(1) = \Omega \otimes \mathbb{C}^2 \\
 \text{Yang bundle} \quad \Theta = \text{Hom}(\Omega(-1), \Omega(1)) = \Omega(-2)
\end{array} \]

Idea from the ledge.

Consider \( \mathbb{C}^2 \) with action of \( SU(1, 1) \).

In other words, you can treat \( \mathbb{C}^2 \) as a Hermitian space.

Now consider
\[ V_0 - V_1 = L_1 s I_1 \]
\[ V_1 - V_2 = L_2 s I_2 \]
\[ I_0 - I_1 = \cos V_0 \]
\[ I_1 - I_2 = C_1 s V_1 \]
\[ I_2 - I_3 = C_2 s V_2 \]

\[ \frac{V_0}{I_0} = \frac{V_0}{I_1 + \cos V_0} = \frac{1}{C_0 s + \frac{I_1}{V_0 + L_1 s I_1}} \]

\[ = \frac{1}{C_0 s + \frac{1}{L_1 s + \frac{V_1}{I_1}}} \]

Converse - suppose \( f(s) \) rational real assume maps RHP into itself.
What you want to do is to link the scattering picture to LC circuits somehow. There should be something you could do. Take an LC 1-port. This should have an obvious type of response - real rational function of $s$, purely imaginary poles, and the should be an equivalent LC ladder obtained from the partial fraction expansion. It might be better to use a string with finite point masses. 

\[ Z_0 = L_1 s + \frac{1}{C_1 s + \frac{1}{L_2 s + \frac{1}{Z_1}}} \]

\[ Z_0 = L_1 s + \frac{1}{C_1 s + \frac{1}{Z_1}} \]

The idea is that you get a rational function $Z_0(s)$ such that $\text{Re}(s) > 0 \Rightarrow \text{Re}(Z_0(s)) > 0$. This should restrict the partial fraction expansion to real terms 

\[ \frac{1}{s^2 + \omega^2} \]

\[ V_0 = L_1 s I_{L_1} + V_1 \]

\[ V_1 = \frac{1}{C_1 s} I_{C_1} \]

\[ I_0 = I_{C_1} + I_{L_2} \]

\[ V_0 = L_1 s I_0 = L_1 s (C_1 s V_1 + \]
You want to start at the scattering end, but maybe this is too hard. Maybe begin with an LC circuit. This is a complicated system of constant coefficients. ODE's. Solutions form a module over the ring \( \mathbb{R} \). You would like to understand the variational approach.

\[ I_0(t) \quad \Omega \quad I_L \]

\[ V_c, I_c \]

\[ V(t) \quad \Omega \quad \Theta \]

\[ V_a(t) \]

Problem principle? How does this involve a variational principle?

\[ H = C' + C \]

\[ \sqrt{V_L + V_c} \]

\[ H/ C \quad \{(I_L) \text{ } | \text{ } I_L = I_c \} \]

\[ \{V_{in} \} \quad \{ I_{out} \} \]

\[ V_L I_L + V_c I_c = \frac{(V_L + V_c) I_{out}}{V_{in}} \]
Today: Legendre transform, Lagrange multipliers. 

Begin with variables \( x, L \) \( \times \) ind. 

Introduce \( \xi \) new ind. \( \xi \) dep. 

Put \( H = x^\xi - L \). At the moment, \( H \) depends on \( x, \xi \). Keep \( \xi \) fixed. Look for a stationary point for \( H \) as a fn of \( x \). 

\[ 0 = \partial_x H = \xi - \partial_x L \]

In good cases, \( \xi = \partial_x L \), can be solved. 

Eg. \( \partial_x \xi = \partial_x^2 L \neq 0 \), then can use implicit function thm. to view \( \xi \) as 

\[ x, L \]
partial unitary \( X \xrightarrow{a \to b} Y \), \( a \frac{a}{a} = b \frac{b}{b} = 1 \frac{1}{x} \)

Then you extend to a Hilbert space \( \mathcal{H} \) by treating \( \mathcal{V}_- \) as incoming and \( \mathcal{V}_+ \) as outgoing.

\[ \mathcal{V}_- \oplus \mathcal{V}_- \oplus \mathcal{V}_- \oplus \mathcal{X} \oplus \mathcal{V}_+ \]

\[ \mathcal{V}_- \oplus \mathcal{V}_- \oplus \mathcal{V}_- \oplus \mathcal{V}_+ \oplus \mathcal{V}_+ \]

\[ \mathcal{H}^2(s, \mathcal{V}) \oplus \mathcal{X} \oplus \mathcal{H}^2(s, \mathcal{V}) \]

Maybe you need to explore Pick functions: analytic functions from \( D \to \mathbb{U} \mathbb{H} \).

