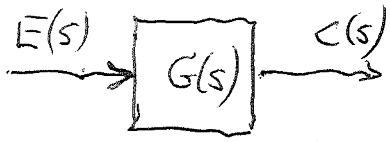
$\alpha 4$ Consider a real f.d. vector space X with a scalar product $(x_1, x_2) = x_1^t x_2$. Let $y^t : x \longmapsto y^t x = (y, x)$ be a nonzero lin. fnl. Short exact sequence $K \rightarrowtail X \xrightarrow{y^t} \mathbb{R}$ where $K = (\mathbb{R}y)^\perp$. Orthogonal splitting $X = K \oplus \mathbb{R}y$.

You want to push forward, or descend, the scalar product on X to the quotient space $y^t : X \twoheadrightarrow \mathbb{R}$. This means lifting $c \in \mathbb{R}$ to an elt of $K^\perp = \mathbb{R}y$, i.e. you want $\lambda \in \mathbb{R}$ such that $y^t(\lambda y) = c$, which gives $\lambda = \dfrac{c}{(y,y)}$, hence the lift is $\lambda y = \dfrac{cy}{(y,y)}$. Then you restrict the scalar product on X to $K^\perp = \mathbb{R}y$ which gives $\left(\dfrac{cy}{(y,y)}, \dfrac{cy}{(y,y)}\right) = \dfrac{c^2}{(y,y)}$.

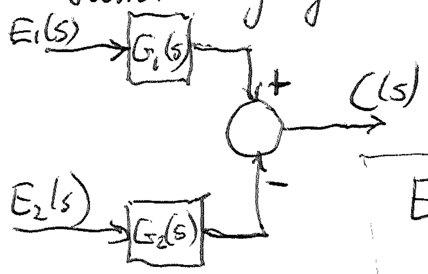
Next, Do the same for scalar product $x_1^t A x_2 = (x_1, x_2)_A$, A pos. def. $K = \{x \mid y^t x = 0 \text{ i.e. } (A^{-1}y, x)_A = 0\} = (\mathbb{R}A^{-1}y)^\perp$ where $\perp = \perp_A$. Take $c \in \mathbb{R}$ choose $\lambda A^{-1}y$ so that $y^t(\lambda A^{-1}y) = c$ i.e. $c = \lambda(y, A^{-1}y)$ or $\lambda = \dfrac{c}{(y, A^{-1}y)}$. Then you have the lifting $\dfrac{cA^{-1}y}{(y, A^{-1}y)}$ which has $\text{norm}_A^2 = \left(\dfrac{cA^{-1}y}{(y,A^{-1}y)}, A\dfrac{cA^{-1}y}{(y,A^{-1}y)}\right) = \dfrac{c^2}{(y, A^{-1}y)}$

p4 ~~[illegible]~~ Make notes from book
Feedback Control Systems, Phillips, Harbor

Signal Flow Graph | equivalent, used to represent graphically
Block Diagram | the transfer fn relation



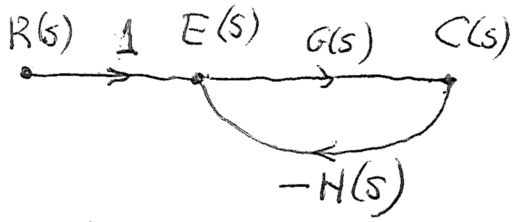
need also the summing junction



The summing junction for signal flow graph is ~~Represented~~ represented by the branches into a node

Ex.
$$E(s) = R(s) - H(s)C(s)$$
$$C(s) = G(s)E(s)$$

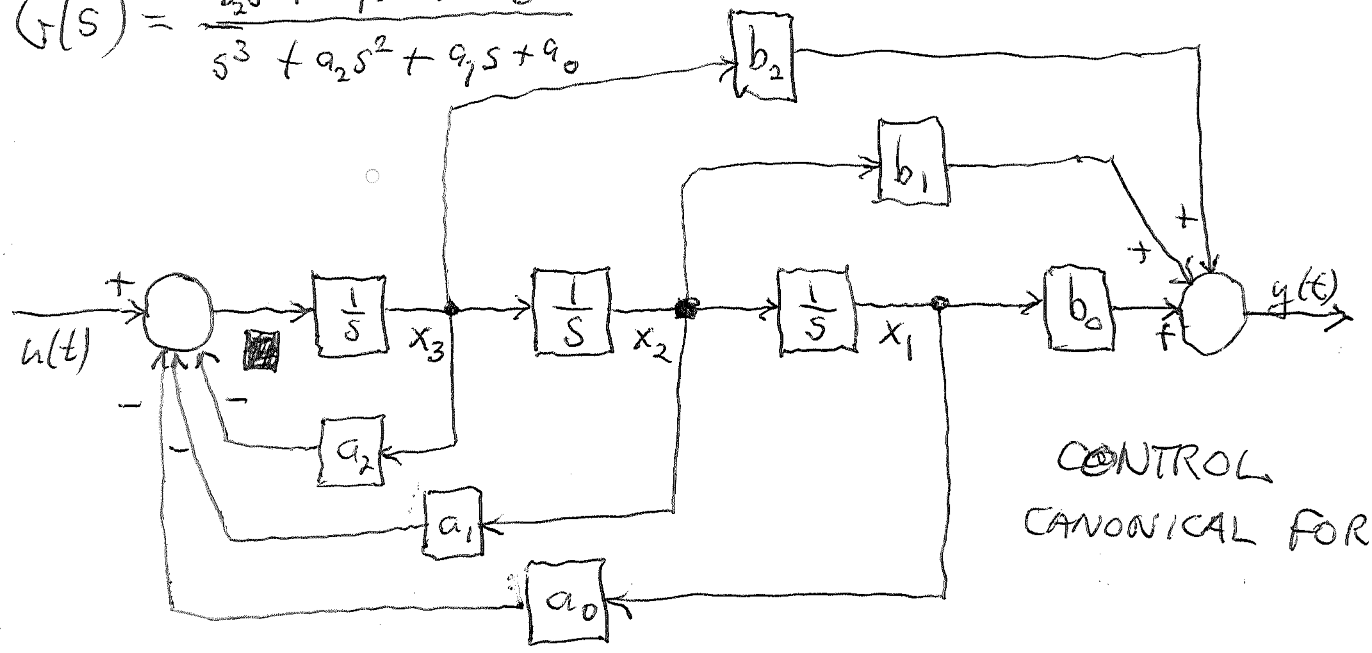
R(s) input sig
C(s) output sig
E(s) int. sig
G(s) H(s) tr fn



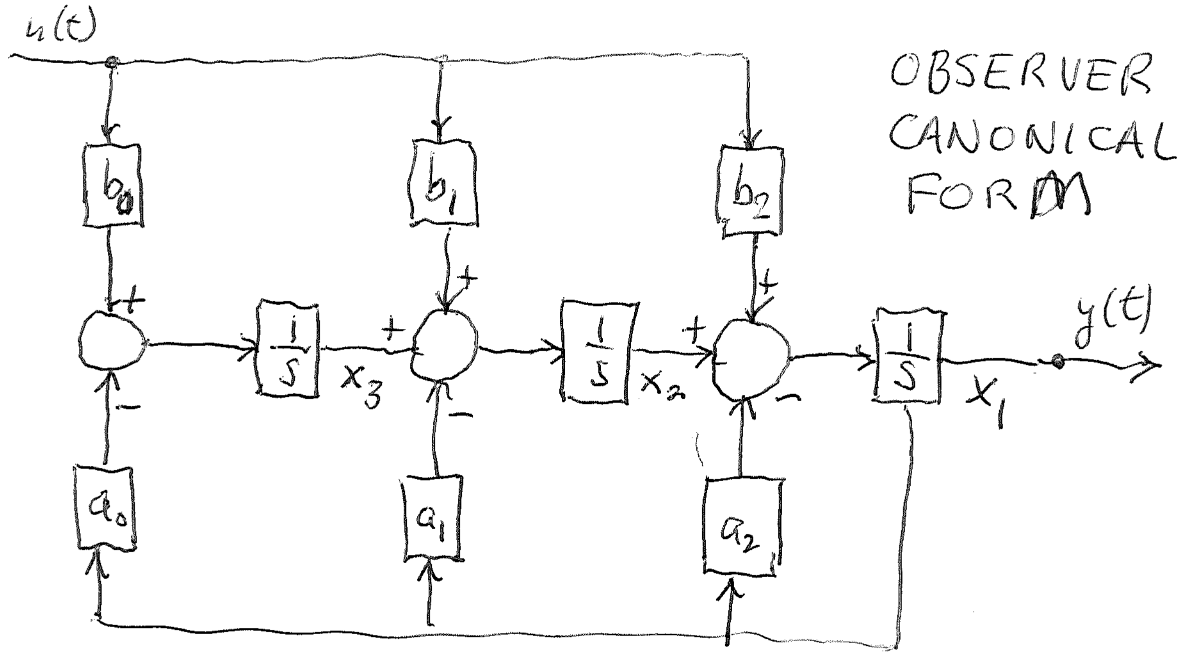
Signal Flow Gph		Block Diag
input node	→	input signal
output node	→	output signal
branch	→	block
node	→	signal

There are two different simulation diagrams (type of block diag or flow graph which realizes a transfer function) e.g. let

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$



CONTROL CANONICAL FORM



OBSERVER CANONICAL FORM

Review: X equipped w $(x_1, x_2) = x_1^t x_2$, pos. symm., $y^t = (y, -) : X \longrightarrow \mathbb{R}$, $y \neq 0$. Calculate the induced scalar product on the quotient $X \xrightarrow{y^t} \mathbb{R}$.
You have $X = \mathbb{R}y^\perp \oplus \mathbb{R}y$, orthog splitting, $\mathbb{R}y^\perp = \operatorname{Ker} y^t$. Take $c \in \mathbb{R}$ and lift it into $\mathbb{R}y$ the orth comp. of $K = \operatorname{Ker} y^t$, say lift c to $x = \lambda y$, where λ is determined by $y^t(\lambda y) = c$, i.e. $\lambda = \frac{c}{(y,y)}$. Then the norm^2 of c is the norm^2 of x: $(x,x) = \lambda^2 (y,y) = \frac{c^2}{(y,y)}$

Next you want to link this to adjoining a branch. Here $X = \bar{C}^0$ is the node potential space, and the scalar product is induced by $\delta : \bar{C}^0 \hookrightarrow C^1$ from the power pos def form $VR^{-1}V$ on C^1. To simplify things suppose the graph is a tree, whence $\delta : \bar{C}^0 \xrightarrow{\sim} C^1$. Also $y^t : X \longrightarrow \mathbb{R}$ is $\varphi \longmapsto \varphi_A$, where a ground O has been chosen $\neq A$ and $\varphi_O = 0$, $\forall \varphi \in \bar{C}^0$.

You now to impose the condition $\varphi_A = c$, i.e. pass to an affine hyperplane in $X = \bar{C}^0$ and also enlarge C^1 to $C^1 \oplus \mathbb{R}$.

84 Try again to understand [redacted] attaching a branch in order to handle an [redacted] external emf applied between two nodes, say A and the ground O. Let X be the node potential space $\tilde{C}^0$ equipped with the power form [redacted] (x_1, x_2) induced from the power form on C^1 via the embedding $\delta : \tilde{C}^0 \hookrightarrow C^1$. To simplify suppose the graph is a tree, so that δ is an isomorphism.

[redacted] Discuss the problem. You have the [redacted] linear nonzero functional $y^t : X \longrightarrow \mathbb{R}$ given by $x = \varphi \longmapsto \varphi_A - \underset{0}{\underset{\shortparallel}{\varphi_O}}$.

You want to fix a value $c \in \mathbb{R}$ and to restrict x to [redacted] the affine hyperplane: $y^t x = c$. In this way the [redacted] x variable has 1 less degree of freedom. So far you haven't changed the number of edges, so you expect to increase H^1 by 1 dim.

You must understand the linear equations which determine the voltages. Recall the Thevenin situation where the inhomogeneous forcing term is any elt $\mathcal{E}$ of C^1, [redacted] and one solves $V - RI = \mathcal{E}$ with V, I subject to the Kirchhoff conditions.

Digress because I appears. The point is that $\mathcal{E} \in C^1$ is arbitrary while $V \in C^1$ satisfies the Kirchhoff voltage condition.

$$\overset{V}{\tilde{C}^0} \rightarrowtail \overset{\mathcal{E}}{C^1} \twoheadrightarrow H^1$$

[redacted] It should be true that $\mathcal{E} = V + (-RI)$ is the orthogonal splitting of this short exact sequence for the power form on C^1.

ε4 I think that you want an analog of the Thevenin idea that the inhomogeneous "forcing" ~~terms~~ should be any elt $\mathcal{E}$ of C^1. ??

$X = \bar{C}^0 \cong C^1$, $X = \mathbb{R}y^{\perp} \oplus \mathbb{R}y$ orthog splitting

$$\begin{array}{c} X \\ y^t \downarrow \\ \mathbb{R} \end{array}$$

Can you specify what you want or need? Go back to the idea that $X = \bar{C}^0$ remains the same after the new edge is added.

Review the problem. You have a Euclidean space $X = \bar{C}^0 \simeq C^1$ with inner product (x_1, x_2) together with an affine constraint $c = y^t x$ which requires x to be in some affine hyperplane.

You want the minimum power configuration x_c subject to this constraint and the power (x_c, x_c) as a function of c.

The Lagrange multiplier method seems to solve this problem by adjoining a new variable λ.

$$F = \tfrac{1}{2}(x,x) + \lambda(c - (y,x))$$

Note that F_λ is quadratic in x, λ because c, y constant

stationary pt:

$$\left.\begin{aligned} \nabla_x F &= x - \lambda y = 0 \\ \partial_\lambda F &= c - (y,x) = 0 \end{aligned}\right\} \text{ linear eqns.}$$

You can treat c as a variable it seems?

So now we have a new idea to develop, starting from the Lagrange multiplier treatment of an affine constraint.

§4 $F = \frac{1}{2}(x,x) + \lambda(c-(y,x))$. This is a quadratic form on $X \oplus \mathbb{R}$ the space of (x,λ). Instead of restricting x to the affine hyperplane $c=(y,x)$ and then finding the critical point & value, you leave X alone but add the parameter λ to force the constraint.

Start with conn R network K with external nodes A and the ground O, $A \neq O$, and let $L = K$ with a branch attached joining A and O. Then you have cocartesian square

$$\begin{tikzcd} \dot{I} \arrow[r] \arrow[d] & I \arrow[d] \\ K \arrow[r] & L \end{tikzcd} \qquad \text{better} \qquad \begin{tikzcd} \dot{I} \arrow[r] \arrow[d, hook] & K \arrow[d, hook] \\ I \arrow[r] & L \end{tikzcd}$$

$$K \hookrightarrow L \twoheadrightarrow L/K$$

$$\begin{tikzcd} \bar{C}^0 K \arrow[r, hook] & C^1 K \arrow[r, two heads] & H^1 K \\ \bar{C}^0 L \arrow[u, hook] \arrow[r, hook] & C^1 L \arrow[u, two heads] \arrow[r, two heads] & H^1 L \arrow[u] \\ 0 & \mathbb{R} = C^1(L,K) \arrow[u] \arrow[r, phantom, "="] & H^1(L,K) \arrow[u] \end{tikzcd}$$

Assume K a tree. Then $H^1K = 0$ and you have

$$\begin{tikzcd} \bar{C}^0 L \arrow[r, hook] \arrow[d, phantom, "\shortparallel"] & C^1 L \arrow[r, two heads] & H^1 L \arrow[d, phantom, "\shortparallel"] \\ \bar{C}^0 K & & \mathbb{R} \end{tikzcd}$$

So from $K \hookrightarrow L \twoheadrightarrow L/K \overset{\shortparallel}{S^1}$ you get where K is a tree

$$\begin{tikzcd}
\tilde{C}^0 K \arrow[r, phantom, "\simeq"] & C^1 K & 0 \\
\tilde{C}^0 L \arrow[u, "\wr"] \arrow[r] & C^1 L \arrow[u] \arrow[r] & H^1 L \\
& \mathbb{R} \arrow[u] \arrow[r, phantom, "="] & \mathbb{R} \arrow[u, phantom, "\wr"]
\end{tikzcd}$$

You actually have ~~an~~ a canon. isomorphism $C^1 L = \begin{pmatrix} C^1 K \\ \mathbb{R} \end{pmatrix}$.

What is your aim? To go from the K picture, with the potential at A fixed and you find the minimum power configuration and power subject to this inhomogeneous constraint, and to proceed to the L picture, which should fit the Thevenin scheme of handling inhomogeneous constraints by an element of $C^1 L$. There is a possible problem with a resistance value for the attached branch, but you hoping that this will be supplied the push forward of the scalar product on $\tilde{C}^0 K$ via the map $\varphi \mapsto \varphi_A : \tilde{C}^0 K \twoheadrightarrow \mathbb{R}$.

Everything should follow from the following data X a Euclidean space, $y^t : X \twoheadrightarrow \mathbb{R}$ nonzero linear functional (use y^t same as $y \in X$).
Then $\tilde{C}^0 L \xrightarrow{\sim} \tilde{C}^0 K \xrightarrow{\sim} C^1 K$ are all canonically isom to X.
Let $\hat{X} = \begin{pmatrix} X \\ \mathbb{R} \end{pmatrix}$ and let $\delta_L = \begin{pmatrix} 1_X \\ y^t \end{pmatrix} : X \longrightarrow \hat{X}$, i.e. δ_L is the graph of $y^t : X \longrightarrow \mathbb{R}$.

04 What do you need to do? You want $F = \frac{1}{2}x^t x + \lambda(c - y^t x)$ to give rise to a scalar product on $\hat{X}$. Better you want $\hat{X}$ to be $C^1 L$, which means you need a scalar product on $\hat{X}$. To be more precise, you want L to be a connected R-network like K, so that you can handle the inhomogeneous constraint à la Thévenin by a branch emf.

Repeat: Given X with scalar product $x^t x$, and a nonzero linear functional $y^t : X \longrightarrow \mathbb{R}$, together with a fixed $c \in \mathbb{R}$. Think of X as the space of node emfs, and the constraint $c = y^t x$ as fixing the voltage drop between 2 nodes by attaching a battery of emf $= c$ between these nodes.

Picture: You have a tree K, and you attach a branch joining the two "external" nodes to obtain a graph L with one loop.

Try $X \xrightarrow{\binom{1}{y^t}} \binom{X}{\mathbb{R}}$ for $\bar{C}^0 L \to C^1 L$

Is $\binom{1}{y^t}$ an isometry? $\binom{X}{\mathbb{R}}$ is equipped with the form $\binom{x}{c}^t \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{y^t y} \end{pmatrix} \binom{x}{c} = x^t x + \frac{c^2}{y^t y}$

$\binom{1}{y^t} : x \longmapsto \binom{x}{y^t x}$ which has norm^2 $= x^t x + \frac{(y^t x)^2}{y^t y} > x^t x$

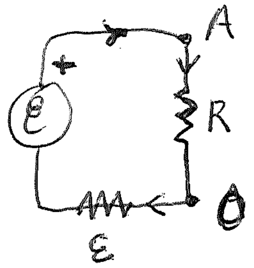
so $\binom{1}{y^t}$ is not an isometry, although it is if you restrict to the hyperplane $y^t x = 0$.