Derive Poisson kernel solving Dirichlet problem for \( D \).

\( u(z) \) holom. for \( |z| < 1 + \varepsilon \). Look

\[ u(z) = \sum_{n \geq 0} a_n r^n e^{in\theta} \]

\[ a_n = \int \frac{e^{-in\theta} u(re^{i\theta})}{2\pi} \, d\theta \]

\( u(z) \) holom. in \( D \)

\[ u(z) = \sum a_n z^n \]

\[ u(re^{i\theta}) = \sum_{n \geq 0} a_n r^n e^{in\theta} \]

leave \( r \) out of it. The important thing is to express \( u(z) \) in terms of the Imag part of \( u \) on the boundary.
So what do you aim for? First of all some sort of sheaf on $\mathbb{P}^1$.

\[ 0 \to \mathcal{O}(-1) \otimes X \to \mathcal{O} \otimes Y \to E \to 0 \]

But this doesn't seem to involve $Y \otimes Y$.

Go back to the circuit:

\[
\begin{align*}
\tilde{C}^0 & \xleftarrow{\cdot} \tilde{C}^0 \xrightarrow{\cdot} C^1 \\
\oplus & \quad \oplus \\
\tilde{C}_0, \text{ext} & \quad \tilde{C}_0 \quad \leftarrow \quad C_1
\end{align*}
\]

\[
\begin{align*}
V^0 \otimes V^1 & \xleftarrow{\cdot} V^2 \xrightarrow{\cdot} D \\
\oplus & \quad \oplus \\
V^0 \otimes V^1 & \quad V^1 \quad \leftarrow \quad D^* \end{align*}
\]

Here kinematics need dynamics which seems to amount to a Lagrangian subspace $L_S$ of $D \oplus D^* \oplus \ldots$ depending on a frequency parameter $s$. So what happens is that you project $L_s$ to a Lagrangian subspace in the symplectic quotient.

\[
W = \begin{bmatrix} V_1 \\ \oplus \\ V^0 \end{bmatrix}, \quad W^\perp = \begin{bmatrix} V_2 \\ \oplus \\ \oplus \end{bmatrix}, \quad \frac{W^\perp}{W} = \begin{bmatrix} V_2/V_1 \\ \oplus \\ V^0/V_1 \end{bmatrix}
\]
Repeat $T^4 \triangleright W^\perp \triangleright W^\perp \triangleright 0$

$L = L^\perp$ Lagrangian and set $L \cap W = 0$

$T$

$L + W \quad W^\perp$

$L \quad (L \cap W \cap W)$

$L \cap W^\perp \quad W$

$\log W^\perp \quad W$

$\log W^\perp \quad W$

$\log W^\perp \quad W$

$\log W^\perp \quad W$

The real question is how to describe the possible $L$'s. The line $L \cap W^\perp$ is equivalent to the Lagrangian subspace $L \cap W^\perp \cap W = (L + W) \cap W^\perp / W$

in $\mathfrak{so}(W^\perp / W)$

\[
T = \begin{pmatrix} D & \ast \\ \ast & D^* \end{pmatrix} \quad V_1 \subset V_2 \subset D \quad W = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}, \quad W^\perp = \begin{pmatrix} V_1^* \\ V_2^* \end{pmatrix}
\]

\[
L = \begin{pmatrix} 1 \\ 0 \end{pmatrix} D \subset \begin{pmatrix} D & \ast \\ \ast & D^* \end{pmatrix}
\]

$L \cap W^\perp = \{ (Q^\xi) \mid i \in V_1, \xi_3 \in V^0 \}$

$V_1 \rightarrow V_2 \rightarrow D$

interested in $\xi \in V_2$

such that $Q\xi \in V^0$.