Let's look at this in a special case, where the tree K is

. Then $X = \{V_A = V_R\}$ with the power function $\frac{V_R^2}{2R}$

suppose we now make the augmented graph

c4

L: 

$$\bar{C}^0 L = \bar{C}^0 K = X = \{ V_A = V_R \}$$

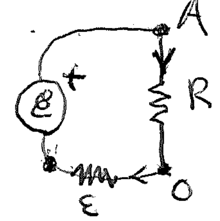
$$C^1 L = \left\{ \begin{pmatrix} V_R \\ \mathcal{E} \end{pmatrix} \right\}$$

$\mathcal{E}$ here is to be the Thevenin voltage source (pure emf) in series with an internal resistance. Next you need $\delta : X = \bar{C}^0 L \longrightarrow C^1 L$. Given V_A the node potential then $\delta : V_A \longmapsto \begin{pmatrix} V_R \\ \mathcal{E} \end{pmatrix}$ where $V_R = V_A$. What is $\mathcal{E}$? You know the current $I = \frac{V_A}{R}$ and $\mathcal{E}$ should be $(\varepsilon + R) I$, thus

$$\mathcal{E} = (\varepsilon + R) I = (\varepsilon + R) \frac{V_A}{R}$$

Note that if $\varepsilon \to 0$ we get $\mathcal{E} = V_A$, which gives the required constraint.

There remains to find the power function on $C^1 L$. Actually it may be more illuminating to find the splitting of $\mathcal{E}$ into a node potential and an orthogonal loop voltage. Review. Consider the network:



What are the appropriate circuit equations.

You have 2 nodes -1 $+1$ loop $= 2$ branches

$$\begin{array}{ccccc} V_A \mapsto \begin{pmatrix} V_A \\ -V_A \end{pmatrix} & & & & \\ \bar{C}^0 & \longrightarrow & C^1 & \longrightarrow & H^1 \\ & & \uparrow \begin{pmatrix} R & 0 \\ 0 & \varepsilon \end{pmatrix} & & \\ \bar{C}_0 & \longleftarrow & C_1 & \longleftarrow & H_1 \\ & & \begin{pmatrix} I \\ +I \end{pmatrix} \longleftarrow & I & \end{array}$$

In C^1 you get approx

$$\begin{pmatrix} V_A \\ -V_A \end{pmatrix} + \begin{pmatrix} RI \\ \varepsilon I \end{pmatrix} = \begin{pmatrix} 0 \\ \mathcal{E} \end{pmatrix}$$

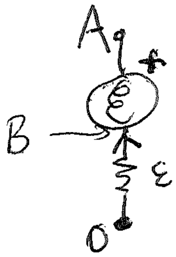
Add $(R + \varepsilon) I = \mathcal{E}$, also you have $V_A + RI = 0$

signs wrong $\quad V_R = V_A - V_0 = RI \qquad \boxed{V_A = RI}$

$$V_e = V_0 - V_A = \varepsilon I - \mathcal{E}$$

$$0 = (R + \varepsilon) I - \mathcal{E}$$

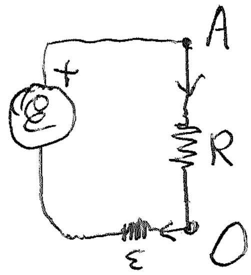
K4 Still don't understand signs



$$\varepsilon I - \mathcal{E} = V_0 - V_A = -V_A$$

way to think: You start at potential V_0 before ε resistor, $V_0 - V_B = \varepsilon I$

$V_B - V_A = -\mathcal{E} \implies V_0 - V_A = \varepsilon I - \mathcal{E}$.



$$\bar{C}^0 \longrightarrow C^1 \overset{(1\ 1)}{\dashrightarrow}$$

$$V_A \mapsto \begin{pmatrix} V_A \\ -V_A \end{pmatrix} = \begin{pmatrix} V_R \\ V_\varepsilon \end{pmatrix} \xleftarrow{\begin{pmatrix} R & 0 \\ 0 & \varepsilon \end{pmatrix}} \begin{pmatrix} I \\ I \end{pmatrix} \longleftarrow\!\shortmid\ I$$

$$\bar{C}_0 \xleftarrow{(1\ -1)} C_1 \qquad H_1$$

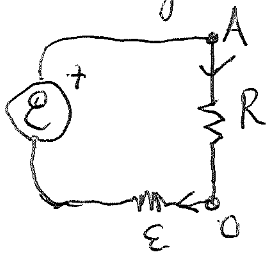
Equations
$$\begin{pmatrix} -V_A \\ +V_A \end{pmatrix} + \begin{pmatrix} RI \\ \varepsilon I \end{pmatrix} = \begin{pmatrix} 0 \\ \mathcal{E} \end{pmatrix}$$

$$\implies \quad V_A = RI \quad \text{and} \quad (R+\varepsilon)I = \mathcal{E}$$

Here seems to be the sign mistake: to add V_A and RI. Correct is

$V_A - V_0 = RI$ or $V_A - RI = V_0$

Look again at



δV_A is $\begin{pmatrix} V_R \\ V_\varepsilon \end{pmatrix}$

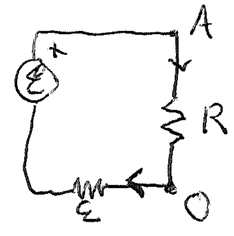
$$V_A - V_0 = RI$$

$$V_0 - \varepsilon I + \mathcal{E} = V_A$$

add $\quad V_0 - V_A = \varepsilon I - \mathcal{E}$

$$0 = (R+\varepsilon)I - \mathcal{E}$$

$\lambda 4$ What should be the power function on C^1L?



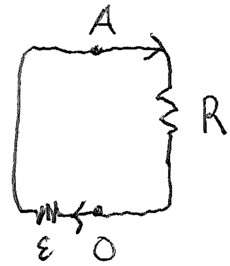
An element of C^1L consists of a voltage drop V_R for the R branch, and a voltage drop V_ε for the ε branch.

How do you handle $\mathcal{E}$? I think you want to set $\mathcal{E} = 0$ first and to get the homogeneous linear system to be nondegenerate. Afterword you introduse $\mathcal{E}$ as an inhomogeneous term.

On the other hand there might be a simple variational way to handle inhomogeneous terms.

Start again with

$$\begin{array}{ccccc} \bar{C}^0 & \hookrightarrow & C^1 & \twoheadrightarrow & H^1 \\ \| & & \| & & \\ \{V_A\} & \xrightarrow{\begin{pmatrix}1\\-1\end{pmatrix}} & \left\{\begin{pmatrix}V_R\\V_\varepsilon\end{pmatrix}\right\} & \xrightarrow{(1\ 1)} & 0 \end{array}$$



and the power function

$$\frac{V_R^2}{2R} + \frac{V_\varepsilon^2}{2\varepsilon} \quad \text{on } C^1$$

Consider the variational problem of minimizing the power subject to the Kirchhoff contraint $V_R + V_\varepsilon = 0$.

Review: The point of departure seems to be the "voltage picture" of a connected R-network. This amounts to a real v.s. C^1 of branch voltages, which is equipped with a positive def. scalar product "the power", and also a subspace $\bar{C}^0 \hookrightarrow C^1$ of "conservative" branch voltages; elements of $\bar{C}^0$ can also be identified with node potentials.

With this data you can form a short exact sequence

$$\bar{C}^0 \hookrightarrow C^1 \twoheadrightarrow H^1,$$

and use the scalar product to split this sequence orthogonally, whence you have induced scalar products on $\bar{C}^0$ and H^1, which combine to give the scaller product on C^1.

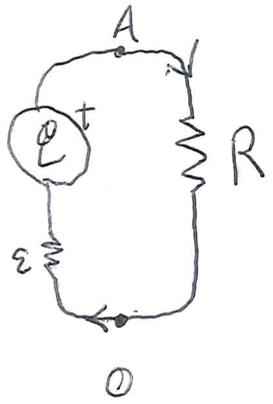
Then any $\varepsilon \in C^1$ splits into a node potential $\varphi \in \bar{C}^0$ and a branch voltage $\perp \bar{C}^0$.

p4 You can combine the voltage picture with the dual "current picture" consisting of current spaces

$$
\begin{tikzcd}
\bar{C}^0 \arrow[r, hook] & C^1 \arrow[r, two heads] & H^1 \\
\bar{C}_0 & C_1 \arrow[l, two heads] \arrow[u, "R", "S"'] & H_1 \arrow[l, hook']
\end{tikzcd}
$$

H_1 is the space of closed currents or "loops", R is isomorphism: $C_1 \xrightarrow{\sim} C^1$ arising from the scalar product on C^1.

Review: 4 Unknowns $\begin{pmatrix} V_R \\ I_R \end{pmatrix}$ $\begin{pmatrix} V_\varepsilon \\ I_\varepsilon \end{pmatrix}$



Constraints

$$I_R = I_\varepsilon \qquad V_\varepsilon = -V_A$$
$$V_\varepsilon + V_R = 0 \qquad V_R = V_A$$

$$\left.\begin{aligned} V_R &= RI \\ -V_\varepsilon &= -\varepsilon I + \mathcal{E} \end{aligned}\right\} \qquad \begin{aligned} V_R &= RI \\ V_\varepsilon &= \varepsilon I - \mathcal{E} \\ \hline 0 &= (R+\varepsilon) I - \mathcal{E} \end{aligned}$$

The question should be whether these linear eqns can be gotten from the voltage picture? From the power form on C^1 together with the Kirchhoff voltage constraints

IDEA: Maxwell's Equations $dA = 0$, $d^*A =$ (charge, current) are half homogeneous linear, specifically $\nabla \cdot B = 0$, $\partial_t B + \nabla \times E = 0$. This suggests that the voltage picture with inhomogeneous terms given only by voltage sources should be a natural object. It's similar to the Lagrangian approach to a harmonic oscillator, where you work in configuration space with a quadratic $L = KE - PE$ + linear term in position.

v4 Maxwell $\Longrightarrow$ only half the equations should be allowed inhomogeneous forcing terms. This suggests that the voltage picture should yield all the information about the state of the network. You would like to treat Thevenin emf's on the branches. Is there some variational problem which handle yield these Thevenin inhomogeneous terms.

$$\begin{array}{ccccc} \bar{C}^0 & \hookrightarrow & C^1 & \longrightarrow & H^1 \\ & & \| & & \\ & & \bar{C}^0 \oplus RI & & \end{array}$$

orthogonal splitting for power,

equations should be $\mathcal{E} = -V + RI$

Example:

$$\begin{array}{ccccc} \bar{C}^0 & & C^1 & & H^1 \\ V_A & \longmapsto \begin{pmatrix} V_A \\ -V_A \end{pmatrix} = & \begin{pmatrix} V_R \\ V_\varepsilon \end{pmatrix} & \xrightarrow{(1\ 1)} & V_R + V_\varepsilon \\ & & & & \| \\ & & & & 0 \end{array}$$

$$\begin{pmatrix} 0 \\ \mathcal{E} \end{pmatrix} = \begin{pmatrix} -V_A \\ V_A \end{pmatrix} + \begin{pmatrix} RI \\ \varepsilon I \end{pmatrix}$$

Variational Problem $\quad F = \dfrac{V_R^2}{2R} + \dfrac{V_\varepsilon^2}{2\varepsilon} - \lambda(V_R + V_\varepsilon)$ yields homogeneous linear equations

$$\frac{V_R}{R} = \lambda, \quad \frac{V_\varepsilon}{\varepsilon} = \lambda, \quad V_R + V_\varepsilon = 0$$

It seems right to interpret λ as the current I and then you get $0 = V_R + V_\varepsilon = (R+\varepsilon)I$, which means you've left out the emf $\mathcal{E}$.

So look at

$$V_R + V_\varepsilon - \mathcal{E} = 0$$

So you should have had $\quad F = \dfrac{V_R^2}{2R} + \dfrac{V_\varepsilon^2}{2\varepsilon} + \lambda(\mathcal{E} - V_R - V_\varepsilon)$

$$0 = \frac{V_R}{R} - \lambda = \frac{V_\varepsilon}{\varepsilon} - \lambda, \quad \mathcal{E} = V_R + V_\varepsilon = (R+\varepsilon)\lambda$$

$$F = \frac{R\lambda^2}{2} + \frac{\varepsilon\lambda^2}{2} = \frac{R+\varepsilon}{2}\frac{\mathcal{E}^2}{(R+\varepsilon)^2} = \frac{\mathcal{E}^2}{2(R+\varepsilon)}$$

critical value

§4 Yesterday you made some progress: treating a branch emf [redacted] as an inhomogeneous constraint using Lagrange multipliers.

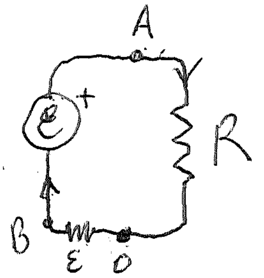
In more detail, given

$$\tilde{C}^0 \hookrightarrow C^1 \twoheadrightarrow H^1$$

together with the power scalar product on C^1, [redacted] and a branch emf $\mathcal{E}$ (internal emf's on the branches), you get a splitting [redacted]

$$\begin{array}{c} \tilde{C}^0 \hookrightarrow C^1 \twoheadrightarrow H^1 \\ \quad\ \| \\ \tilde{C}^0 \oplus (\tilde{C}^0)^\perp \end{array}$$

Review example. [redacted] First find the circuit eqns. involving both voltages + currents.



[redacted] variables $\begin{pmatrix} V_R \\ V_\varepsilon \end{pmatrix}$, $\begin{pmatrix} I_R \\ I_\varepsilon \end{pmatrix}$ Ohm $V_R = RI_R$, $V_\varepsilon = \varepsilon I_\varepsilon$

Kirchhoff: $I_R = I_\varepsilon$, $V_R + V_\varepsilon = 0$.

You've made a mistake because $\mathcal{E}$ should occur as part of an inhomogeneous constraint. [redacted]

$$\tilde{C}^0 \hookrightarrow C^1 \twoheadrightarrow H^1$$

$$V_A \xmapsto{\begin{pmatrix} 1 \\ -1 \end{pmatrix}} \begin{pmatrix} V_A \\ -V_A \end{pmatrix} = \begin{pmatrix} V_R \\ V_\varepsilon - \mathcal{E} \end{pmatrix} \xmapsto{(1\ 1)} V_R + V_\varepsilon - \mathcal{E}$$

$$V_0 - V_B = \varepsilon I = V_\varepsilon$$
$$V_B - V_A = -\mathcal{E}$$
$$V_A - V_0 = V_R$$

[redacted] Variational Method

$$F = \frac{V_R^2}{2R} + \frac{V_\varepsilon^2}{2\varepsilon} + \lambda(\mathcal{E} - V_R - V_\varepsilon)$$

$$\frac{V_R}{R} = \lambda, \quad \frac{V_\varepsilon}{\varepsilon} = \lambda, \quad \mathcal{E} = V_R + V_\varepsilon = \lambda(R+\varepsilon)$$

$$F = \frac{\lambda^2 R}{2} + \frac{\lambda^2 \varepsilon}{2} = \frac{\lambda^2}{2}(R+\varepsilon) = \frac{\mathcal{E}^2}{2(R+\varepsilon)}$$

i.e. the response to the applied $\mathcal{E}$ is that of a resistance $R+\varepsilon$.

04 Let's check the calculation by finding the critical point satisfying the constraint $V_R + V_\varepsilon = \mathcal{E}$. Let V_R be the independent variable, $V_\varepsilon = \mathcal{E} - V_R$. ~~What~~ You want the critical point for

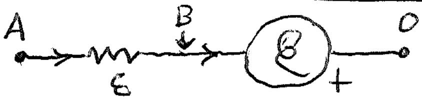
$$\frac{1}{2}\begin{pmatrix} V_R \\ \mathcal{E}-V_R \end{pmatrix}^t \begin{pmatrix} \frac{1}{R} & 0 \\ 0 & \frac{1}{\varepsilon} \end{pmatrix}\begin{pmatrix} V_R \\ \mathcal{E}-V_R \end{pmatrix} \quad \text{which means}$$

$$(1 \ \ -1)\begin{pmatrix} \frac{1}{R} & 0 \\ 0 & \frac{1}{\varepsilon} \end{pmatrix}\begin{pmatrix} V_R \\ V_\varepsilon \end{pmatrix} = 0 \qquad \text{i.e. } \frac{V_R}{R} = \frac{V_\varepsilon}{\varepsilon} = \lambda$$

call this ratio λ. Then the critical value is

$$\frac{1}{2}\begin{pmatrix} \lambda R \\ \lambda \varepsilon \end{pmatrix}\begin{pmatrix} R^{-1} & 0 \\ 0 & \varepsilon^{-1} \end{pmatrix}\begin{pmatrix} \lambda R \\ \lambda \varepsilon \end{pmatrix} = \frac{1}{2}\lambda^2(R+\varepsilon)$$

where $\mathcal{E} = V_R + V_\varepsilon = \lambda(R+\varepsilon)$ and so the critical value in terms of $\mathcal{E}$ is $\dfrac{1}{2}\dfrac{\mathcal{E}^2}{R+\varepsilon}$.

Consider a branch  . A state of the branch is given by a pair V, I satisfying a variant of Ohm's Law.

$$V_A - \varepsilon I = V_B \qquad V_A - V_B = \varepsilon I \qquad V_A - 0 = \varepsilon I - \mathcal{E}$$

$$V_B + \mathcal{E} = 0 \qquad V_B - 0 = -\mathcal{E}$$

Conclude $\boxed{V_{\text{branch}} = \varepsilon I - \mathcal{E}}$ state equation.

Now you want this to arise from a quadratic function of V involving $\mathcal{E}$ as a constant.

$$F = \frac{V^2}{2\varepsilon} + \frac{V\mathcal{E}}{\varepsilon} \qquad \frac{dF}{dV} = \frac{V}{\varepsilon} + \frac{\mathcal{E}}{\varepsilon} = \frac{\varepsilon I}{\varepsilon} = I$$

$\pi 4$ Special case of attaching a branch with emf to handle a constraint.

$C^1K = \{V_R\}$ with power $\dfrac{V_R^2}{2R}$

$\delta \uparrow$

$\bar{C}^0K = \{V_A\}$. $V_R = V_A - V_0 = V_A$

Your constraint is $-V_A = \mathcal{E}$

You want the critical point +value for $\dfrac{V_R^2}{2R}$ subject to $\mathcal{E} + V_R = 0$. The critical point seems to be $V_R = -\mathcal{E}$ critical value is $\dfrac{\mathcal{E}^2}{2R}$.

Lagrange mult.

$$F = \frac{V_R^2}{2R} - \lambda(\mathcal{E} + V_R) \quad\Bigg|\quad 0 = \frac{\partial F}{\partial V_R} = \frac{V_R}{R} - \lambda \quad\Bigg|\quad \frac{\partial F}{\partial \lambda} = \mathcal{E} + V_R = 0$$

$$\lambda R = V_R = -\mathcal{E}$$

$$F = \frac{V_R^2}{2R} = \frac{\mathcal{E}^2}{2R}$$

The other method consists of attaching an $\mathcal{E}$ branch, then solving the appropriate linear equations, The new point is that for a branch with $\mathcal{E}$ Ohm's Law needs to be changed

$V = V_A - V_O = \varepsilon I - \mathcal{E}$

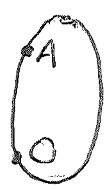
$V = \varepsilon I - \mathcal{E}$ power $\dfrac{1}{2\varepsilon}(V^2 + 2V\mathcal{E}) = \dfrac{1}{2\varepsilon}\left[(V+\mathcal{E})^2 - \mathcal{E}^2\right]$

$V + \mathcal{E} = \varepsilon I$ so if the constant $\mathcal{E}^2$ is ignored

you have power $= \dfrac{1}{2\varepsilon}(V+\mathcal{E})^2 = \dfrac{1}{2}\varepsilon I^2$.

§4 Next, you want to start with a connected R-network K with external nodes $A \neq O$, ~~then~~ and to understand how to attach a pure emf $\mathcal{E}$ branch joining A to O

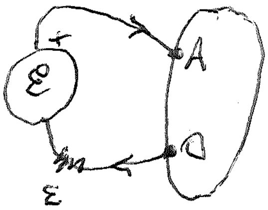
K



$$\begin{array}{ccc} & \bar{C}^0K & \xrightarrow{\delta} C^1K \\ \varphi \downarrow & \downarrow & \\ \varphi_A - \varphi_O & \mathbb{R} & \end{array}$$

Here you impose the constraint $\varphi_A - \varphi_O = \mathcal{E}$

The augmented network L has a small resistance ε on the $\mathcal{E}$ branch which you want to go to zero.



$$\varphi_O - \varphi_A = \varepsilon I - \mathcal{E}$$

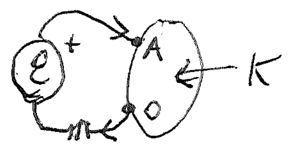
The power on C^1K should be an orthogonal ~~direct~~ sum of ~~[illegible]~~ the power for each branch.

Let's describe C^1L. All branches except the $\mathcal{E}$ branch are described by the power function $\frac{1}{2}\frac{V^2}{R}$ whose derivative $\partial_V\left(\frac{1}{2}\frac{V^2}{R}\right) = \frac{V}{R}$ is the current.

Next look at the $\mathcal{E}$ branch where the power function is $P = \frac{1}{2\varepsilon}(V+\mathcal{E})^2$, up to an additive constant, and the derivative $\frac{\partial P}{\partial V} = \frac{1}{\varepsilon}(V+\mathcal{E})$ ~~is~~ the current.

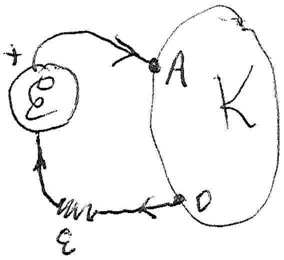
Begin with $\bar{C}^0K = \bar{C}^0L$, but $C^1L = C^1K \oplus \mathbb{R}$ where $\mathbb{R} = \{V_e\}$. Then $\delta_L : \bar{C}^0L \longrightarrow C^1L$ should have two components, one from $\delta_K : \bar{C}^0K \longrightarrow C^1K$ and the other should take a node potential $\varphi \in \bar{C}^0K$ ~~[illegible]~~ to $V_e = \varphi_O - \varphi_A = \varepsilon I_e - \mathcal{E}$. So you should be getting the usual ~~[illegible]~~ homogeneous linear circuit equations: $V = RI$ for all branches in K. For the e branch you get inhomogeneous term $-\mathcal{E}$.

σ 4   [redacted] Begin with K $\overset{\text{connected}}{\wedge}$ R-network, external node pair
A, O   You have $\mathbb{R} \twoheadleftarrow \bar{C}^0 K \overset{\delta}{\hookrightarrow} C^1 K$. Now attach branch
$\varphi_A - \varphi_0 \leftarrow\!\shortmid \varphi$



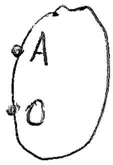
The linear fcn. $\varphi \mapsto \varphi_A - \varphi_0$ is [redacted] given by pairing φ with the node current $[A] - [0] \in \bar{C}_0 K$. But $\varphi \mapsto \varphi_0 - \varphi_A$ is $\delta_L \varphi$ evaluated on the [redacted] new branch. You should know that $C^1 L = C^1 K \oplus \mathbb{R}$ and $\delta_L = \begin{pmatrix} \delta_K \\ [0]-[A] \end{pmatrix} : \bar{C}^0 K \longrightarrow \begin{pmatrix} \bar{C}^0 K \\ \mathbb{R} \end{pmatrix}$

Begin with a connected R-network K, an "external" node pair A, O so that you have $\mathbb{R} \twoheadleftarrow \bar{C}^0 K \hookrightarrow C^1 K$
$\varphi_A - \varphi_0 \leftarrow\!\shortmid \varphi$ $\wr$ $C_1 K$

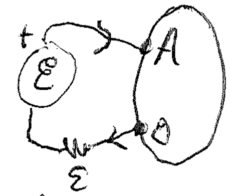


You ~~next~~ attach a branch joining A to O consisting of a pure emf $\mathcal{E}$ in series with a resistance ε. This gives an R-network L having the same node potential space: $\bar{C}^0 K = \bar{C}^0 L$. What is $C^1 L$ [redacted] and the coboundary map $\delta_L : \bar{C}^0 L \longrightarrow \bar{C}^1 L$? $C^1 L$ is the [redacted] direct sum over the branches of the branch voltage lines, such a line becomes $\mathbb{R}$ upon orienting the branch. Also there is quadratic form on [redacted] each branch voltage line given by the power $\frac{V^2}{R}$ (up to $\frac{1}{2}$?).

Program: Today you want to settle the Thevenin business. ~~[illegible]~~ It should be simple. K:



L:

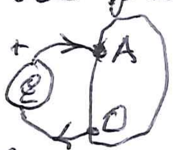


~~Accompanying Thevenin~~

You want the response to the applied emf $\mathcal{E}$, i.e. [redacted] the current I flowing for each $\mathcal{E}$. Formula

$$I = \frac{\mathcal{E}}{R + \varepsilon}$$ where R $\overset{\text{Thevenin}}{\wedge}$ equivalent resistance

τ4 Here's the problem:

You have connected K: , form L: , you want the current response $I_\mathcal{E}$ to $\mathcal{E}$. Return to old idea of solving the circuit equations for K, modified by allowing node currents supported at (O, A).

$$
\begin{tikzcd}
\mathbb{R} & \tilde{C}^0K \arrow[l, two heads, "-\varphi_A+\varphi_O \leftarrow\!\shortmid\, \varphi"'] \arrow[r, hook] \arrow[d, phantom, "\wr"] & C^1K \arrow[d, phantom, "\wr"] \\
\mathbb{R} \arrow[r, hook, "-[A]+[O]"] & \tilde{C}_0K & C_1K \arrow[l, two heads]
\end{tikzcd}
$$

You perhaps want to arrange this

$$\tilde{C}^0K \xrightarrow{\left(\delta_K,\ (-[A]+[O])^*\right)} \begin{pmatrix} C^1K \\ \mathbb{R} \end{pmatrix}$$

$$\tilde{C}_0K \xleftarrow{\left(\partial_K \quad -[A]+[O]\right)} \begin{pmatrix} C_1K \\ \mathbb{R} \end{pmatrix} \qquad ? \; ?$$

Go over this again to get it clearer. Starting with K and the "external" node pair (A, O) you know there is a Thevenin equivalent circuit  which gives the same relation between the current and voltage associated to this node pair. You expect that this resistance should be used for the new branch in the augmented graph.

IDEA. You recall shorting A, O by a zero resistance wire. This gives a different R-network from K – NO, not unless you shrink the wire to a point, in which case the loop number is unchanged.

So attach the wire – what happens to the circuit equations?

u4 Consider a connected $\mathbb{R}$-network K with a given external node pair $(A, 0)$. You should now be able to exhibit the cochain short exact sequence for the augmented graph as follows. Recall that we have the s.e.s. for K

$$\begin{array}{ccccc} \bar{C}^0K & \overset{\delta_K}{\hookrightarrow} & C^1K & \overset{\pi_K}{\twoheadrightarrow} & H^1K \\ f\big\downarrow & & & & \\ \mathbb{R} & & & & \end{array}$$

and the linear fnl given by $f(\varphi) = \varphi_0 - \varphi_A$

Then $\bar{C}^0K = \bar{C}^0L$, $C^1L = \begin{pmatrix} C^1K \\ \mathbb{R} \end{pmatrix}$

and $\delta_L = \begin{pmatrix} \delta_K \\ f \end{pmatrix} : \bar{C}^0L \longrightarrow \begin{pmatrix} C^1K \\ \mathbb{R} \end{pmatrix} = C^1L$. We want to describe $\pi_L : C^1L \longrightarrow H^1L$. Picture

$$\begin{tikzcd}
\bar{C}^0K \arrow[r, "\delta_K"] \arrow[ddr, "f"'] & C^1K \arrow[r, "\pi_K"] \arrow[ddr, "-f'"] & H^1K \\
 & \oplus & \oplus \\
 & \mathbb{R} \arrow[r, "1"] & \mathbb{R}
\end{tikzcd}$$

where f' is a linear fnl on C^1K extending f on $\bar{C}^0K$: $f'\delta_K = f$.

Let's check exactness: $\begin{pmatrix} \pi_K & 0 \\ -f' & 1 \end{pmatrix}\begin{pmatrix} \delta_K \\ f \end{pmatrix} = \begin{pmatrix} \pi_K\delta_K \\ -f'\delta_K + f \end{pmatrix} = 0.$

On the other hand let $\begin{pmatrix} V \\ c \end{pmatrix} \in \begin{matrix} C^1K \\ \oplus \\ \mathbb{R} \end{matrix} = C^1L$ satisfy

$\pi_L\begin{pmatrix} V \\ c \end{pmatrix} = \begin{pmatrix} \pi_K & 0 \\ -f' & 1 \end{pmatrix}\begin{pmatrix} V \\ c \end{pmatrix} = 0$, i.e. $\pi_K V = 0$, $c = +f'V$. By exactness for K one have $V = \delta\varphi$, $\varphi \in \bar{C}^0K$.

[redacted]

Then $\delta_L\varphi = \begin{pmatrix} \delta_K\varphi \\ f\varphi \end{pmatrix} = \begin{pmatrix} V \\ +f'\delta_K\varphi \end{pmatrix} = \begin{pmatrix} V \\ f'V \end{pmatrix} = \begin{pmatrix} V \\ c \end{pmatrix}$

Note: this involves a choice of $f' : C^1K \to \mathbb{R}$ such that $f'\delta_K\varphi = f\varphi$. This means that you want a current f' in K such that $\partial f' = [0] - [A]$.

$\phi 4$ previous work K2. Given $V' \overset{\delta}{\hookrightarrow} V \twoheadrightarrow V''$ and $\lambda' : V' \to \mathbb{R}$ construct

$$\begin{tikzcd}
V' \arrow[r, hook] \arrow[d, equal] & V \arrow[r, two heads] & V'' \\
V' \arrow[r, hook, "\binom{\delta}{-\lambda'}"] & \begin{pmatrix} V \\ \mathbb{R} \end{pmatrix} \arrow[u] \arrow[r, two heads] & W \arrow[u] \\
& \mathbb{R} \arrow[u, hook] \arrow[r, equal] & \mathbb{R} \arrow[u, hook]
\end{tikzcd}$$

W is cofibre prod

$$\begin{tikzcd}
V' \arrow[r, "\delta"] \arrow[d, "\lambda'"'] & V \arrow[d] \arrow[dl, dashed, "\lambda"'] \\
\mathbb{R} \arrow[r] & W
\end{tikzcd}$$

so [redacted] extending λ' to λ: $\lambda\delta = \lambda'$ should gives a retract of W onto $\mathbb{R}$.

Generalization

$$\begin{tikzcd}
V' \arrow[r, tail, "\delta"] \arrow[d, equal] & V \arrow[r, two heads] & V'' \\
V' \arrow[r, tail, "\binom{\delta}{-\lambda'}"] & \begin{pmatrix} V \\ U \end{pmatrix} \arrow[u, "(1\,0)"] \arrow[r, two heads] & W \arrow[u] \\
& U \arrow[u, hook, "\binom{0}{1}"] \arrow[r, equal] & U \arrow[u, hook]
\end{tikzcd}
\qquad
\begin{tikzcd}
C^0K \arrow[r, tail, "\delta"] \arrow[d, equal] & C^1K \arrow[r] & H^1K \\
C^0L \arrow[r, tail, "\binom{\delta}{-\lambda'}"] & \begin{pmatrix} C^1K \\ \oplus \\ U \end{pmatrix} \arrow[u] \arrow[r] & H^1L \arrow[u] \\
& U \arrow[u] \arrow[r, equal] & U \arrow[u, "\lambda"]
\end{tikzcd}$$

Hope: These vector ~~space~~ diagrams fit with Lagrange multipliers.

X4 What you learned yesterday:

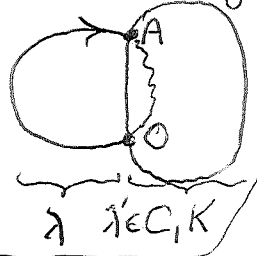
$$\begin{tikzcd} V' \arrow[r, hook, "\delta"] \arrow[d, equal] & V \arrow[r, two heads] & V'' \\ V' \arrow[r, hook, "\binom{\delta}{-\lambda}"] & \begin{pmatrix} V \\ U \end{pmatrix} \arrow[u] \arrow[r, two heads] & W \arrow[u] \\ & U \arrow[u, hook] \arrow[r, equal] & U \arrow[u, hook] \end{tikzcd} \qquad \begin{tikzcd} C^0K \arrow[r, hook] \arrow[d, equal] & C^1K \arrow[r, two heads] & H^1K \\ C^0L \arrow[r, hook, "\binom{\partial_K}{-\lambda}"] & \begin{pmatrix} C^1K \\ U \end{pmatrix} \arrow[u] \arrow[r, two heads] & H^1L \arrow[u] \\ & U \arrow[u, hook] \arrow[r, equal] & U \arrow[u, hook] \end{tikzcd}$$

Thus W is the cofibre product

$$\begin{tikzcd} V' \arrow[r, hook, "\delta"] \arrow[d, "\lambda"'] & V \arrow[d] \arrow[dl, dashed, "\lambda'"] \\ U \arrow[r, hook] & W \end{tikzcd}$$

You get a retract of W onto U by extending λ to $\lambda' : V \to U$ s.t. $\lambda'\delta = \lambda$.

Now you need to handle the quadratic form. But first you should understand the Kirchhoff voltage constraints [redacted] arising from $\begin{pmatrix} C^1K \\ \mathbb{R} \end{pmatrix} \twoheadrightarrow H^1L$. Put another way each linear functional on H^1L, this is the same as a closed current / loop current, gives such a constraint. From loop currents in K you get the voltage law in K, but there's the constraint given by $\lambda' : C^1K \to \mathbb{R}$.

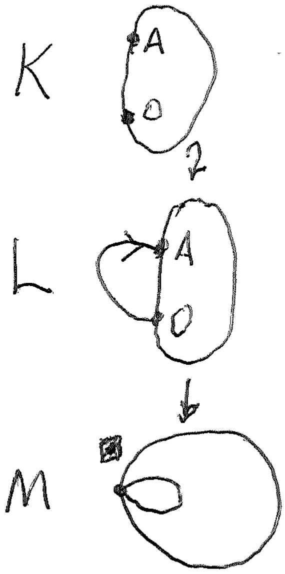


Recall $\lambda'\delta = \lambda : \varphi \longmapsto \varphi_0 - \varphi_A$

Viewed as chains: $\partial\lambda' = \lambda$.

Next you want to introduce the power quadratic form on C^1K, then produce the appropriate quadratic form on $C^1L = \begin{pmatrix} C^1K \\ \mathbb{R} \end{pmatrix}$.

ψ4 Thevenin idea of shorting (O,A)



$$\begin{tikzcd}
\tilde{C}^0K \arrow[r, hook] & C^1K \arrow[r, two heads] & H^1K \\
 & & \mathbb{R} \arrow[dl] \arrow[d] \\
\tilde{C}^0L \arrow[uu, equal] \arrow[r, hook] \arrow[dr] & C^1L \arrow[uu] \arrow[r, two heads] \arrow[d] & H^1L \arrow[uu] \\
 & \mathbb{R} & \\
\tilde{C}^0M \arrow[uu, hook] \arrow[r, hook] & C^1M \arrow[uu, hook] \arrow[r, two heads] & H^1M \arrow[uu, equal]
\end{tikzcd}$$

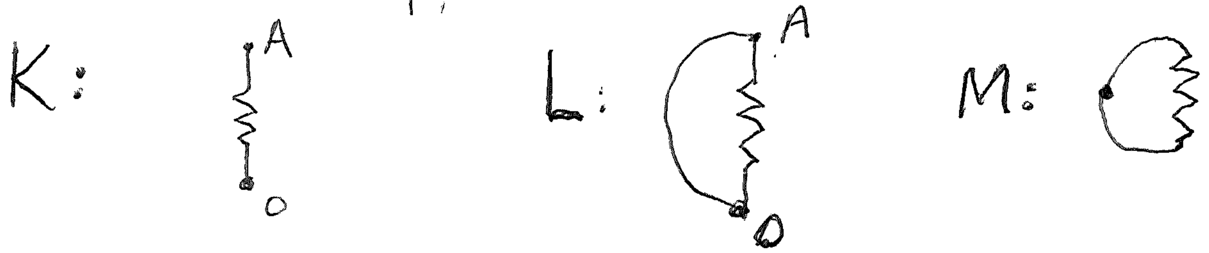
$$\begin{tikzcd}
\tilde{C}^0K \arrow[r, hook] & C^1K \arrow[r, two heads] & H^1K & \\
\tilde{C}^0L \arrow[u, equal] \arrow[r, hook] & C^1L \arrow[u] \arrow[r, two heads] & H^1L \arrow[u] & \mathbb{R} \arrow[l] \\
\tilde{C}^0M \arrow[u, hook] \arrow[r, hook] & C^1M \arrow[u, hook] \arrow[r, two heads] & H^1M \arrow[u, equal] &
\end{tikzcd}$$

← nullity 1

codim 1 →

[redacted] Note that K and M [redacted] are honest R-networks with >0 resistances for each edge. ? ?

Take some examples.



So it should be clear that $K \to M$ collapses the nodes (A,O), but doesn't affect the edges of these graphs. Conclude $C^1M \xrightarrow{\sim} C^1K$ as v.s. with quadratic form.

It looks like [redacted] the basic [redacted] sequence might be

$$\tilde{C}^0M \xhookrightarrow{\alpha} C^1L \xrightarrow{\beta} H^1K$$

but there seems to be 2 dimensional homology: $\operatorname{Ker}\beta / \operatorname{Im}\alpha$.

w4 IDEA You are reminded of cohomology with compact support, maybe intersection homology, which can carry positive quadratic forms, nondegenerate at least.

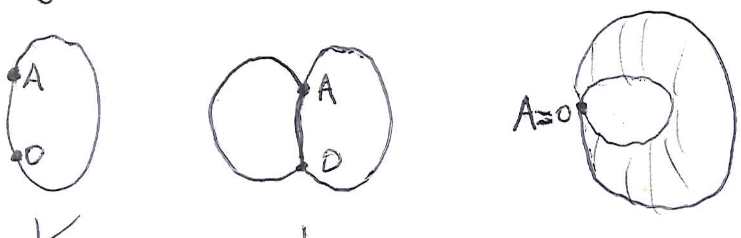
At this point you feel that the quadratic form calculations should be done in the space $C^1M \xrightarrow{\sim} C^1K$ with its positive form.