$\delta V_1 \leftarrow \delta V_2 \leftarrow D$

probably means that $\xi$ is stationary with variations $\xi + \delta V_1$
making \( L \rightarrow (L + W)/W = \log W^+ \cap W/W \) into some sort of correspondence.

Take \( T^4 \), \( W \) line. Generic situation is when \( \log W = 0 \) equiv. \( L + W^+ = 0 \). But singular case is when \( \log W \neq 0 \) i.e. \( W \cap L \subseteq W^+ \).

\[ \log W^+ \]
\[ L + W^+ \]
\[ L \]
\[ W^+ \]
\[ \log W \]
\[ W \]

\[ \log W \]

\[ T^4 \cap W^+ \cap W = 0 \quad L \subseteq T \]

Generic case \( \log W = 0 \) equiv. \( L + W^+ = T \).

In this case \( \log W^+ \) is a line in \( L \) and one has a point \( \log W^+ \subseteq L \) in the flag bundle over \( SG(T) \), call that \( SG'(T) \). Do you have a map \( SG'(T) \rightarrow SG(W^+/W) \)?
$T$ symplectic, $W$ isotropic, $W^\perp/W$ the symplectic quotient of $T$. Example: $T = \left( \begin{array}{cc} D & \ast \\ \ast & D^\ast \end{array} \right)$ with $\left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right)$, $W = \left( \begin{array}{c} V_1 \\ V_2 \end{array} \right)$ where $V_1 \subset V_2 \subset D$

Then $W^\perp = \left( \begin{array}{c} V_1 \\ 0 \end{array} \right)$ and $V_2^\circ = \left( \begin{array}{c} D \\ V_1 \end{array} \right)$ where $V_2^\circ = \left( \begin{array}{c} V_2 \\ V_1 \end{array} \right)$

Here $V_i^\circ = \{ x \in V_i^* | \lambda x = 0 \}$, and $W^\perp$ refers to annihilator for the symplectic form. Note

$$
\frac{W^\perp}{W_0} = \left( \begin{array}{c} V_2/V_1 \\ V_1/V_2 \end{array} \right) = \left( \begin{array}{c} V_2/V_1 \\ V_1/V_2 \end{array} \right)
$$

Note that the $W_i$ of this form are the isotropic subspaces preserved by $\varepsilon = \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right)$.

Now add to $T, W$ as above a Lagrangian subspace $L \subset T$. In above: $L = \Gamma_Q = \left( \begin{array}{c} \Gamma \\ \ast \end{array} \right)$ where $Q : D \to D^\ast$ is symmetric. Such "quadratic forms" $Q$ yield all $L$ projecting nonsingularly on $D$. A generic $Q$ should be nondegenerate when restricted to $D, V_1, V_2$ and therefore to yield a splitting of the filtration $0 \subset V_1 \subset V_2 \subset D$ and also nondegenerate forms on the layers.

$$
0 \subset V_1 \subset V_2 \subset D
$$

\[ \Gamma \]

\[ \varepsilon \]

\[ \ast \]

\[ \Gamma_Q \]

\[ \ast \]

\[ \ast \]

\[ \ast \]

\[ \ast \]

Thus quadratic form induced by $Q$ on $V_2/V_1$ yields a Lagrangian subspace of $W^\perp/W$. \[ \ast \]
No over unit circle stuff where you use hermitian forms.

$P_1 C$, $SU(1,1)$ symmetries of $\mathbb{C}^2$ equipped with the hermitian form

$$
\begin{pmatrix}
\bar{z}_1 \\
\bar{z}_2
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
z_1 \\
z_2
\end{pmatrix}
= (\bar{z}_1, \bar{z}_2)(z_1, z_2) = |z_1|^2 - |z_2|^2
$$

$g \in SU(1,1)$ means $g^* (1 \ 0) g = (1 \ 0)$

$$
g^* = \begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix} g^{-1} \begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
$$

Assume $\det(g) = 1$. 