IDEA You are also reminded of ignoring nil-modules where the canonical factorization thru the image is used.

$$\begin{tikzcd}
\bar{C}^0M \arrow[dr, hook] & & C^1L \arrow[dr, two heads] & & H^1K \\
& C^1M \arrow[ur, hook] \arrow[dr, two heads] & & H^1L \arrow[ur] & \\
& & H^1M \arrow[ur, phantom, "="] & &
\end{tikzcd}$$

??

Review yesterday's advance



K L M

M is obtained from L by collapsing the branch attached

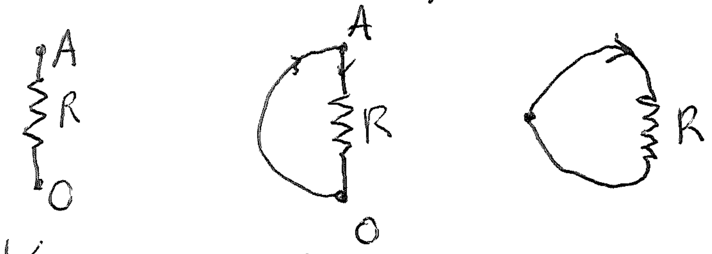
The natural map from K to M is bijective on the branches and preserves resistances. Thus $C^1K \xrightarrow{\sim} C^1M$; the isom. respects the power form. You don't see a candidate for the resistance of the attached branch to L. You think that stationary point calculations should involve the space $C^1K = \bar{C}^1M$ with its pos. definite power form. How to proceed?

IDEA: 2 dim'l space $\operatorname{Ker}\beta / \operatorname{Im}\alpha$

$$\bar{C}^0K \xhookrightarrow{\alpha} C^1L \xrightarrow{\beta} H^1K$$

α5 Does this 2 dim'l space, consisting roughly of an extra node current (linear fnl on $\tilde{C}^0$ for K, L) and an extra loop voltage for L, M, have a nice interpretation, say symplectic, or involving Lagrange multipliers in some way?

Look at the simplest case:

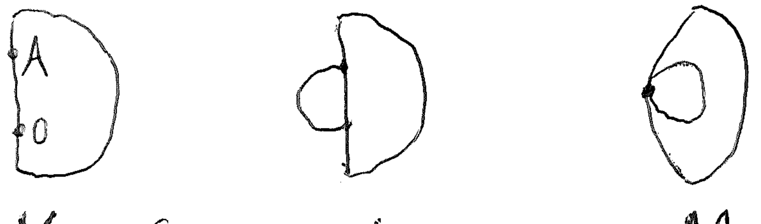


$$\begin{tikzcd}
K \arrow[r] & L \arrow[r] & M \\
\text{dim } 1\ \tilde{C}^0K \arrow[r, hook] & C^1K \ (1) \arrow[r, two heads] & H^1K = 0 \\
(1)\ \tilde{C}^0L \arrow[u, equal] \arrow[r, hook] & C^1L \ (2) \arrow[u] \arrow[r, two heads] & H^1L \ (1) \arrow[u, "\text{nullity } 1"'] \\
0 = \tilde{C}^0M \arrow[u, hook, "\text{codim } 1"] \arrow[r, hook] & C^1M \arrow[u, hook] \arrow[r, two heads] & H^1M \ (1) \arrow[u, equal] \\
 & (1) &
\end{tikzcd}$$

[redacted]

Let's focus upon the positive quadratic space $C^1K = C^1M$. This will induce [redacted] pos. def. forms on $\tilde{C}^0K, H^1K, \tilde{C}^0M, H^1M$. So you get pos. def. forms on $\tilde{C}^0L = \tilde{C}^0K$ and $H^1L = H^1M$. So only C^1L is lacking.

Picture



$M = K/_{A=0}$

$$K \hookrightarrow L \rightarrow M$$

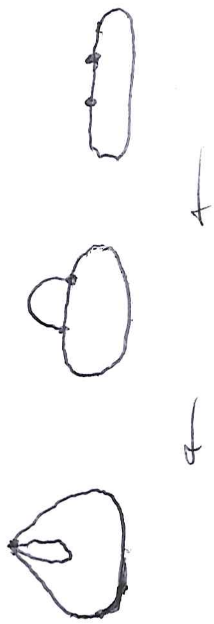
[redacted]

The point: K, M are connected R-networks with $C^1M \xrightarrow{\sim} C^1K$ as quadratic spaces. What happens in the Thevenin picture where you have $\mathcal{E} \in C^1$, $\mathcal{E}$ is a family of branch emf's, being split into $V \in \bar{C}^0$ and RI, $I \in H_1$? This is some kind of Hodge Decomposition

---

Let's go over the situation beginning with 3 short exact sequence

$$\begin{tikzcd}
\bar{C}^0K \arrow[r, hook] & C^1K \arrow[r, two heads] & H^1K \\
\bar{C}^0L \arrow[u, equal] \arrow[r, hook] \arrow[dr, phantom, "\text{cocartesian}"] & C^1L \arrow[u, hook, "\text{nullity}=1"'] \arrow[r, two heads] \arrow[ur, phantom, "\text{cartesian}"] & H^1L \arrow[u, hook, "\text{nullity}=1"'] \\
\bar{C}^0M \arrow[u, hook, "\text{codim } 1"] \arrow[r, hook] & C^1M \arrow[u, hook, "\text{codim } 1"'] \arrow[r, two heads] & H^1M \arrow[u, equal]
\end{tikzcd}$$



What do you want to do?

You have this quadratic space $C^1K \xleftarrow{\sim} C^1M$

Inside this quadratic space is where you split a Thevenin branch voltage $\mathcal{E}$ into its "harmonic" components. But the splitting depends upon the subspace $\bar{C}^0$ you use. There are 2 candidates, namely: $\bar{C}^0M \xhookrightarrow{\text{cod } 1} \bar{C}^0K$.

Something to do is to understand the two Hodge decompositions

$$\begin{tikzcd}
\bar{C}^0K \arrow[r, hook] & C^1K \arrow[r, two heads] & H^1K \\
\bar{C}^0M \arrow[u, hook, "\text{codim } 1"] \arrow[r, hook] & C^1M \arrow[u, "\wr"] \arrow[r, two heads] & H^1M \arrow[u, hook, "\text{nullity } 1"']
\end{tikzcd}$$

 Return to

K: 

$$\begin{array}{ccccc} \bar{C}^0K & \hookrightarrow & C^1K & \twoheadrightarrow & H^1K \\ \text{codim } 1 \uparrow & & \uparrow S & & \uparrow \text{nullity } 1 \\ \bar{C}^0M & \hookrightarrow & C^1M & \twoheadrightarrow & H^1M \end{array}$$

M: 

It seems that what you have here is a 3 step filtration

$$0 \subset \bar{C}^0M \subset \bar{C}^0K \subset C^1K$$

together with the quadratic form on C^1K. Recall that $\bar{C}^0M = \operatorname{Ker}\{\bar{C}^0K \xrightarrow{\lambda} \mathbb{R}\}$, $\varphi \mapsto \varphi_A - \varphi_0$ and that you want to minimize the power over $\lambda^{-1}(c)$ which is ~~a~~ an affine hyperplane $\parallel \bar{C}^0M$ in $\bar{C}^0K$.

You want to understand the meaning of these graphs. The graphs K, M are connected R-networks with pos def. power, and this leads to orthogonal splittings:

$$C^1K = \bar{C}^0K \oplus (\bar{C}^0K)^{\perp}$$

$$V = V_0 - V_A$$
$$= RI - \mathcal{E}$$
$$\boxed{V - RI = -\mathcal{E}}$$

$$\begin{array}{ccccc} \bar{C}^0K & \hookrightarrow & C^1K & \twoheadrightarrow & H^1K \\ & & S \uparrow\downarrow R & & \\ \bar{C}_0K & \twoheadleftarrow & C_1K & \hookleftarrow & H_1K \end{array}$$

Idea last night is to use the Thevenin equivalent resistance associated to the terminals $(A, 0)$. This you understand in the voltage current picture, meaning you see a 1-current I which is closed except at $(A, 0)$, and a 1-voltage V ~~which~~ with constraint between $(A, 0)$.

§5

$$\begin{tikzcd} \mathbb{R} & \tilde{C}^0K \arrow[l, "\rho^t"'] \arrow[r, hook] \arrow[d, "\wr"] & C^1K \arrow[d, dash, "\wr"] \\ \mathbb{R} \arrow[r, hook, "\rho"] & \tilde{C}_0K & C_1K \arrow[l, two heads] \end{tikzcd}$$

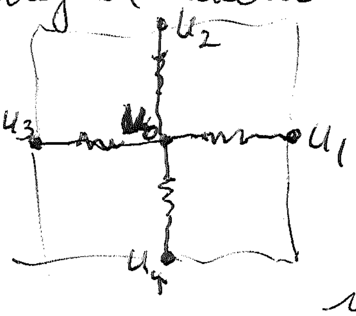
Idea of Poisson's equation $\Delta\varphi = \rho$. Discuss this: you have potential function φ, a norm $\|d\varphi\|^2$ yielding d^*d, a Laplacian. ρ charge density or 0-current, get $\varphi \longmapsto \rho^t\varphi$ which you want to represent: $(d\psi, d\varphi) = (\psi, \rho)$.

Better:

$$\begin{tikzcd} \mathbb{R} & \tilde{C}^0K \arrow[l, two heads, "\rho^t"'] \arrow[r, hook, "\delta"] \arrow[d, "\delta^t\delta"] & C^1K \arrow[d, dash, "R^{-1}\wr"'] \\ \mathbb{R} \arrow[r, hook, "\rho"] & \tilde{C}_0K & C_1K \arrow[l, "\delta^t"'] \end{tikzcd}$$

Solve $\delta^t\delta u = \rho$, Poisson's eqn. u is a node potential, δu is its gradient: the collection of branch voltage drops, $R^{-1}\delta u$ is the induced family of branch currents, $\delta^t R^{-1}\delta u$ is the resulting family of node currents. So when you solve $\delta^t R^{-1}\delta u = \rho$, where $\rho = [A] - [0]$, then u should be harmonic except at $(A, 0)$; harmonic should be a sort of 0 power condition.

Let's explore "harmonic" further. Recall the integral lattice with 1 ohm branches, look at the variation in the power arising from δu_0 an infinitesimal change in u_0



$$\delta \sum_{i=1}^{4} \frac{(u_0 - u_i)^2}{2} = \sum_1^4 (u_0 - u_i)\,\delta u_0.$$ This vanish where $4u_0 = \sum_1^4 u_i$ or $u_0 = \frac{1}{4}\sum_{i=1}^4 u_i$

which looks like a discrete version of harmonic.
More generally consider an R-network K and look at $\delta(\text{Power})$ corresponding to δu_N for some node N.

$$\text{Power} = \frac{1}{2}\sum_b \frac{V_b^2}{R_b}, \quad \text{where } b \text{ runs over the branches.}$$

As u_N varies what terms in the power change? You only have to consider b joining N to a different node M_b. The contribution to the variation is

$$\delta\left(\frac{1}{2}\sum_{\substack{b \\ b:\, N\to M_b}} \frac{(u_N - u_{M_b})^2}{R_b}\right) = \left(\sum_b \frac{u_N - u_{M_b}}{R_b}\right)\delta u_N$$

Note that $\dfrac{u_N - u_{M_b}}{R_b} = I_b$ so $\dfrac{\delta(\text{power})}{\delta u_N} = \sum\limits_{b:\,N\to M_b} I_b$.

So you see that stationary power wrt δu_N is the same as the Kirchhoff node current condition at N.

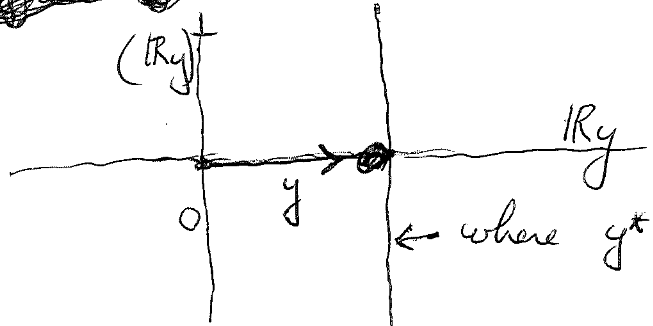
Next go back to Thevenin's idea of putting emfs in the branches. This means we can have voltage sources present without affecting the Kirchhoff conditions. Therefore we find that the power is stationary wrt variations δu_N for all nodes N iff u is harmonic iff the Kirchhoff node current condition holds.

This is nice, but you are still missing the Thev. equivalent resistance of a $K = {}^{A}_{O}\!\supset$ with an external node $A \neq$ ground O.

§5 You need to go over X Euclidean $y \in X$, $y^*: X \to \mathbb{R}$. $(\mathbb{R}y)^\perp = \mathrm{Ker}\{y^*: X \to \mathbb{R}\}$, $X = \mathbb{R}y \oplus \mathbb{R}y^\perp$

You want the norm^2 induced by push forward wrt y^*.

$x = \lambda y$



Try again. X with $\frac{1}{2}\|x\|^2$, $y \in X$, $y^*: X \to \mathbb{R}$

X splits into $\mathbb{R}y$ and $(\mathbb{R}y)^\perp = \mathrm{Ker}(y^*)$. Now let $c \neq 0$, $c \in \mathbb{R}$, and find the x such that $y^* x = c$ and $x \perp (\mathbb{R}y)^\perp$

i.e. $x = \lambda y$. $\Rightarrow$ $c = y^* x = y^* \lambda y$ $\therefore \lambda = \frac{c}{\|y\|^2}$

$x = \frac{cy}{\|y\|^2}$ and $\frac{1}{2}\|x\|^2 = \frac{c^2\|y\|^2}{2\|y\|^4} = \frac{c^2}{2\|y\|^2}$

~~Still~~ confused

$$\begin{array}{ccccc} \mathbb{R} & \xleftarrow{\rho^t} & \bar{C}^0 K & \overset{\delta}{\hookrightarrow} & C^1 K \\ & & \downarrow \Delta & & \downarrow R^{-1} \\ \mathbb{R} & \xrightarrow{\rho} & \bar{C}_0 K & \xleftarrow{\delta^t} & C_1 K \end{array}$$

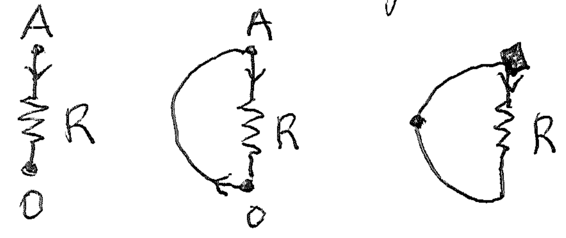
You want $u \in \bar{C}^0 K$ s.t. $\Delta u = \rho$, implies u harmonic away from $\mathrm{supp}(\rho)$.

The node potential corresponding to the node current ρ is $u = \Delta^{-1}\rho$, and the power function associated to this node potential and the node current (0-chain) $\rho = [A] - [0]$ should be $\frac{1}{2}\rho^t \Delta^{-1} \rho$. This checks.

So now you basically understand the Thevenin equivalent resistance, and you can look again at attaching a branch joining nodes $(A, 0)$. Start

$\eta 5$ with simplest case

[redacted]



K L M

[redacted] IDEAS to use:

<u>pushforward</u>

<u>Thevenin equivalent</u> R

[redacted]

$$\begin{tikzcd}
 & C^1L & \\
\bar{C}^0K \arrow[r, hook, "\delta"] \arrow[dr, "-\delta^t"'] & C^1K \arrow[r, two heads] \arrow[dr] & H^1K \\
 & \oplus & \\
 & R \arrow[r, hook] & H^1L \arrow[uu]
\end{tikzcd}$$

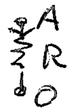
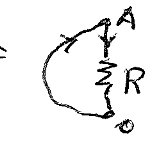
? ?

[redacted] [redacted] [redacted]

It seem that C^1L might be a pushout

$$\begin{tikzcd}
\bar{C}^0K \arrow[r, hook] & C^1K \arrow[r, two heads] & H^1K \\
\bar{C}^0L \arrow[u, equal] \arrow[r, hook] & C^1L \arrow[u, "\text{null}=1"'] \arrow[r, two heads] & H^1L \arrow[u, "\text{null}=1"']
\end{tikzcd}$$

$$\begin{tikzcd}
{} & & \\
\bar{C}^0L \arrow[u] \arrow[r, hook] & C^1L \arrow[r, two heads] & H^1L \\
\bar{C}^0M \arrow[u, hook, "\text{cod}=1"] \arrow[r, hook] \arrow[ur, phantom, "\text{pushout}"] & C^1M \arrow[u, hook, "\text{cod}=1"'] \arrow[r, two heads] & H^1M \arrow[u, equal]
\end{tikzcd}$$

Θ5 Look at L in the case K =  L = 

$$\tilde{C}^0 L \overset{\delta}{\hookrightarrow} C^1 L \twoheadrightarrow H^1 L$$

$$\{u_A\} \underset{\binom{1}{-1}}{\longmapsto} \begin{pmatrix} u_A \\ -u_A \end{pmatrix} = \begin{pmatrix} V_R \\ V_e \end{pmatrix} \xmapsto{(1\ 1)} V_R + V_e = 0$$

$$I_R - I_e = 0 \longleftarrow \begin{pmatrix} I_R \\ I_e \end{pmatrix} = \begin{pmatrix} I \\ I \end{pmatrix} \underset{\binom{1}{1}}{\longleftarrow\!\!\!\shortmid} I$$

$$\tilde{C}_0 L \xleftarrow{(1\ -1)} C_1 L \longleftrightarrow H_1 L$$

What are you writing? 4 variables V_R I_R V_e I_e

2 Kirchhoff constraints $V_R + V_e = 0$, $I_R = I_e$. Finally Ohm: $\begin{pmatrix} V_R \\ V_e \end{pmatrix} = \begin{pmatrix} R & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} I \\ I \end{pmatrix}$ yields 2 relations $V_R = RI_R$, $V_e = 0$.

These four equations have only the zero solution:

$V_e = 0 \implies V_R = 0 \implies I_R = 0 \implies I_e = 0$.

Is there a Hodge decomposition of some sort?

First notice that by linear algebra, when you have 4 linear homogeneous equations in 4 unknowns

$$V_R + V_e = 0$$
$$I_R - I_e = 0 \qquad \text{i.e.} \quad AX = 0$$
$$V_R - RI_R = 0$$
$$V_e = 0$$

having only the zero solution, then $AX = Y$ has a unique soln. $X = A^{-1}Y$ for each Y.

Does this amount to some kind of Hodge decomposition? Ideally you want a splitting of $C^1 L$ into $\delta(\tilde{C}^0 L)$ and an appropriate complement. But there's the degeneracy of the power form to contend with.

Past approach: put $\varepsilon > 0$ resistance on the e branch. The power is $\frac{1}{2}\left(\frac{V_R^2}{R} + \frac{V_e^2}{\varepsilon}\right)$. Alternately use the current picture, where the power is $\frac{1}{2}\left(R I_R^2 + \varepsilon I_e^2\right)$.

What should happen is that the power form on C^1L restricts to a nondegenerate form on H_1L, namely $\frac{1}{2}(R+\varepsilon)I^2$. The power on C^1L restricts to the form $\frac{1}{2}\left(\frac{u_A^2}{R} + \frac{(-u_A)^2}{\varepsilon}\right) = \frac{1}{2}\left(\frac{1}{R}+\frac{1}{\varepsilon}\right)u_A^2$, which looks problematic as $\varepsilon \downarrow 0$.

Review.

$$\tilde{C}^0L \hookrightarrow C^1L \twoheadrightarrow H^1L$$

$$u_A \xmapsto{\begin{pmatrix}1\\-1\end{pmatrix}} \begin{pmatrix}u_A\\-u_A\end{pmatrix} = \begin{pmatrix}V_R\\V_e\end{pmatrix} \xmapsto{(1\ \ 1)} V_R + V_e = 0$$

$$\tilde{C}_0L \twoheadleftarrow C_1L \hookleftarrow H_1L$$

$$0 = I_R - I_e \xleftarrow{(1\ -1)} \begin{pmatrix}I_R\\I_e\end{pmatrix} = \begin{pmatrix}I\\I\end{pmatrix} \xleftarrow{\begin{pmatrix}1\\1\end{pmatrix}} I$$

The Kirchhoff subspace is 2 diml. The fact that the Ohm + Kirchhoff spaces are transversal, both are 2 diml and the intersection is 0, means that you get a splitting of $C^1L \oplus C_1L$. Also you should get a positive definite form on the Kirchhoff spaces. This needs checking, understanding better.

Review: Begin with a phase space, i.e. the direct sum $\begin{pmatrix}C^1\\C_1\end{pmatrix}$ of a vector space and its dual. Consider the subspace given by the graph $\begin{pmatrix}1\\T\end{pmatrix}C^1$ of a linear map $T: C^1 \to C_1$. Such a T is

K5 . equivalent to a bilinear form x^tTy on C^1.

You have decided to review symplectic conventions concerning "phase space", that is, the direct sum $\begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$ of a vector space and its dual.
Consider a harmonic oscillator: $q \in C^1$ is position, $p \in C_1$ is momentum. P.E. $= \frac{1}{2} q^t k q$, K.E. $= \frac{1}{2} p^t m^{-1} p$,
Hamiltonian (function) $H = \frac{1}{2}\begin{pmatrix} q \\ p \end{pmatrix}^t \begin{pmatrix} k & 0 \\ 0 & m^{-1} \end{pmatrix} \begin{pmatrix} q \\ p \end{pmatrix}$, the Hamiltonian flow is

$$\begin{pmatrix} \dot q \\ \dot p \end{pmatrix} = \begin{pmatrix} \frac{\partial H}{\partial p} \\ -\frac{\partial H}{\partial q} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & m^{-1} \\ -k & 0 \end{pmatrix}}_{X\text{, the generator of Ham. flow}} \begin{pmatrix} q \\ p \end{pmatrix}$$

You propose to determine the sign of the symplectic form A by $AX = H$ i.e.

$$A \begin{bmatrix} 0 & m^{-1} \\ -k & 0 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & m^{-1} \end{bmatrix} \Longrightarrow A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Proof that the Ham. flow respects A and the symmetric form H: First, recall that an invertible operator g preserves a bilinear form $B: C^1 \longrightarrow C_1$ when $(g\xi)^t B (g\xi) = \xi^t g^t B g \xi$, i.e. $g^t B g = B$, infinitesimal $X^t B + BX = 0$. Now $H^t = H = AX$ $\Longrightarrow X^t A^t = H^t = H = AX$, so $X^t A + AX = 0$ as $A^t = -A$. Then $X^t H + HX = X^t A X + AXX = (X^t A + AX)X = 0$.
Thus X respects A and H. Note that interchanging q, p leads to

$$A \begin{bmatrix} 0 & -k \\ m^{-1} & 0 \end{bmatrix} = \begin{bmatrix} m^{-1} & 0 \\ 0 & k \end{bmatrix} \Longrightarrow A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

λ5

Example

$$\begin{tikzcd}
\bar{C}^0 \arrow[r, hook, "\begin{pmatrix}1\\-1\end{pmatrix}"] & C^1 \arrow[r, two heads, "(1\ \ 1)"] & H^1 \\
\bar{C}_0 & C_1 \arrow[l, two heads, "(1\ \ -1)"] \arrow[u, "\begin{pmatrix}R&0\\0&0\end{pmatrix}"'] & H_1 \arrow[l, hook', "\begin{pmatrix}1\\1\end{pmatrix}"']
\end{tikzcd}$$

$$\begin{pmatrix}V_R\\V_e\end{pmatrix} \longmapsto V_R+V_e=0$$

$$0 = I_R - I_e \longleftarrow\!\shortmid \begin{pmatrix}I_R\\I_e\end{pmatrix}$$ [redacted]

Simpler is:

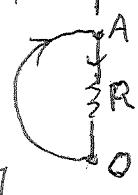
$$\begin{tikzcd}
\bar{C}^0 \arrow[r, hook, "\delta"] & C^1 \arrow[r, two heads, "\varepsilon^t"] & H^1 \\
\bar{C}_0 & C_1 \arrow[l, two heads, "\delta^t"'] \arrow[u, leftrightarrow, "\Gamma"] & H_1 \arrow[l, hook', "\varepsilon"']
\end{tikzcd}$$

Kirchhoff space $K = \bar{C}^0 \oplus H_1$ is a Lagrangian subspace of $C^1 \oplus C_1$. Why? The annihilator of $\delta\bar{C}^0 = C^1 \oplus \operatorname{Ker}\delta^t$, the annihilator of $\varepsilon H_1 = C_1 \oplus \operatorname{Ker}\varepsilon^t$, so the annihilator of $\delta\bar{C}^0 + \varepsilon H_1$ is $\operatorname{Ker}\delta^t \cap \operatorname{Ker}\varepsilon^t = K$.

$$\begin{tikzcd}
\bar{C}^0 \arrow[r, hook] & C^1 \arrow[r, two heads] & H^1 \\
\bar{C}_0 & C_1 \arrow[l, two heads] \arrow[u, leftrightarrow, "\Omega"] & H_1 \arrow[l, hook']
\end{tikzcd}$$

Ω = Ohm correspondence between C^1 and C_1

Example



$K: \left| \begin{array}{l} V_R + V_e = 0 \\ I_R - I_e = 0 \end{array}\right.$

$\Omega: \left| \begin{array}{l} V_R - RI_R = 0 \\ V_e \qquad = 0 \end{array}\right.$

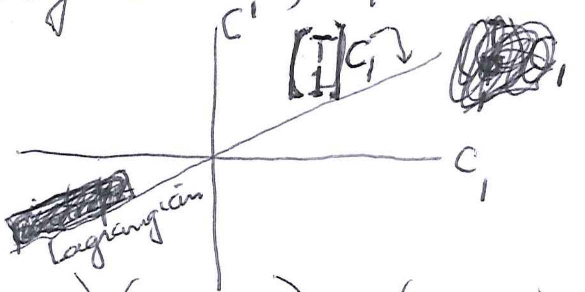
[redacted]

You have $K \cap \Omega = 0$ as $V_e = 0 \Rightarrow V_R = 0 \Rightarrow I_R = 0 \Rightarrow I_e = 0$. Therefore you have $K \oplus \Omega \xrightarrow{\sim} \begin{pmatrix}C^1\\C_1\end{pmatrix}$; [redacted] this splitting of the branch state space should [redacted] yield for every internal branch emf $\mathcal{E}$ the resulting [redacted] Kirchhoff state.

$\mu 5$ In the example  you have a state space of 4 variables V_R, V_e, I_R, I_e together with a Kirchhoff space $K = \bar{C}^0 \oplus H_1$ defined by $V_R + V_e = 0$, $I_R = I_e$ and an Ohm space Ω defined by $V_R = RI_R$, $V_e = 0$. One has $K \cap \Omega = 0$, so that K and Ω are complementary: $K \oplus \Omega = \begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$.

Question: What is significant about a splitting of a symplectic ~~vector~~ space into two Lagrangian subspaces? Such splittings should be related to symplectic isomorphisms. For example if you choose a linear isom $K \simeq C^1$ and the dual $C_1 \simeq K^\vee = \Omega$, then you get a symplectic autom. of $\begin{pmatrix} C^1 \\ C_1 \end{pmatrix}$ carrying $C^1 \simeq K$, $C_1 \simeq K^\vee = \Omega$.

Example.



Check $\begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix}$ is a symplectic autom for $T = T^t$

$$\begin{pmatrix} 1 & 0 \\ T^t & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ T^t & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & T \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & \underbrace{-T^t + T}_{0} \end{pmatrix}$$

Let's discuss the splitting in the example at the top.

$$\begin{tikzcd}
\bar{C}^0 \arrow[r, hook, "\delta = \binom{1}{-1}"] & C^1 \arrow[r, two heads, "\varepsilon^t = (1\ 1)"] & H^1 \\
 & \Omega \arrow[u] \arrow[d] & \\
H_1 \arrow[r, hook, "\varepsilon = \binom{1}{1}"] & C_1 \arrow[r, two heads, "\delta^t = (1\ -1)"] & \bar{C}_0
\end{tikzcd}$$

K

You know that Ω and K are complementary, and hence $\Omega \xrightarrow[\sim]{\begin{bmatrix} \varepsilon^t \\ \delta^t \end{bmatrix}} \begin{bmatrix} H^1 \\ \bar{C}_0 \end{bmatrix}$

For the purposes of Thevenin theory you want an emf $\mathcal{E}$ on the e branch, that is, an "input" $\begin{pmatrix} \mathcal{E} \\ 0 \end{pmatrix} \in C^1$. Let's see what it means to split $\begin{pmatrix} \mathcal{E} \\ 0 \end{pmatrix}$ into K and Ω components.

Given $\begin{pmatrix} \mathcal{E} \\ 0 \end{pmatrix} \in C^1$, move it to $(1\ 1)\begin{pmatrix} \mathcal{E} \\ 0 \end{pmatrix} = \mathcal{E} \in H^1$, choose $I \in H_1$ such that $(1\ 1)\begin{pmatrix} R & 0 \\ 0 & R_e \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} I = \mathcal{E}$, i.e. $I = \dfrac{\mathcal{E}}{R + R_e}$.

Next subtract from $\begin{pmatrix}\mathcal{E}\\0\end{pmatrix}$ the non conservative part arising from the current I:

$$\begin{pmatrix}\mathcal{E}\\0\end{pmatrix} - \begin{pmatrix}R\\R_e\end{pmatrix}\frac{\mathcal{E}}{R+R_e} = \mathcal{E}\left(\begin{bmatrix}1\\0\end{bmatrix} - \begin{bmatrix}R/R+R_e\\R_e/R+R_e\end{bmatrix}\right)$$

$$= \mathcal{E}\begin{bmatrix}R_e/R+R_e\\-R_e/R+R_e\end{bmatrix}$$

So it seems that one has the decomposition

$$\begin{pmatrix}\mathcal{E}\\0\end{pmatrix} = \underbrace{\begin{bmatrix}R_e\\-R_e\end{bmatrix}I}_{\in\bar{C}^0} + \begin{bmatrix}R\\R_e\end{bmatrix}I \quad , \quad I = \frac{\mathcal{E}}{R+R_e}$$

$$\underset{\sim}{R}\,I$$

this is the Ω part.

The nice thing here is that you can let $R_e \downarrow 0$, whence you get $\begin{bmatrix}\mathcal{E}\\0\end{bmatrix} = \begin{bmatrix}R\cdot I\\0\cdot I\end{bmatrix}$.

$$\begin{aligned}
\begin{bmatrix}1\\0\end{bmatrix} &= \begin{bmatrix}R\\R_e\end{bmatrix}\frac{1}{R+R_e} + \begin{bmatrix}R_e\\-R_e\end{bmatrix}\frac{1}{R+R_e} \longrightarrow \begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\0\end{bmatrix}\\
\underbrace{\begin{bmatrix}0\\1\end{bmatrix}}_{\in C^1} &= \underbrace{\begin{bmatrix}R\\R_e\end{bmatrix}\frac{1}{R+R_e}}_{\text{from } \Omega + H^1} + \underbrace{\begin{bmatrix}-R\\R\end{bmatrix}\frac{1}{R+R_e}}_{\bar{C}^0} \longrightarrow \begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}-1\\1\end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\begin{bmatrix}1\\0\end{bmatrix} &= \begin{bmatrix}R^{-1}\\-R_e^{-1}\end{bmatrix}\frac{1}{R^{-1}+R_e^{-1}} + \begin{bmatrix}R_e^{-1}\\R_e^{-1}\end{bmatrix}\frac{1}{R^{-1}+R_e^{-1}} \longrightarrow \begin{bmatrix}0\\-1\end{bmatrix} + \begin{bmatrix}1\\1\end{bmatrix}\\
\underbrace{\begin{bmatrix}0\\1\end{bmatrix}}_{\in C_1} &= \underbrace{\begin{bmatrix}-R^{-1}\\R_e^{-1}\end{bmatrix}\frac{1}{R^{-1}+R_e^{-1}}}_{\text{from } \Omega + \bar{C}_0} + \underbrace{\begin{bmatrix}R^{-1}\\R^{-1}\end{bmatrix}\frac{1}{R^{-1}+R_e^{-1}}}_{H_1} \longrightarrow \begin{bmatrix}0\\1\end{bmatrix} + \begin{bmatrix}0\\0\end{bmatrix}
\end{aligned}$$

Notice that in the cases $\begin{pmatrix}\mathcal{E}\\0\end{pmatrix}\in C^1$, $\begin{pmatrix}0\\\mathcal{J}\end{pmatrix}\in C_1$ the $R_e \downarrow 0$ limit is the same.

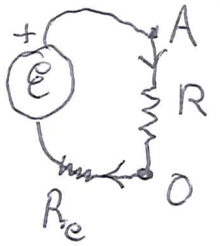
§5 Record for later reference the mistake (p 145) where you say that an emf $\mathcal{E}$ on the e branch corresponds to an "input" $\begin{pmatrix}\mathcal{E}\\0\end{pmatrix} \in C^1$. Because $C^1 = \left\{\begin{pmatrix}V_R\\V_e\end{pmatrix}\right\}$, the input should be $\begin{pmatrix}0\\\mathcal{E}\end{pmatrix}$.

Questions • What's interesting about two Lagrangian subspaces of a symplector vs S which are transversal?

The only thing I can see is that one gets a phase space picture for S i.e $S = K \oplus \Omega$, where either K or Ω can be taken to be position (or configuration) space and the other to be momentum. But there's still no dynamics yet.

In our situation where $S = \begin{bmatrix}C^1\\C_1\end{bmatrix}$, $K = \begin{bmatrix}\tilde{C}^0\\H_1\end{bmatrix}$, and Ω is the Ohm correspondence, then the splitting $S = K \oplus \Omega$ allows us to take any branch state $\begin{bmatrix}\mathcal{E}\\I\end{bmatrix} \in \begin{bmatrix}C^1\\C_1\end{bmatrix}$ and to project it onto a Kirchhoff state using the Ohm relations.

• Do you get the desired current response from an external emf applied between two nodes?



$$K: \left| \begin{array}{l} V_R + V_e = 0 \\ I_R = I_e \end{array}\right. \qquad \Omega: \left| \begin{array}{l} V_R = R I_R \\ V_e - R_e I_e = -\mathcal{E} \end{array}\right.$$

$$0 = V_R + V_e = (R + R_e) I - \mathcal{E}$$

$$\begin{bmatrix}0\\\mathcal{E}\end{bmatrix} = \begin{bmatrix}R_e\\R\end{bmatrix} I + \begin{bmatrix}-R_e\\R_e\end{bmatrix} I, \qquad I = \frac{\mathcal{E}}{R+R_e}, \text{ so if}$$

$$\begin{bmatrix}0\\1\end{bmatrix} = \underbrace{\begin{bmatrix}R_e\\R\end{bmatrix}\frac{1}{R+R_e}}_{\Omega \cdot I} + \underbrace{\begin{bmatrix}-R_e\\R_e\end{bmatrix}\frac{1}{R+R_e}}_{\tilde{C}^0}$$

you let $R_e \downarrow 0$, you get the desired current response in the e branch.

Note: the splitting ↑ has the limit as $R_e \uparrow \infty$: $\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}-1\\1\end{bmatrix}$

05 • Next question: Positivity and Power.

You know these are important when all branches have $R>0$. !

Observation: When $K\oplus\Omega \xrightarrow{\sim} \begin{bmatrix} C^1 \\ C_1 \end{bmatrix}$, since $K = \begin{bmatrix} \bar{C}^0 \\ H_1 \end{bmatrix}$ you get $\Omega \xrightarrow{\sim} \begin{bmatrix} H^1 \\ \bar{C}_0 \end{bmatrix}$.

So it seems that Ω has a canonical splitting into voltage and current components. You would like to have $\Omega = \begin{bmatrix} \Omega\cap C^1 \\ \Omega\cap C_1 \end{bmatrix}$.

Let's display the splitting $K\oplus\Omega = \begin{bmatrix} C^1 \\ C_1 \end{bmatrix}$ neatly

$$\begin{tikzcd}
\bar{C}^0 \arrow[r, hook, "\delta = \binom{1}{-1}"] & C^1 \arrow[r, two heads, "\varepsilon^t = (1\ 1)"] & H^1 \\
& \Omega \arrow[u] \arrow[d] & \\
H_1 \arrow[r, hook, "\varepsilon = \binom{1}{1}"] & C_1 \arrow[r, two heads, "\delta^t = (1\ -1)"] & \bar{C}_0
\end{tikzcd}$$

voltage side

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} R \\ R_e \end{bmatrix}\frac{1}{R+R_e} + \begin{bmatrix} R_e \\ -R_e \end{bmatrix}\frac{1}{R+R_e}$$

$$\underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{C^1} = \underbrace{\begin{bmatrix} R \\ R_e \end{bmatrix}\frac{1}{R+R_e}}_{C^1\cap\Omega} + \underbrace{\begin{bmatrix} -R \\ R \end{bmatrix}\frac{1}{R+R_e}}_{\bar{C}^0}$$

current side

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} R^{-1} \\ -R_e^{-1} \end{bmatrix}\frac{1}{R^{-1}+R_e^{-1}} + \begin{bmatrix} R_e^{-1} \\ R_e^{-1} \end{bmatrix}\frac{1}{R^{-1}+R_e^{-1}}$$

$$\underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{C_1} = \underbrace{\begin{bmatrix} -R^{-1} \\ R_e^{-1} \end{bmatrix}\frac{1}{R^{-1}+R_e^{-1}}}_{C_1\cap\Omega} + \underbrace{\begin{bmatrix} R^{-1} \\ R^{-1} \end{bmatrix}\frac{1}{R^{-1}+R_e^{-1}}}_{H_1}$$

It might help to add the current picture calculations. Begin with $\begin{bmatrix}1\\0\end{bmatrix} \in C_1$ move to $[1\ \ -1]\begin{bmatrix}1\\0\end{bmatrix} = 1 \in \bar{C}_0$, choose $u \in \bar{C}^0$ so that $[1\ \ -1]\begin{pmatrix}R^{-1} & 0\\ 0 & R_e^{-1}\end{pmatrix}\begin{bmatrix}1\\-1\end{bmatrix}u = (R^{-1}+R_e^{-1})u$ equals 1 $\Rightarrow$ $u = \dfrac{1}{R^{-1}+R_e^{-1}}$. Then remove

$$\begin{bmatrix}1\\0\end{bmatrix} - \begin{bmatrix}R^{-1} & 0\\ 0 & R_e^{-1}\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix} u = \begin{bmatrix}1\\0\end{bmatrix} - \begin{bmatrix}R^{-1}\\-R_e^{-1}\end{bmatrix}\frac{1}{R^{-1}+R_e^{-1}}$$

which gives the decomposition

$$\underbrace{\begin{bmatrix}1\\0\end{bmatrix}}_{C_1} = \underbrace{\begin{bmatrix}R^{-1}\\-R_e^{-1}\end{bmatrix}\frac{1}{R^{-1}+R_e^{-1}}}_{C_1 \cap \Omega} + \underbrace{\begin{bmatrix}R_e^{-1}\\R_e^{-1}\end{bmatrix}\frac{1}{R^{-1}+R_e^{-1}}}_{H_1}$$

At this point you understand a little better the decomposition of $\begin{bmatrix}C^1\\C_1\end{bmatrix}$ into $K \oplus \Omega$, but it's still messy.