$g = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 

$g^{-1} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$
g^* = \begin{pmatrix}
\bar{a} & \bar{c} \\
\bar{b} & \bar{d}
\end{pmatrix} = \begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix} = \begin{pmatrix} d & b \\ -c & a \end{pmatrix}
$$

$$
\therefore \quad \bar{a} = d, \quad \bar{b} = c
$$

Hermitian form above applied to $P_1(\mathbb{C}^2)$ vanishes on $|z| = 1$. 

$> 0$ on $|z| < 1$ 

$< 0$ on $|z| > 1$.

$\gamma$ complex v.s. with hermitian form.

The idea maybe is that you go between $P_1(\mathbb{R}) \subset P_1 C$ and $|z|^2 = 1$. Over $P_1 C$ you have a canonical $\mathcal{O}(-1) \rightarrow \mathcal{O} \rightarrow \mathcal{O}(1)$. If we restrict to the reals then ??
Example.

\[
\begin{pmatrix}
C_1 \\
C_0
\end{pmatrix} = T^4
\]

no loops.

constraint \( I_L = I_C \)

You have variables making a symplectic space \( T \) of dim 4

where \( V_L, I_L \) are "dual variables" \( V_C, I_C \)

constraint \( I_L = I_C \) is a linear ful, on \( T \)

represented by \( V_L + V_C \) (up to signs)

Next have Lagrangian subspace \( L_5 \).
Question: Can you fit the transmission line into your subquotient of a polarized Hilb space picture?

Transmission line = simplest Dirac eqn., 2-drink space-time lift + right movers, harmonic oscillator appears upon bosonization.

Idea (from `98)  The response function arising from an LC circuit with external nodes should be a kind of Gelfand $R \ldots \text{quasi-determinant}$

Q: Is there some link between the response functions (impedance function $Z(s)$) for LC circuits, more generally subquotients of a polarized Hilb space, and the response functions (scattering operators $S(\omega)$) for partial unitary operators?
\[
\begin{pmatrix}
P_n \\
q_n
\end{pmatrix}
= \frac{1}{k_n} \begin{pmatrix}
h_n & 0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
z & 0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
P_{n-1} \\
q_{n-1}
\end{pmatrix}
\]
form you want to take

\[
\begin{pmatrix}
P_x \\
q_x
\end{pmatrix}
= \begin{pmatrix}
1 & h_x dx \\
-i k dx & 0
\end{pmatrix}
\begin{pmatrix}
1 + i k dx & 0 \\
0 & 1 - i k dx
\end{pmatrix}
\begin{pmatrix}
P_x dx \\
q_x dx
\end{pmatrix}
\]

\[
\frac{\partial}{\partial x} \begin{pmatrix}
P_x \\
q_x
\end{pmatrix}
= \begin{pmatrix}
0 & h_x \\
-i \frac{\partial}{\partial x} & 0
\end{pmatrix}
\begin{pmatrix}
P_x \\
q_x
\end{pmatrix}
\]

There is something one can do for \( k \) real.

It seems that it might be possible to link partial unitaries.
Power? $P = VI$ in a direct current $R$ circuit so you should proceed following Böttcher-Weigl.

Graph $\Gamma$ with resistance $R_e$ assigned to each edge $e$. Orient edges. Introduce variables $V_e, I_e$ for each edge, subject to the relations $[V_e^2]$ conservative equiv $\sum_e V_e$ along any cycle is zero.

Ohm's Law $V_e = R_e I_e$ for each edge.

\[\begin{align*}
\tilde{C}_0 &\leftarrow \tilde{C}_0 \xrightarrow{\mathcal{D}} C^1 \\
\tilde{C}_0 &\xrightarrow{1/R} \quad \tilde{C}_0 = C_0 = \frac{C_n}{\mathcal{O}(x)}
\end{align*}\]

Let the internal modes be free.