Here's another approach. Begin with $\begin{bmatrix}C^1\\C_1\end{bmatrix}$ a 4 dim hyperbolic symplectic space and the Lagrangian subspace $\begin{bmatrix}\bar{C}^0\\H_1\end{bmatrix} = K$. Let's try to understand the possible Ω which are complementary to K. $\dim \Omega = 2$.

Q: Is it possible that a splitting $K \oplus \Omega \xrightarrow{\simeq} \begin{bmatrix}C^1\\C_1\end{bmatrix}$ with $K = \begin{bmatrix}\bar{C}^0\\H_1\end{bmatrix}$ yields a splitting of the voltage and the current short exact sequences?

IDEA: Is there an analogy in the symplectic theory of "retract", where instead of: $\begin{bmatrix}W_+\\W_-\end{bmatrix} \xleftarrow{\begin{bmatrix}\beta_+ & 0\\ 0 & \beta_-\end{bmatrix}} \begin{bmatrix}V\\V\end{bmatrix} \xleftarrow{\begin{bmatrix}\alpha_+ & 0\\ 0 & \alpha_-\end{bmatrix}} \begin{bmatrix}W_+\\W_-\end{bmatrix}$ a retract of a free $\mathbb{Z}/2$ module you have a retract of a hyperbolic space $\begin{bmatrix}V\\V^\vee\end{bmatrix}$?

p5 Start with the short exact voltage sequence ⊕ the dual short exact current; Then K is a Lagrangian subspace of the symplectic space S, and S/K is naturally isom to K^*.

$$\underbrace{\begin{bmatrix} \tilde{C}^0 \\ H_1 \end{bmatrix}}_{K} \hookrightarrow \underbrace{\begin{bmatrix} C^1 \\ C_1 \end{bmatrix}}_{S} \twoheadrightarrow \underbrace{\begin{bmatrix} H^1 \\ \tilde{C}_0 \end{bmatrix}}_{S/K}$$

You want to understand Lagrangian complements L to K.

Start again with a different notation. Consider a short exact sequence direct sum with the dual exact sequence arranged as follows:

$$\underbrace{\begin{bmatrix} A \\ C^* \end{bmatrix}}_{K} \hookrightarrow \underbrace{\begin{bmatrix} B \\ B^* \end{bmatrix}}_{S} \twoheadrightarrow \begin{bmatrix} C \\ A^* \end{bmatrix}$$

Thus we have a hyperbolic symplectic space S together with a special type of Lagrangian subspace K: compatible with the hyperbolic grading. We follow Lafforgue's convention of Roman + Greek letters: $\begin{bmatrix} b \\ \beta \end{bmatrix} \in \begin{bmatrix} B \\ B^* \end{bmatrix}$. The symplectic form on S is $\begin{bmatrix} b_1 \\ \beta_1 \end{bmatrix}^t \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} b_2 \\ \beta_2 \end{bmatrix} = -b_1^t\beta_2 + \beta_1^t b_2$. Let's check that K is isotropic. $\begin{bmatrix} b_1 \\ \beta_1 \end{bmatrix}, \begin{bmatrix} b_2 \\ \beta_2 \end{bmatrix} \in K$ means $\left(\begin{matrix} b_1 \\ b_2 \end{matrix}\right. \in A$ and $\left(\begin{matrix} \beta_1 \\ \beta_2 \end{matrix}\right. \in B^*$ is 0 on A. so it's clear. It should now follow that the symplectic form on S induces a nondegenerate pairing between K and $S/K = \begin{bmatrix} C \\ A^* \end{bmatrix}$. Let's use a wedge notation

$$\begin{bmatrix} b_1 \\ \beta_1 \end{bmatrix} \wedge \begin{bmatrix} b_2 \\ \beta_2 \end{bmatrix} = -b_1^t\beta_2 + \beta_1^t b_2$$ for the symplectic form on S.

Then following $\Lambda^2 B = A \otimes C$ in spirit, the induced pairing between K and S/K should be given by

$$\begin{bmatrix} a_1 \\ \gamma_1 \end{bmatrix} \wedge \begin{bmatrix} c_2 \\ \alpha_2 \end{bmatrix} = -a_1^t\alpha_2 + \gamma_1^t c_2$$

Clear,

σ5 Next project is to understand properly the Lagrangian complements Ω to K in S. You should know that these form an affine space whose associated vector space is the symmetric maps from K to S/K.

Let's consider a special type of Ω, where the hyperbolic structure, i.e. grading into voltage and current, is preserved. This means that you lift C into B and lift A^* into B^* subject to the condition that these lifts $\tilde{C}$, $\tilde{A^*}$ are orthogonal for the symplectic form.

Let
$$\underbrace{\begin{bmatrix} A & C \\ C^* & A^* \end{bmatrix}}_{S \;=\; K \quad \Lambda}$$
be the hyperbolic symplectic space with the symplectic form

$$\begin{bmatrix} a_1 & c_1 \\ \gamma_1 & \alpha_1 \end{bmatrix} \wedge \begin{bmatrix} a_2 & c_2 \\ \gamma_2 & \alpha_2 \end{bmatrix} = \begin{matrix} -a_1^t\alpha_2 - c_1^t\gamma_2 \\ +\gamma_1^t c_2 + \alpha_1^t a_2 \end{matrix}$$

There's an equivalence between Lagrangian subspaces Ω of S complementary to K and graphs of symmetric maps $K \leftarrow \Lambda$. Look at the maps preserving the horizontal grading $A \xleftarrow{u} C$, $C^* \xleftarrow{v} A^*$. The graph of this map consists of elements

$$\begin{bmatrix} u(c_1) & c_1 \\ v(\alpha_1) & \alpha_1 \end{bmatrix} \wedge \begin{bmatrix} u(c_2) & c_2 \\ v(\alpha_2) & \alpha_2 \end{bmatrix} = \begin{matrix} -u(c_1)^t\alpha_2 \\ -c_1^t v(\alpha_2) \\ +v(\alpha_1)^t c_2 \\ +\alpha_1^t u(c_2) \end{matrix} \left| \begin{matrix} -c_1^t u^t \alpha_2 \\ -c_1^t v \alpha_2 \\ +\alpha_1^t v^t c_2 \\ +\alpha_1^t u c_2 \end{matrix} \right.$$

$$u(c_1)^t\alpha_1 = (uc_1)^t\alpha_1 = c_1^t u^t \alpha_1 \quad \text{etc.}$$

So you find that the graph is Lagrangian iff $u^t + v = 0$. Now you want to know that this is equivalent to u, v being symmetric

I5

$$\begin{bmatrix} B \\ B^* \end{bmatrix} = \begin{bmatrix} A \\ C^* \end{bmatrix} \oplus \begin{bmatrix} C \\ A^* \end{bmatrix}$$

$$S = K \oplus \Lambda$$

S is a hyperbolic symplectic v.s. when equipped with the skew symmetric pairing: denoted by $\wedge$

$$\begin{bmatrix} b_1 \\ \beta_1 \end{bmatrix} \wedge \begin{bmatrix} b_2 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}^t \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} b_2 \\ \beta_2 \end{bmatrix} = b_1^t \beta_2 - b_2^t \beta_1$$

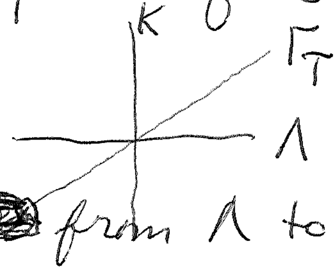
Thus in terms of the 4 components of S one has

$$\underbrace{\begin{bmatrix} a_1 + c_1 \\ \gamma_1 + \alpha_1 \end{bmatrix} \wedge \begin{bmatrix} a_2 + c_2 \\ \gamma_2 + \alpha_2 \end{bmatrix}}_{\|} = (a_1 + c_1)^t(\gamma_2 + \alpha_2) - (\gamma_1 + \alpha_1)^t(a_2 + c_2)$$
$$= a_1^t \alpha_2 + c_1^t \gamma_2 - \gamma_1^t c_2 - \alpha_1^t a_2$$

$$\begin{bmatrix} a_1 \\ \gamma_1 \end{bmatrix} \wedge \begin{bmatrix} c_2 \\ \alpha_2 \end{bmatrix} + \begin{bmatrix} c_1 \\ \alpha_1 \end{bmatrix} \wedge \begin{bmatrix} a_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} a_1 \\ \gamma_1 \end{bmatrix} \wedge \begin{bmatrix} c_2 \\ \alpha_2 \end{bmatrix} - \begin{bmatrix} a_2 \\ \gamma_2 \end{bmatrix} \wedge \begin{bmatrix} c_1 \\ \alpha_1 \end{bmatrix}$$

Note the 2×2 determinant pattern.

Next we determine the Lagrangian subspaces of S which are complementary to K. By:



subspaces complementary to K are the graphs Γ_T of maps T from Λ to K:

$$\begin{bmatrix} A \\ C^* \end{bmatrix} \xleftarrow{T = \begin{bmatrix} u & u' \\ v' & v \end{bmatrix}} \begin{bmatrix} C \\ A^* \end{bmatrix}.$$

But Γ_T Lagrangian should be equivalent to T symmetric, provided you identify K with the dual of Λ appropriately u5 The basic pairing between K and Λ should be

$$* \quad \begin{bmatrix} a \\ \gamma \end{bmatrix} \wedge \begin{bmatrix} c \\ \alpha \end{bmatrix} = \begin{bmatrix} a \\ \gamma \end{bmatrix}^t \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} c \\ \alpha \end{bmatrix} = a^t\alpha - \gamma^t c$$

Let $T : \begin{bmatrix} A \\ C^* \end{bmatrix} \xleftarrow{\begin{bmatrix} u & u' \\ v' & v \end{bmatrix}} \begin{bmatrix} C \\ A^* \end{bmatrix}$. You want to understand when T is symmetric wrt the pairing above, that is, when

$$\begin{bmatrix} c' \\ \alpha' \end{bmatrix}^t \cancel{\begin{bmatrix} a \\ \gamma \end{bmatrix}^t} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} u & u' \\ v' & v \end{bmatrix} \begin{bmatrix} c \\ \alpha \end{bmatrix} = \begin{bmatrix} c' \\ \alpha' \end{bmatrix}^t \begin{bmatrix} v' & v \\ -u & -u' \end{bmatrix} \begin{bmatrix} c \\ \alpha \end{bmatrix}$$

is equal to $\begin{bmatrix} c' \\ \alpha' \end{bmatrix}^t \begin{bmatrix} v'^t & -u^t \\ v^t & -u'^t \end{bmatrix} \begin{bmatrix} c \\ \alpha \end{bmatrix}$ YES.

$$T : \begin{tikzcd} A & C \arrow[l, "u"'] \arrow[dl, "v'"] \\ C^* & A^* \arrow[l, "v"] \arrow[ul, "u'"'] \end{tikzcd} \qquad \Gamma_T \ni \begin{bmatrix} uc + u'\alpha & c \\ v'c + v\alpha & \alpha \end{bmatrix}$$

$$\begin{bmatrix} uc_1 + u'\alpha_1 & c_1 \\ v'c_1 + v\alpha_1 & \alpha_1 \end{bmatrix} \wedge \begin{bmatrix} uc_2 + u'\alpha_2 & c_2 \\ v'c_2 + v\alpha_2 & \alpha_2 \end{bmatrix}$$

$$= (uc_1)^t\alpha_2 + (u'\alpha_1)^t\alpha_2 + c_1^t v'c_2 + c_1^t v\alpha_2$$
$$-(v'c_1)^t c_2 - (v\alpha_1)^t c_2 - \alpha_1^t u c_2 - \alpha_1^t u'\alpha_2$$

$$= c_1^t \overset{3}{u^t}\alpha_2 + \alpha_1^t \overset{1}{u'^t}\alpha_2 + c_1^t \overset{2}{v'} c_2 + c_1^t \overset{3}{v}\alpha_2$$
$$- c_1^t \underset{2}{v'^t} c_2 - \alpha_1^t \underset{4}{v^t} c_2 - \alpha_1^t \underset{4}{u} c_2 - \alpha_1^t \underset{1}{u'}\alpha_2$$

1. $u'^t = u'$
2. $v'^t = v'$
3. $u^t + v = 0$
4. $v^t + u = 0$

So Γ_T isotropic means $\begin{bmatrix} v' & v \\ -u & -u' \end{bmatrix}$ symmetric

But Γ_T isotropic should be equivalent to T symmetric which means that you take two elements $\begin{bmatrix} c_1 \\ \alpha_1 \end{bmatrix}, \begin{bmatrix} c_2 \\ \alpha_2 \end{bmatrix}$ apply T to the second and interpret the first as a linear functional on $\begin{bmatrix} A \\ C^* \end{bmatrix}$ using the symplectic form $\wedge$.

$$\begin{bmatrix} c_1 \\ \alpha_1 \end{bmatrix} \wedge \begin{bmatrix} u & u' \\ v' & v \end{bmatrix} \begin{bmatrix} c_2 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} c_1 \\ \alpha_1 \end{bmatrix}^t \begin{bmatrix} 0 & +1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} u & u' \\ v' & v \end{bmatrix} \begin{bmatrix} c_2 \\ \alpha_2 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 \\ \alpha_1 \end{bmatrix}^t \begin{bmatrix} v' & v \\ -u & -u' \end{bmatrix} \begin{bmatrix} c_2 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} c_2 \\ \alpha_2 \end{bmatrix}^t \begin{bmatrix} v'^t & -u^t \\ v^t & -u'^t \end{bmatrix} \begin{bmatrix} c_1 \\ \alpha_1 \end{bmatrix}$$

which indeed is symmetric under $1 \leftrightarrow 2$ iff $\begin{bmatrix} v' & v \\ -u & -u' \end{bmatrix}$ is symmetric.

Look at the special case where $T = \begin{bmatrix} u & u' \\ v' & v \end{bmatrix}$ respects the voltage-current grading, that is $u' = 0$ and $v' = 0$. Then

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} u & 0 \\ 0 & v \end{bmatrix} = \begin{bmatrix} 0 & v \\ -u & 0 \end{bmatrix} = \begin{bmatrix} 0 & v \\ -u & 0 \end{bmatrix}^t$$

iff $-u = v^t$. In this case

$$T : \begin{bmatrix} A \\ C^* \end{bmatrix} \xleftarrow{\begin{bmatrix} u & 0 \\ 0 & -u^t \end{bmatrix}} \begin{bmatrix} C \\ A^* \end{bmatrix}$$

Review the problem: You consider an abstract version of a network consisting of a short exact sequence of f.d. $\mathbb{R}$ v.s. and the dual sequence:

$$A \hookrightarrow B \twoheadrightarrow C$$

$$C^* \hookrightarrow B^* \twoheadrightarrow A^*$$

Put $S = \begin{bmatrix} B \\ B^* \end{bmatrix}$, $K = \begin{bmatrix} A \\ C^* \end{bmatrix}$, $S/K = \begin{bmatrix} C \\ A^* \end{bmatrix}$. S is the hyperbolic symplectic space associated to the v.s. V with skew-form $\begin{bmatrix} b_1 \\ \beta_1 \end{bmatrix} \wedge \begin{bmatrix} b_2 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ \beta_1 \end{bmatrix}^t \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} b_2 \\ \beta_2 \end{bmatrix} = b_1^t \beta_2 - \beta_1^t b_2$.

X5 $\quad K$ is a Lagrangian subspace S and the quotient S/K is canon. isom. to K^*.

Next you need a version of the Ohm's Law relations between voltage + current for the branches. The typical example is a positive symmetric operator $R^{-1}: B \xrightarrow{\sim} B^*$, but you want to handle degenerate situations.

Now the graphs $\begin{bmatrix} R^{-1} \\ 1 \end{bmatrix} B = \begin{bmatrix} 1 \\ R \end{bmatrix} B^*$ coincide and yield a Lagrangian subspace Ω of S, which is complementary to K.

The appropriate substitute for the Ohm relations seems to be a Lagrangian subspace $\Omega \subset S$ such that $\Omega \hookrightarrow S \longrightarrow S/K$ is an isom. Then you get $S = K \oplus \Omega$, a kind of Hodge decomposition.

You need to understand such an Ω better.

▶ Once you choose a "basepoint" Lagrangian complement Λ to K: $S = K \oplus \Lambda$, then another one Ω is specified by an operator $T: \Lambda \longrightarrow K$ which is symmetric in a suitable sense.

Go back to our starting point:

$$\begin{bmatrix} A \\ C^* \end{bmatrix} \hookrightarrow \begin{bmatrix} B \\ B^* \end{bmatrix} \twoheadrightarrow \begin{bmatrix} C \\ A^* \end{bmatrix}$$

$$K \qquad\qquad S \qquad\qquad S/K \simeq K^*$$

IDEA: Is there anything cohomologically interesting about the choice of basepoint, mod 2 maybe because of the symmetry given by $*$:

$$K \hookrightarrow S \twoheadrightarrow K^*$$

$$K \hookrightarrow S^* \twoheadrightarrow K^* \quad ?$$

Go back to the starting point:

$$\begin{bmatrix} A \\ C^* \end{bmatrix} \hookrightarrow \begin{bmatrix} B \\ B^* \end{bmatrix} \twoheadrightarrow \begin{bmatrix} C \\ A^* \end{bmatrix}$$

$$K \qquad\qquad S \qquad\qquad S/K = K^*$$

Aim: To understand Lagrangian $\Omega \subset S$ s.t. Ω is a complement to K: $S = K \oplus \Omega$.
There should be an important class of such Ω, namely those $\Omega \subset \begin{bmatrix} B \\ B^* \end{bmatrix}$ which respect the voltage-current grading, i.e. $\Omega = \begin{bmatrix} \Omega \cap B \\ \Omega \cap B^* \end{bmatrix}$. Such an Ω should arise by choosing a splitting of the s.e.s. $A \hookrightarrow B \twoheadrightarrow C$, i.e. a lifting $\tilde{C} \subset B$ of C, and then $\tilde{C} = \Omega \cap B$. Using duality the splitting chosen should yield a splitting of the dual s.e.s., i.e. a lifting $\widetilde{A^*} \subset B^*$ of A^*, and then $\widetilde{A^*} = \Omega \cap B^*$.

Let's fix a homogeneous Ω as above, call it Λ, and let's work with the corresponding splitting

$$S = K \oplus \Lambda = \begin{bmatrix} A \\ C^* \end{bmatrix} \oplus \begin{bmatrix} C \\ A^* \end{bmatrix}$$

The homogeneous Ω to consider are graphs of operators $\begin{bmatrix} A \\ C^* \end{bmatrix} \xleftarrow{\begin{bmatrix} 0 & u' \\ v' & 0 \end{bmatrix}} \begin{bmatrix} C \\ A^* \end{bmatrix}$ satisfying $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & u' \\ v' & 0 \end{bmatrix} = \begin{bmatrix} v' & 0 \\ 0 & -u' \end{bmatrix}$ is symmetric

i.e. $u' : A \leftarrow A^*$ and $v' : C^* \leftarrow C$ symmetric.

So it seems now that you have a complete analogue of the picture occurring in a R-network with >0 resistances for the branches. Namely, any Ω complementary to K will yield compatible splittings of the voltage and current s.e.s.

$$\begin{tikzcd} A \arrow[r, hook] & B \arrow[l, dashed, bend right] \arrow[r, two heads] & C \arrow[l, dashed, bend right] \end{tikzcd}$$

$$\begin{tikzcd} C^* \arrow[r, hook] & B^* \arrow[l, dashed, bend left] \arrow[r, two heads] & A^* \arrow[l, dashed, bend left] \end{tikzcd}$$

You also seem to get symmetric maps

$$C^* \xleftarrow{v'} C \;, \qquad A \xleftarrow{u'} A^*$$

which means Lagrangian subspaces in the hyperbolic spaces $\begin{bmatrix} C \\ C^* \end{bmatrix}$ and $\begin{bmatrix} A \\ A^* \end{bmatrix}$. ◎

IDEA. In studying
$$\underset{K}{\begin{bmatrix} A \\ C^* \end{bmatrix}} \hookrightarrow \underset{S}{\begin{bmatrix} B \\ B^* \end{bmatrix}} \twoheadrightarrow \underset{S/K = K^*}{\begin{bmatrix} C \\ A^* \end{bmatrix}}$$
you can choose a pos. def. scalar product on B. Then you have $B \xrightarrow{\sim} B^*$, $b \mapsto b^t$, and induced isos. $A \xrightarrow{\sim} A^*$, $\boxed{A^\perp = C} = C^*$. So you have a Euclidean picture

$$\begin{bmatrix} B \\ B \end{bmatrix} = \begin{bmatrix} A \\ C \end{bmatrix} \oplus \begin{bmatrix} C \\ A \end{bmatrix} \qquad S = K \oplus K^*$$

where the skew form is given by a skew symmetric operator in "4 dimensions".

6α Return to our abstract network

$$\begin{bmatrix} A \\ C^* \end{bmatrix} \hookrightarrow \begin{bmatrix} B \\ B^* \end{bmatrix} \twoheadrightarrow \begin{bmatrix} C \\ A^* \end{bmatrix}$$

$$K \qquad\qquad S \qquad\qquad S/K = K^*$$

and suppose given a splitting compatible with duality, so that

$$\begin{bmatrix} B \\ B^* \end{bmatrix} = \begin{bmatrix} A \\ C^* \end{bmatrix} \oplus \begin{bmatrix} C \\ A^* \end{bmatrix}$$

$$S = K \oplus K^*$$

So S has two obvious splittings into complementary Lagrangian subspaces. Review the formula for the symplectic form on S

$$\begin{bmatrix} b_1 \\ \beta_1 \end{bmatrix} \wedge \begin{bmatrix} b_2 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ \beta_1 \end{bmatrix}^t \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} b_2 \\ \beta_2 \end{bmatrix} = b_1^t \beta_2 - \beta_1^t b_2$$

$$\parallel$$

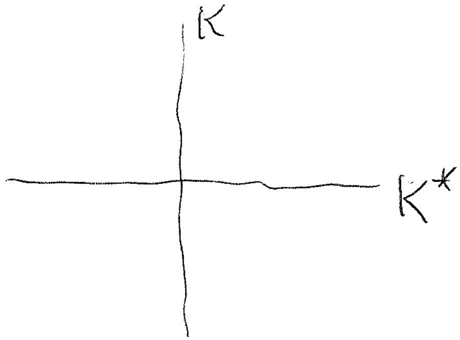
$$\begin{bmatrix} a_1 + c_1 \\ \gamma_1 + \alpha_1 \end{bmatrix} \wedge \begin{bmatrix} a_2 + c_2 \\ \gamma_2 + \alpha_2 \end{bmatrix} = \begin{bmatrix} a_1 \\ \gamma_1 \end{bmatrix} \wedge \begin{bmatrix} c_2 \\ \alpha_2 \end{bmatrix} + \begin{bmatrix} c_1 \\ \alpha_1 \end{bmatrix} \wedge \begin{bmatrix} a_2 \\ \gamma_2 \end{bmatrix}$$

$$= a_1^t \alpha_2 - \gamma_1^t c_2 + c_1^t \gamma_2 - \alpha_1^t a_2$$

$$= \begin{bmatrix} a_1^t & \gamma_1^t & c_1^t & \alpha_1^t \end{bmatrix} \begin{bmatrix} & & & 1 \\ & & -1 & \\ & 1 & & \\ -1 & & & \end{bmatrix} \begin{bmatrix} a_2 \\ \gamma_2 \\ c_2 \\ \alpha_2 \end{bmatrix}$$

Here you are writing

$$S = \begin{bmatrix} K \\ K^* \end{bmatrix} = \begin{bmatrix} A \\ C^* \\ C \\ A^* \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} A \\ C^* \end{matrix}}\right\} K \\ \left.\vphantom{\begin{matrix} C \\ A^* \end{matrix}}\right\} K^* \end{matrix}$$



6β   You can also write

$$S = \begin{bmatrix} B \\ B^* \end{bmatrix} = \begin{bmatrix} A \\ C \\ C^* \\ A^* \end{bmatrix}$$

$$b_1^t \beta_2 - \beta_1^t b_2 = \left[a_1^t \; c_1^t \; \gamma_1^t \; \alpha_1^t \right] \left[\begin{array}{cc|cc} & & & 1 \\ & & 1 & \\ \hline & -1 & & \\ -1 & & & \end{array}\right] \begin{bmatrix} a_2 \\ c_2 \\ \gamma_2 \\ \alpha_2 \end{bmatrix}$$

IDEA. This "4 dim'l" picture reminds you about the two hermitian forms arising in the wave equation.

At the moment you need to work on your conjecture in the case of the symplectic space $S = \begin{bmatrix} B \\ B^* \end{bmatrix}$ and the Lagrangian subspace $K = \begin{bmatrix} A \\ C^* \end{bmatrix}$, that a complementary Lagrangian subspace Ω to K is equivalent to the following:

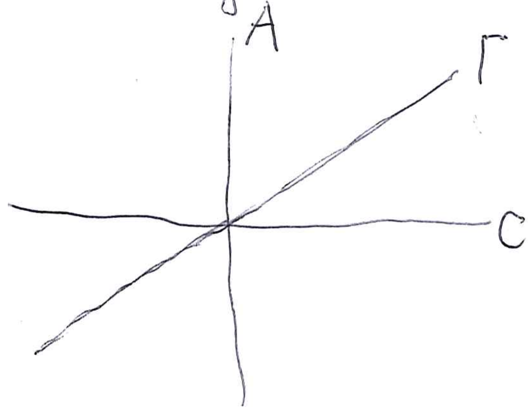
1) Splittings of the voltage and current s.e.s compatible with duality:

$$\begin{bmatrix} B \\ B^* \end{bmatrix} = \begin{bmatrix} A \\ C^* \end{bmatrix} \oplus \begin{bmatrix} C \\ A^* \end{bmatrix}$$

2) Symmetric maps $C^* \leftarrow C$, $A \leftarrow A^*$.

Let's begin with the (voltage) s.e.s. together with a splitting

$$\begin{tikzcd} & \Gamma \arrow[d, hook] \arrow[dr, "\sim"] & \\ A \arrow[r, hook] \arrow[dr, "\sim"'] & B \arrow[r, two heads] \arrow[d] & C \\ & \Delta & \end{tikzcd}$$



 Now dualize to get the (current) s.e.s. with the splitting:

$$\begin{tikzcd}
 & \Delta^* \arrow[d, hook] \arrow[dr, "\sim"] & \\
C^* \arrow[r, hook] \arrow[dr, "\sim"'] & B^* \arrow[r, two heads] \arrow[d] & A^* \\
 & \Gamma^* &
\end{tikzcd}$$

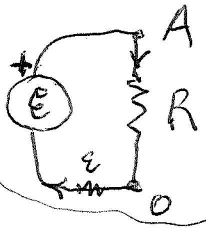
You would like to put these two splitting diagrams together

$$\begin{tikzcd}
 & \begin{bmatrix} \Gamma \\ \Delta^* \end{bmatrix} \arrow[d, hook] \arrow[dr, "\sim"] & \\
\begin{bmatrix} A \\ C^* \end{bmatrix} \arrow[r, hook] \arrow[dr, "\sim"'] & \begin{bmatrix} B \\ B^* \end{bmatrix} \arrow[r, two heads] \arrow[d] & \begin{bmatrix} C \\ A^* \end{bmatrix} \\
 & \begin{bmatrix} \Delta \\ \Gamma^* \end{bmatrix} &
\end{tikzcd}$$

What does this mean? You have the hyperbolic space of B with the complementary Lagrangian subspaces $\begin{bmatrix} A \\ C^* \end{bmatrix}$ and $\begin{bmatrix} \Gamma \\ \Delta^* \end{bmatrix}$.

So far you have respected the voltage-current grading, but you should be able to add symmetric maps $C^* \leftarrow C$, $A \leftarrow A^*$?

68 Now you should go back to the [redacted] example where you've adjoined [redacted] a pure emf branch. Review the equations.



There are 2 edges hence 4 variables V_R, V_e, I_R, I_e subject to Kirchhoff conditions $V_R + V_e = 0$, $I_R = I_e$ and two [redacted] Ohm's Law conditions. For the R edge you have $V_R = RI_R$, but the e edge might cause problems.

To proceed put in a small resistance $R_e = \varepsilon$ on the e edge. Then Ohm says: $\mathcal{E}$ [redacted] $= \varepsilon I + RI$. The problem now is to understand the [redacted] Ohm[redacted] conditions [redacted] using Lagrangian subspaces.

[redacted]

How do you handle things? It's probably a bad idea to introduce ε. Instead [redacted] consider the voltage + current s.e.s.

$$\bar{C}^0 \hookrightarrow C^1 \twoheadrightarrow H^1$$

$$H_1 \hookrightarrow C_1 \twoheadrightarrow \bar{C}_0$$

You need a Lagrangian complement Ω to the Kirchhoff space K. [redacted] A branch with [redacted] resistance R is described by a [redacted] hyperbolic plane $\left\{\begin{bmatrix} V \\ I \end{bmatrix}\right\}$ equipped with the Lagrangian subspace $V = RI$. This makes sense for $R = 0$ and $R = \infty$, but you

6ε need to make $\Omega \cap K = 0$.

The equations for K are $V_R + V_e = 0$, $I_R = I_e$ and and for Ω are $V_R = RI_R$, $V_e = 0$. $\therefore$ only the zero solution, so $\Omega \cap K = 0$.

Next you want the corresponding splitting of $S = \begin{bmatrix} C^1 \\ C_1 \end{bmatrix} = K \oplus \Omega$

$$\begin{tikzcd}
\bar{C}^0 \arrow[r, hook, "\begin{bmatrix} 1 \\ -1 \end{bmatrix}"] & C^1 \arrow[r, two heads, "\begin{bmatrix} 1 & 1 \end{bmatrix}"] & H^1 \\
\varphi \arrow[r, mapsto] & \begin{bmatrix} 1 \\ -1 \end{bmatrix}\varphi = \begin{bmatrix} V_R \\ V_e \end{bmatrix} \arrow[r, mapsto] & V_R + V_e \\
H_1 \arrow[r, hook, "\begin{bmatrix} 1 \\ 1 \end{bmatrix}"] & C_1 \arrow[uu, bend right, "\begin{bmatrix} R & 0 \\ 0 & 0 \end{bmatrix}"'] \arrow[r, two heads, "\begin{bmatrix} 1 & -1 \end{bmatrix}"] & \bar{C}_0 \\
I \arrow[r, mapsto] & \begin{bmatrix} 1 \\ 1 \end{bmatrix} I = \begin{bmatrix} I_R \\ I_e \end{bmatrix} \arrow[r, mapsto] & I_R - I_e
\end{tikzcd}$$

Now you want to take $\begin{bmatrix} 0 \\ -\varepsilon \end{bmatrix} \in C^1$ and split it into K and Ω components. Actually you should perhaps take $\begin{bmatrix} 0 \\ -\varepsilon \\ 0 \\ 0 \end{bmatrix}$ in $\begin{bmatrix} C^1 \\ C_1 \end{bmatrix}$. You push this into $\begin{bmatrix} -\varepsilon \\ 0 \end{bmatrix} \in \begin{bmatrix} H^1 \\ \bar{C}^0 \end{bmatrix}$. To find $\omega \in \Omega$ with the same image in $\begin{bmatrix} H^1 \\ \bar{C}^0 \end{bmatrix}$. This means finding I such that $\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} R & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} I =$

$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} RI \\ 0 \end{bmatrix} = RI$ is equal to $-\varepsilon$.

So $I = \dfrac{-\varepsilon}{R}$ and $\begin{bmatrix} 0 \\ -\varepsilon \end{bmatrix} - \begin{bmatrix} R & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ I \end{bmatrix} = \begin{bmatrix} \overset{+\varepsilon}{-RI} \\ -\varepsilon \end{bmatrix}$, which is the image of a $\varphi \in \bar{C}^0$. (Perhaps ε should be $-\varepsilon$ above) Yes.

6.5 What you want here is the splittings of the two s.e.s. and also quadratic forms on H^1 and $\bar{C}^0$. (So far you have split $\begin{bmatrix}0\\-\mathcal{E}\end{bmatrix}$ into $\begin{bmatrix}R & 0\\0 & 0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\underbrace{I}_{= -\frac{\mathcal{E}}{R}} + \begin{bmatrix}\mathcal{E}\\-\mathcal{E}\end{bmatrix} = \begin{bmatrix}-\mathcal{E}\\0\end{bmatrix} + \begin{bmatrix}\mathcal{E}\\-\mathcal{E}\end{bmatrix}$)

This is confusing. You need an intelligent way to handle the splitting $S = K \oplus \Omega$. You know that Ω amounts to a homogeneous splitting wrt the voltage-current grading together with quadratic forms on the "wings".

Picture

$$\begin{tikzcd}
 & & H_1 \arrow[dl, "{\begin{bmatrix}R\\0\end{bmatrix}}"'] \arrow[dr, "R", "\wr"'] & \\
\bar{C}^0 \arrow[r, hook, "{\begin{bmatrix}1\\-1\end{bmatrix}}"] & C^1 \arrow[rr, two heads, "{[1\ 1]}"] & & H^1 \\
H_1 \arrow[r, hook, "{\begin{bmatrix}1\\1\end{bmatrix}}"] \arrow[d, "R"', "\wr"] & C_1 \arrow[u, "{\begin{bmatrix}R & 0\\0 & 0\end{bmatrix}}"'] \arrow[rr, two heads, "{[1\ -1]}"] \arrow[dl, "{[R\ 0]}"] & & \bar{C}_0 \\
H^1 & & &
\end{tikzcd}$$

Notice that in this situation you get a splitting of the voltage s.e.s. given by $I \longmapsto \begin{bmatrix}1\\1\end{bmatrix} I \longmapsto \begin{bmatrix}R & 0\\0 & 0\end{bmatrix}\begin{bmatrix}I\\I\end{bmatrix} = \begin{bmatrix}RI\\0\end{bmatrix}$ from H_1 to C^1. There doesn't seem to be a similar map $\bar{C}^0 \longrightarrow C_1$, however, there is a unique splitting of the current s.e.s. compatible with the voltage splitting and duality.

Notice that if $V \in H^1$, then $V \mapsto \begin{bmatrix}R\\0\end{bmatrix} R^{-1} V = \begin{bmatrix}V\\0\end{bmatrix} \in C^1$ gives the desired lifting of H^1 into C^1. Similarly

$\underset{[1\ 0]}{\underset{\shortparallel}{R^{-1}[R\ 0]}} : C_1 \longrightarrow H^1 \longrightarrow H_1$ is a retract of C_1 onto H_1

$\begin{bmatrix}I_R\\I_e\end{bmatrix} \mapsto R I_R \mapsto I_R = I$

6η

[redacted] So the picture of the splittings of the voltage + current splittings becomes

$$
\begin{tikzcd}
 & H^1 \arrow[d, "{\begin{bmatrix}1\\0\end{bmatrix}}"] \arrow[dr, "1", "\sim"'] & \\
\bar{C}^0 \arrow[r, hook, "{\begin{bmatrix}1\\-1\end{bmatrix}}"] & C^1 \arrow[r, two heads, "{\begin{bmatrix}1 & 1\end{bmatrix}}"] & H^1 \\
H_1 \arrow[r, hook, "{\begin{bmatrix}1\\1\end{bmatrix}}"] \arrow[dr, "1"', "\sim"] & C_1 \arrow[u, "{\begin{bmatrix}R & 0\\0 & 0\end{bmatrix}}"'] \arrow[r, two heads, "{\begin{bmatrix}1 & -1\end{bmatrix}}"] \arrow[d, "{\begin{bmatrix}1 & 0\end{bmatrix}}"'] & \bar{C}_0 \\
 & H_1 &
\end{tikzcd}
$$

Unfortunately the voltage + current splittings described here are not expressed in the same form - for voltage you have a subspace, for current you have a quotient space.