By positivity of the quadratic form in $C^1$, you get an induced positive form on the external applied voltage. Why does this imply that there's a unique solution of the equations:

\[\begin{align*}
\tilde{C}_0^\bullet &= R_e I_e \\
\mathcal{I}^\bullet &= 0 \quad \text{on internal nodes.}
\end{align*}\]
Start where?

\[ V_L = L \frac{d}{dt} I_L \]
\[ I_C = C \frac{d}{dt} V_C \]
\[ I_o = I_L = I_C \]
\[ V_o = V_L + V_C \]

\[ V_L, I_L \]
\[ V_C, I_C \]

Three vertices, two edges.

\[ V_L = V_o - V_1 \]
\[ V_C = V_1 \]
\[ I_o = I_L \]
\[ I_L = I_C \]

Node 1 internal

\[ \dot{C} \rightarrow \dot{C} \rightarrow C \rightarrow C^1 \]

\[ V_o, V_1 \rightarrow V_h, V_c \]

\[ \frac{1}{L_s} \left( V_o - V_1 \right) + C_s V_1^2 \]

\[ O = \frac{1}{L_s} (V_o - V_1) (-1) + C_s V_1 \]

\[ \frac{1}{L_s} \frac{V_o}{V_1} = \frac{1}{L_s} \left( \frac{1}{C_s} \right) V_1 \]

\[ V_1 = \frac{V_o}{1 + L_s C_s^2} \]

\[ V_o - V_1 = V_o \left( 1 - \frac{1}{1 + L_s C_s^2} \right) = \frac{V_o}{1 + L_s C_s^2} \]
from an LC 1-port you get an impedance \( Z(s) \) of a specific form, e.g.
\[
\alpha \frac{s}{s^2 + \omega_0^2} \quad \alpha_0 > 0.
\]

\( Z \) maps RHP to itself, \( \text{iR to iR} \)

\[
V \xrightarrow{\text{\footnotesize \text{\( f^+ \)}}} H_+ + H_-
\]

\[
\text{\footnotesize \text{\( f^- \)}}
\]

here have \( s \| f^+ \|^2 + s^{-1} \| f^- \|^2 \)
on \( V \) you get \( sf \)

need to work on an example

\[\begin{array}{c}
\text{\rotatebox{90}{---}}
\end{array}\]

\[\begin{array}{c}
\text{\rotatebox{90}{---}}
\end{array}\]

\[\begin{array}{c}
\text{\rotatebox{90}{---}}
\end{array}\]

idea. Hyman Bass \& a \( Z \) function of a graph.

But a graph is a correspondence of some sort.

\[\text{\footnotesize \text{\( \Delta \)}}\]

Is there a \( Z \) function for a correspondence?

Iterate the correspondence \( n \)-fold intersect with \( \Delta \)?
explain what you mean by variational principle
with how to proceed
You have

a symplectic quotient of
\( C' \oplus C_1 \)

which arises
from a subquotient of \( C' \) — this
is kinematics. There is quadratic form
\( Q_5 \) on \( C_1 \) — this is dynamics.

The response for is the induced form
on the subquotient (ext modes)

Confusing part. Why symplectic
reduction of a Lagrangian subspace is
related to a variational principle.
List problems as of July 15, 02

continuous version of LC circuit.

\[ I_0 \rightarrow \frac{I_1}{C_1} \rightarrow V_1 \rightarrow \frac{I_2}{C_2} \rightarrow V_2 \]

\[ V_0 - V_1 = L_1 s I_1 \]
\[ I_1 - I_2 = C_1 s V_1 \]
\[ V_1 - V_2 = L_2 s I_2 \]

\[ \frac{V_0}{I_1} = \frac{L_1 s I_1 + V_1}{I_1} = L_1 s + \frac{V_1}{I_1} \]

\[ \frac{I_2}{V_1} = C_1 s + \frac{1}{L_2 s \frac{V_2}{I_2}} \]

Here the variables you consider are \( V_0, I_1, V_1, I_2, V_2 \).

What is your aim? For an LC 1-port you have a symplectic quotient picture:

\[ \{ V_e \} \leftarrow C^o \overset{\delta}{\longrightarrow} C' \]

\[ \bar{C}^o = \{ V_x \} \]

\[ \{ I_e \} \rightarrow \bar{C}^o \leftarrow C_1 \]

\[-2 \chi V = \lambda s \chi \]
\[-2 \chi I = \varphi s \chi \]

\[ 56 \times 38 \text{ cm} \]

Sharpe
R-965
3.7 x 55.5 x 54
John Lewis

Whitelaw
MY 277
32 x 52 x 48
Dixons

S. 953?
\[
\frac{1}{k} \begin{pmatrix}
1 & h \\
h & 1
\end{pmatrix} \begin{pmatrix}
z^{1/2} & 0 \\
0 & z^{-1/2}
\end{pmatrix} \begin{pmatrix} 
(1 & 1)
-1 & 1
\end{pmatrix} = \frac{1}{\gamma^2} \begin{pmatrix} 
1 & 1+1 \\
-1 & 1-1
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 1 \\
h & 1
\end{pmatrix} \begin{pmatrix} 
1 & 1+0 \\
-1 & 0-1
\end{pmatrix} = \begin{pmatrix}
1 & 1 \\
-1 & -1
\end{pmatrix} \begin{pmatrix} 
z^{1/2} & 1 \\
0 & 1-1
\end{pmatrix}
\]

\[
\frac{1}{\gamma^2} \begin{pmatrix}
1 & 1 \\
h & 1
\end{pmatrix} \begin{pmatrix}
z^{1/2} & 0 \\
0 & z^{-1/2}
\end{pmatrix} \begin{pmatrix} 
1 & 1 \\
-1 & -1
\end{pmatrix} = \frac{1}{\gamma^2} \begin{pmatrix}
z^{1/2} & z^{-1/2} \\
z^{1/2} & z^{-1/2}
\end{pmatrix} \begin{pmatrix}
(\cos \Theta) & i \sin \Theta \\
i \sin \Theta & \cos \Theta
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & \tan \Theta \\
\tan \Theta & 1
\end{pmatrix} \approx \begin{pmatrix}
1 & S \\
S & 1
\end{pmatrix}
\]

Note: The highlighted part. Hope that this transformation provides the link between partial unitaries and LC circuits. The first part of the problem is the frequency parameter \( \gamma \) versus \( S \) in \( i\mathbb{R} \).

\[
S = \frac{z^{1/2} - z^{-1/2}}{z^{1/2} + z^{-1/2}} = \frac{z - 1}{z + 1} \quad z = \frac{1 + s}{1 - s}
\]

\[
S = \frac{z + 1 - 2}{z + 1} = 1 - \frac{2}{z + 1} \quad \frac{-s + 1}{2} = \frac{1}{z + 1} \quad z + 1 = \frac{2}{1 - s} \quad z = \frac{2}{1 - s} - 1 = \frac{2 - 1 + s}{1 - s} = \frac{1 + s}{1 - s}
\]
\[ \tilde{C}^o \leftarrow \tilde{C}^o \xrightarrow{\delta} C^1 \]

On \( \tilde{C}^o \) one has the quad form \( SV^t R^{-1} SV \), the restriction of \( (V^t)^* R^{-1} V^t \) on \( C^1 \). Next you deduce \( SV^t R^{-1} SV \) \( \sim \) from \( \tilde{C}^o \) to \( C^1 \) the quotient \( \tilde{C}_{\text{ext}} = \tilde{C}^o / C^o \)

\[ C^o \xrightarrow{\text{int}} \tilde{C}^o \xrightarrow{\delta} \tilde{C}^o \xrightarrow{\text{ext}} \tilde{C}_{\text{ext}} \]

Given \( V^o \in \tilde{C}^o \) you project \( V^o \) \( \perp \to C^o \)
Given \( X \overset{a}{\underset{b}{\longrightarrow}} Y \) \( \alpha \cdot \beta = 1 - b \cdot \beta \)

Unit vector basis for \( V \): \( \overset{\lambda}{\bar{b}} a_x^* + \overset{\lambda}{\bar{c}} h \overset{\lambda}{\bar{c}} ^* = c_h \)

\[
(\lambda - c_h)^{-1} = (\lambda - c_0 - \delta)^{-1} = (\lambda - c_0)^{-1} + (\lambda - c_0)^{-1} \delta (\lambda - c_0)^{-1} + \ldots
\]

\[
(\lambda - ba^*)^{-1} + (\lambda - ba^*)^{-1} h \overset{\lambda}{\bar{c}} ^*(\lambda - ba^*)^{-1} + \ldots
\]

You are missing something.\[
(\lambda a - b) x = - u_+ + \overset{\lambda}{\bar{c}} x \quad x = a^*(\lambda - b a^*)^{-1} \overset{\lambda}{\bar{c}}^-
\]

\[
(\lambda - \lambda a b) x = a^* x \quad \overset{\lambda}{\bar{c}}^{-1} = - (\lambda a - b) a^*(\lambda - b a^*)^{-1} \overset{\lambda}{\bar{c}}^-
\]

\[
G_h = G_0 + G_0 \delta G_0 + \ldots
\]

\[
\overset{\lambda}{\bar{c}}^{-1} = \overset{\lambda}{\bar{c}}^{-1} G_0 \overset{\lambda}{\bar{c}}^{-1} + \overset{\lambda}{\bar{c}}^{-1} h \overset{\lambda}{\bar{c}}^{-1} G_0 \overset{\lambda}{\bar{c}}^{-1} + \overset{\lambda}{\bar{c}}^{-1} G_0 \overset{\lambda}{\bar{c}}^{-1} \overset{\lambda}{\bar{c}}^{-1} G_0 \overset{\lambda}{\bar{c}}^{-1} + \overset{\lambda}{\bar{c}}^{-1}
\]

\[
= \left( \overset{\lambda}{\bar{c}}^{-1} G_0 \overset{\lambda}{\bar{c}}^{-1} \right) \frac{1}{1 - h \overset{\lambda}{\bar{c}}^{-1} G_0 \overset{\lambda}{\bar{c}}^{-1}}
\]

\[
\overset{\lambda}{\bar{c}}^{-1} G_0 \overset{\lambda}{\bar{c}}^{-1} = \overset{\lambda}{\bar{c}}^{-1} (\lambda - b a^*)^{-1} \overset{\lambda}{\bar{c}}^{-1} = \overset{\lambda}{\bar{c}}^{-1} (1 - \lambda^{-1} b a^*)^{-1} \overset{\lambda}{\bar{c}}^{-1} = S_0 (\lambda^{-1})
\]

\[
\overset{\lambda}{\bar{c}}^{-1} G_h \overset{\lambda}{\bar{c}}^{-1} = S_0 (\lambda^{-1}) \frac{1}{1 - h S_0 (\lambda^{-1})}
\]

\[
S_0 (\lambda^{-1}) = \frac{1 + h S_0}{1 - h S_0}
\]

\[
1 + \frac{h S_0}{1 - h S_0} = \frac{h S_0}{1 - h S_0}
\]

You're missing the meaning of \( \overset{\lambda}{\bar{c}}^{-1} G_h \overset{\lambda}{\bar{c}}^{-1} \).
\[ \text{ch is a perturbation of } ba^* \]

\[ S(z) \text{ recall what this is. } \]

\[ \begin{array}{cccc}
X & \rightarrow & Y & \rightarrow E_z \\
\frac{z-a}{z-b} & \rightarrow & a^* & \rightarrow X
\end{array} \]

\[ y(z) = y - (z-a-b) a^* (z-ba^*)^{-1} y \]

\[ = (z-ba^*) - (z-a-b) a^* (z-ba^*)^{-1} y = (1-aa^*) \frac{z}{z-ba^*} y \]

Maybe there's a simpler version with:

\[ W = \begin{pmatrix} a \\ b \end{pmatrix} X \subseteq \langle y \rangle \quad W^\perp = W \oplus \langle V_+ \rangle \]

You want \( \langle z \rangle \bigcap W^\perp \)

\[ \begin{pmatrix} y \\ z^y \end{pmatrix} = \begin{pmatrix} ax \\ bx + v_+ \end{pmatrix} \]

\[ z (aX + v_+) = bx + v_- \]

\[ (2a-b)X = -zv_+ + v_- \]

\[ (z-a^*b)X = a^* v_- \]

\[ X = a^* (z-ba^*)^{-1} v_- \]

\[ zv_+ = v_- - (z-a-b) a^* (z-ba^*)^{-1} v_- = (z-ba^*) - (z-a^*b) a^* (z-ba^*)^{-1} v_- \]

\[ (z^y - ba^*) (z-ba^*)^{-1} v_- \]

\[ zv_+ = z (1-aa^*) (z-ba^*)^{-1} v_- \]