Better:

$$
\begin{tikzcd}
 & \Gamma \arrow[d, "{\begin{bmatrix}1\\0\end{bmatrix}}"'] \arrow[dr, "1", "\sim"'] & \\
\bar{C}^0 \arrow[r, hook, "{\begin{bmatrix}1\\-1\end{bmatrix}}"] \arrow[dr, "1"', "\sim"] & C^1 \arrow[r, two heads, "{\begin{bmatrix}1 & 1\end{bmatrix}}"] \arrow[d, "{\begin{bmatrix}0 & -1\end{bmatrix}}"] & H_1 \\
 & \Delta &
\end{tikzcd}
\qquad
\begin{tikzcd}
 & \Delta^* \arrow[d, "{\begin{bmatrix}0\\-1\end{bmatrix}}"'] \arrow[dr, "1", "\sim"'] & \\
H_1 \arrow[r, hook, "{\begin{bmatrix}1\\1\end{bmatrix}}"] \arrow[dr, "1"'] & C_1 \arrow[r, two heads, "{\begin{bmatrix}1 & -1\end{bmatrix}}"] \arrow[d, "{\begin{bmatrix}1 & 0\end{bmatrix}}"] & \bar{C}_0 \\
 & \Gamma^* &
\end{tikzcd}
$$

$$\begin{bmatrix}1\\-1\end{bmatrix}\begin{bmatrix}0 & -1\end{bmatrix} \text{[redacted]} + \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1 & 1\end{bmatrix}$$

$$= \begin{bmatrix}0 & -1\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 1\\0 & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

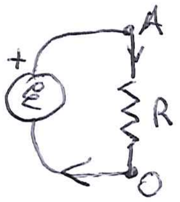
$$\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix} + \begin{bmatrix}0\\-1\end{bmatrix}\begin{bmatrix}1 & -1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\1 & 0\end{bmatrix} + \begin{bmatrix}0 & 0\\-1 & +1\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

Also record the shorter pictures

$$\bar{C}^0 \underset{[0\ \ -1]}{\overset{\begin{bmatrix}1\\-1\end{bmatrix}}{\rightleftarrows}} C^1 \underset{[1\ \ 1]}{\overset{\begin{bmatrix}1\\0\end{bmatrix}}{\leftrightarrows}} H^1 \qquad\qquad H_1 \underset{[1\ \ 0]}{\overset{\begin{bmatrix}1\\1\end{bmatrix}}{\rightleftarrows}} C_1 \underset{[1\ \ -1]}{\overset{\begin{bmatrix}0\\-1\end{bmatrix}}{\leftrightarrows}} \bar{C}_0$$

Let's review the situation in the simple case



State space $\begin{bmatrix} C^1 \\ C_1 \end{bmatrix}$ variables V_R, V_e, I_R, I_e

Kirchhoff: $V_R + V_e = 0, \quad I_R = I_e$

Ohm: $V_e = 0, \quad V_R = R I_R$

So $K \cap \Omega = 0$, assuming $R \neq 0$. You believe that Ω yields splittings of the voltage + current short exact sequences, compatible with duality, and also symmetric bilinear forms

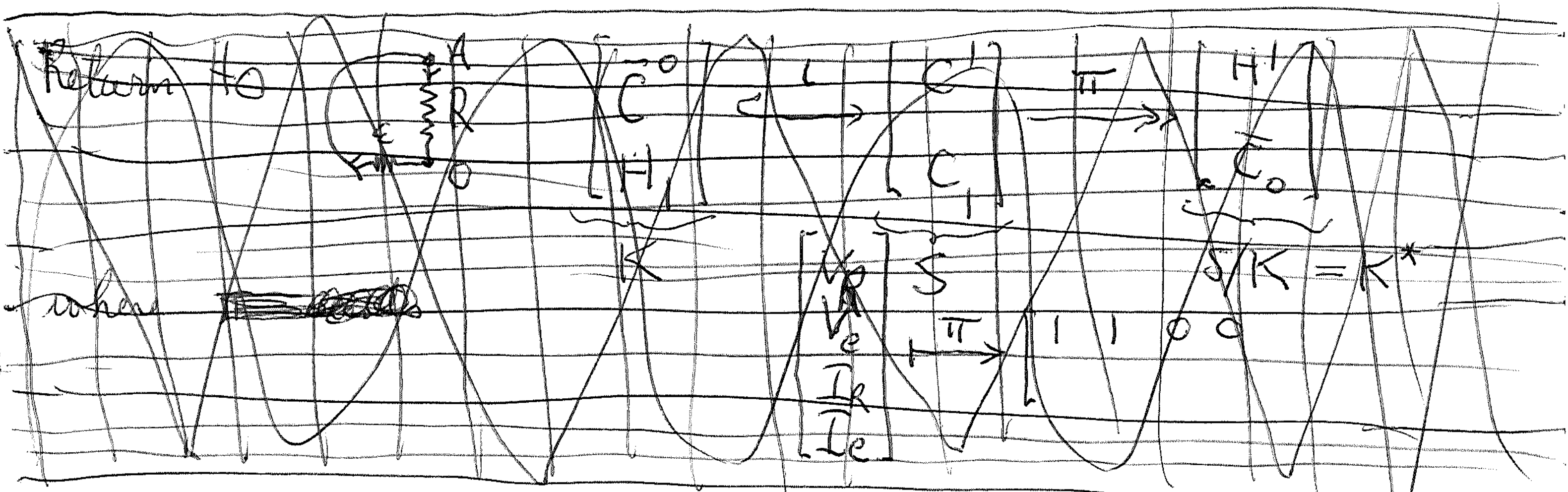
$H_1 \leftarrow H^1$ and $\bar{C}^0 \leftarrow \bar{C}_0$. It seems that the former is the inverse of the obvious map $R : H_1 \xrightarrow{\sim} H^1$. Is there an obvious candidate for the latter? Notice that the map goes from node currents to node potentials.

The important thing is the splitting $\underbrace{S = K \oplus \Omega}_{\text{HODGE DECOMPOSITION}}$, that is, the projection operators on S. This is the output; it's related to Hadamard's finite part and the Birkhoff decomposition for loop groups. Connections with scattering, stuff you did with grid spaces, Wiener-Hopf?.

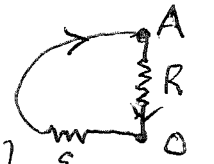
IDEA: Grothendieck motif used to define λ-ring, where one has a triple involving the free λ-ring generated by A. Question is whether you can do something similar with symplectic spaces and the hyperbolic functor.

6 L IDEA which seems familiar: Work with Euclidean spaces instead of dual pairs V, V^*. More precisely, given a dual pair V, V^* choose a positive scalar product [redacted] $V \xrightarrow{\sim} V^*$, and then handle bilinear forms as operators. Thus a symplectic vector space is represented by a Euclidean space with invertible skew-symmetric operator. Then choose the Euclidean structure better (by polar decomp.) you get a Euclidean space with complex structure J. The hyperbolic symplectic space associated a Euclidean space V is the ~~complexification~~ complexification $\mathbb{C} \otimes_{\mathbb{R}} V$ of V.

Q: What are Lagrangian subspaces of [redacted] the hermitian complex space V, J? The symplectic form should be the imaginary part of the hermitean scalar product. Thus a real subspace $L \subset V$ is isotropic when the hermitian form is real on L.



6K Hodge decomposition $S = K \oplus \Omega$



$$S = \begin{bmatrix} C^1 \\ C_1 \end{bmatrix} \ni \begin{bmatrix} V_R \\ V_e \\ I_R \\ I_e \end{bmatrix}, \quad K = \begin{bmatrix} \bar{C}^0 \\ H_1 \end{bmatrix} \ni \begin{bmatrix} \varphi \\ I \end{bmatrix}, \quad S/K = \begin{bmatrix} H^1 \\ \bar{C}_0 \end{bmatrix} = K^* \ni \begin{bmatrix} u \\ J \end{bmatrix}$$

exact sequence
$$\begin{bmatrix} \bar{C}^0 \\ H_1 \end{bmatrix} \xhookrightarrow{i} \begin{bmatrix} C^1 \\ C_1 \end{bmatrix} \xrightarrow{\pi} \begin{bmatrix} H^1 \\ \bar{C}_0 \end{bmatrix}$$

where
$$i \begin{bmatrix} \varphi \\ I \end{bmatrix} = \begin{bmatrix} \varphi \\ -\varphi \\ I \\ I \end{bmatrix}, \quad \pi \begin{bmatrix} V_R \\ V_e \\ I_R \\ I_e \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_R \\ V_e \\ I_R \\ I_e \end{bmatrix} = \begin{bmatrix} u \\ J \end{bmatrix}$$

Next one has Ω which is the graph of the resistance $\begin{bmatrix} R & 0 \\ 0 & \varepsilon \end{bmatrix} : C_1 \longrightarrow C^1$; this is symmetric so its graph Ω is Lagrangian. Thus Ω has the elements

$$\begin{bmatrix} V_R \\ V_e \\ I_R \\ I_e \end{bmatrix} = \begin{bmatrix} R & 0 \\ 0 & \varepsilon \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_R \\ I_e \end{bmatrix} = \begin{bmatrix} R I_R \\ \varepsilon I_e \\ I_R \\ I_e \end{bmatrix} \quad \text{for } \begin{bmatrix} I_R \\ I_e \end{bmatrix} \in C_1$$

The restriction of π to Ω is

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} R & 0 \\ 0 & \varepsilon \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_R \\ I_e \end{bmatrix} = \begin{bmatrix} R & \varepsilon \\ 1 & -1 \end{bmatrix} \begin{bmatrix} I_R \\ I_e \end{bmatrix} = \begin{bmatrix} u \\ J \end{bmatrix}$$

which has inverse
$$\begin{bmatrix} I_R \\ I_e \end{bmatrix} = \frac{1}{+R+\varepsilon} \begin{bmatrix} +1 & +\varepsilon \\ +1 & -R \end{bmatrix} \begin{bmatrix} u \\ J \end{bmatrix}$$

Then ~~one~~ has the lifting of $\begin{bmatrix} u \\ J \end{bmatrix} \in S/K$ into Ω given by

$$\begin{bmatrix} R & 0 \\ 0 & \varepsilon \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{R+\varepsilon} \begin{bmatrix} 1 & \varepsilon \\ 1 & -R \end{bmatrix} \begin{bmatrix} u \\ J \end{bmatrix}$$ and we conclude that

6)
$$\begin{bmatrix} R & 0 \\ 0 & \varepsilon \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{R+\varepsilon} \begin{bmatrix} 1 & \varepsilon \\ 1 & -R \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

is the projection operator on S with kernel K and image Ω.

Review the network 

, states are $\begin{bmatrix} V_R \\ V_e \\ I_R \\ I_e \end{bmatrix} \in \begin{bmatrix} C^1 \\ C_1 \end{bmatrix} = S$

Kirchhoff constraints: $V_R + V_e = 0$, $I_R - I_e = 0$ define a Lagrangian subspace $K = \begin{bmatrix} \bar{C}_0 \\ H_1 \end{bmatrix} \subset S$ such that $S/K = K^* = \begin{bmatrix} H^1 \\ \bar{C}_0 \end{bmatrix}$. The Ohm subspace Ω of S is given by $V_R = RI_R$, $V_e = 0$. It is the graph of the resistance [redacted] map $\begin{bmatrix} I_R \\ I_e \end{bmatrix} \mapsto \begin{bmatrix} R & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I_R \\ I_e \end{bmatrix} = \begin{bmatrix} RI_R \\ 0 \end{bmatrix}$ from C_1 to C^1. Thus

$$\Omega \ni \begin{bmatrix} R & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_R \\ I_e \end{bmatrix} = \begin{bmatrix} RI_R \\ 0 \\ I_R \\ I_e \end{bmatrix} \quad \forall\, I_R, I_e$$

Ω is a Lagrangian subspace of S because the resistance map $C_1 \to C^1$ is symmetric. [redacted]

Assume $R \neq 0$. Then the relations defining K & Ω imply that $K \cap \Omega = 0$, & hence $S = K \oplus \Omega$. [redacted] This can be viewed (or called) a Hodge decomposition where the positivity of the resistance map has been relaxed. (Maybe better to say it is an analog or generalization of the Hodge decomposition where the resistance map is >0.

Next you want to see how this decomposition leads to splittings of the voltage and current s.e.s. You also want to find the associated [redacted] symmetric maps $H^1 \to H_1$ and $\bar{C}_0 \to \bar{C}^0$.

Let's construct the Hodge decomposition by showing that $\Omega \subseteq S \xrightarrow{\pi} S/K$ is an isom. π can be viewed as the Kirchhoff constraint map:

$$\begin{bmatrix} V_R \\ V_e \\ I_R \\ I_e \end{bmatrix} \longmapsto \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_R \\ V_e \\ I_R \\ I_e \end{bmatrix} = \begin{bmatrix} V_R + V_e \\ I_R - I_e \end{bmatrix} = \begin{bmatrix} u \\ J \end{bmatrix} \in \begin{bmatrix} H^1 \\ \bar{C}_0 \end{bmatrix}$$

Restricting to Ω gives

$$\begin{bmatrix} I_R \\ I_e \end{bmatrix} \longmapsto \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} RI_R \\ 0 \\ I_R \\ I_e \end{bmatrix} = \begin{bmatrix} RI_R \\ I_R - I_e \end{bmatrix} = \begin{bmatrix} R & 0 \\ 1 & -1 \end{bmatrix} \overbrace{\begin{bmatrix} I_R \\ I_e \end{bmatrix}}^{= \begin{bmatrix} u \\ J \end{bmatrix}}$$

Then $\begin{bmatrix} R & 0 \\ 1 & -1 \end{bmatrix}^{-1} = \frac{1}{+R} \begin{bmatrix} +1 & 0 \\ +1 & -R \end{bmatrix}$ since $R \neq 0$, showing that $\Omega \xrightarrow{\sim} S/K$. Picture:

$$\begin{tikzcd}
\bar{C}^0 \arrow[r, hook] & C^1 \arrow[r, two heads] & H^1 \arrow[r, phantom, "\ni"] & u \arrow[d] & \\
& \Omega \arrow[u] \arrow[d] \arrow[r, "\sim"] & \begin{bmatrix} H^1 \\ \bar{C}_0 \end{bmatrix} \arrow[u] \arrow[d] \arrow[r, phantom, "\ni"] & \begin{bmatrix} u \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ J \end{bmatrix} \\
H_1 \arrow[r, hook] & C_1 \arrow[r, two heads] & \bar{C}_0 \arrow[rr, phantom, "\ni"] & & J \arrow[u]
\end{tikzcd}$$

Lift H^1 and $\bar{C}_0$ into Ω in the way indicated, using the splitting of Ω given the isom. $\Omega \xrightarrow{\sim} \begin{bmatrix} H^1 \\ \bar{C}_0 \end{bmatrix}$. This amounts to

$$\begin{bmatrix} u \\ J \end{bmatrix} \longmapsto \frac{1}{R} \begin{bmatrix} 1 & 0 \\ 1 & -R \end{bmatrix} \begin{bmatrix} u \\ J \end{bmatrix} = \begin{bmatrix} I_R \\ I_e \end{bmatrix} \longmapsto \begin{bmatrix} R & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_R \\ I_e \end{bmatrix} \in \Omega$$

$$\begin{bmatrix} u \\ J \end{bmatrix} \longmapsto \begin{bmatrix} R & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{R} \begin{bmatrix} 1 & 0 \\ 1 & -R \end{bmatrix} \begin{bmatrix} u \\ J \end{bmatrix} = \begin{bmatrix} R & 0 \\ 0 & 0 \\ 1 & 0 \\ 1 & -R \end{bmatrix} \frac{1}{R} \begin{bmatrix} u \\ J \end{bmatrix}$$

Thus we get the lifting of $u \in H^1$ to $\begin{bmatrix} R \\ 0 \\ 1 \\ 1 \end{bmatrix} \frac{1}{R} u \in S$ and the lifting of J in $\bar{C}_0$ to $\begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} J \in S$. Take the former and project it into C^1 to get $\begin{bmatrix} 1 \\ 0 \end{bmatrix} u \in C^1$. Take the latter and project it into C_1 to get $\begin{bmatrix} 0 \\ -1 \end{bmatrix} J \in C_1$. Notice this checks with $[1 \;\; 1]\begin{bmatrix} 1 \\ 0 \end{bmatrix} u = u$, $[1 \;\; -1]\begin{bmatrix} 0 \\ -1 \end{bmatrix} J = J$.

Apparently you get the following splittings for the voltage + current s.e.s.

$$\bar{C}^0 \xleftrightarrow{\begin{bmatrix} 1 \\ -1 \end{bmatrix}} C^1 \xleftrightarrow{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} H^1$$

$$H_1 \xleftrightarrow{\begin{bmatrix} 1 \\ 1 \end{bmatrix}} C_1 \xleftrightarrow{\begin{bmatrix} 0 \\ -1 \end{bmatrix}} \bar{C}_0$$

These liftings should be u, v in

$$\begin{bmatrix} u & u' \\ v' & v \end{bmatrix} : \begin{bmatrix} \bar{C}^0 \\ H_1 \end{bmatrix} \longleftarrow \begin{bmatrix} H^1 \\ \bar{C}_0 \end{bmatrix}$$

The symmetric maps u', v' should be obtained by taking the current part of $\begin{bmatrix} R \\ 0 \\ 1 \\ 1 \end{bmatrix} \frac{1}{R}$ which is $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{1}{R}$ and the voltage part of $\begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$ which is zero. Thus $u' = 0$ and it looks like $v' = \frac{1}{R} : H^1 \longrightarrow H_1$

6 §   Q: Can you interpret a quadratic space in "triple" terms using the hyperbolic functor?

Recall
$$H(V) = \begin{bmatrix} V \\ V^* \end{bmatrix} \ni \begin{bmatrix} v \\ \varphi \end{bmatrix} \longmapsto \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$H(V)^* = \begin{bmatrix} V \\ V^* \end{bmatrix}^* = \begin{bmatrix} V^* \\ V \end{bmatrix} \ni \begin{bmatrix} \varphi \\ v \end{bmatrix}$$

so that the symmetric form on $H(V)$ is

$$\begin{bmatrix} v_1 \\ \varphi_1 \end{bmatrix}, \begin{bmatrix} v_2 \\ \varphi_2 \end{bmatrix} \longmapsto \begin{bmatrix} v_1 \\ \varphi_1 \end{bmatrix}^t \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v_2 \\ \varphi_2 \end{bmatrix} = v_1^t \varphi_2 + \varphi_1^t v_2$$

(It's probably clearer to define the symmetric form of $H(V)$ via
$$H(V) = \begin{bmatrix} V \\ V^* \end{bmatrix} \xrightarrow{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}} \begin{bmatrix} V^* \\ V \end{bmatrix} = \begin{bmatrix} V \\ V^* \end{bmatrix}^* = H(V)^* \ ?)$$

Suppose $T: V \longrightarrow V^*$ is symmetric. Then

$$\begin{tikzcd}
V \arrow[r, "\begin{bmatrix} 1 \\ T \end{bmatrix}"] \arrow[d, "2T"'] & H(V) \arrow[d, "S"] & \ni & \begin{bmatrix} v \\ Tv \end{bmatrix} \arrow[d, mapsto] \\
V^* & H(V)^* \arrow[l, "\begin{bmatrix} 1 & T^t \end{bmatrix}"'] & \ni & \begin{bmatrix} Tv \\ v \end{bmatrix}
\end{tikzcd}$$

$$2Tv = (T+T^t)v \longleftarrow \begin{bmatrix} Tv \\ v \end{bmatrix}$$

If $2T$ is invertible it looks like the quadratic space (V, T) is a retract of the hyperbolic space $\left(H(V), \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right)$. Certainly you should know how to use the invertibility of $2T$ to split $H(V)$ into V and an orthogonal complement.

60 Calculate the Hodge decomposition in the case of

4 variables V_R, V_e, I_R, I_e

Kirchhoff : $V_R + V_e = I_R - I_e = 0$ " defines K

Ohm : $V_R = RI_R$, $V_e = 0$. " Ω

$$\begin{tikzcd} \begin{bmatrix} \tilde{C}^0 \\ H_1 \end{bmatrix} \arrow[r, "\begin{bmatrix} 1 \\ -1 \end{bmatrix}", shift left] \arrow[r, "\begin{bmatrix} 1 \\ 1 \end{bmatrix}"', shift right] & \begin{bmatrix} C^1 \\ C_1 \end{bmatrix} \arrow[r, "{[1 \ 1]}", shift left] \arrow[r, "{[1 \ -1]}"', shift right] & \begin{bmatrix} H^1 \\ \tilde{C}_0 \end{bmatrix} \\ K \arrow[u, phantom, "\shortparallel"] \arrow[r, hook] & S \arrow[u, phantom, "\shortparallel"] \arrow[r, two heads, "\pi"] & S/K = K^* \arrow[u, phantom, "\shortparallel"] \\ & \Omega \arrow[u, phantom, "\cup"] \arrow[ur] & \end{tikzcd}$$

Let $\begin{bmatrix} \tilde{C}^0 \\ H_1 \end{bmatrix}$ have coords $\begin{bmatrix} \varphi \\ I \end{bmatrix}$

" $\begin{bmatrix} H^1 \\ \tilde{C}_0 \end{bmatrix}$ " " $\begin{bmatrix} U \\ J \end{bmatrix}$

$$\pi \begin{bmatrix} V_R \\ V_e \\ I_R \\ I_e \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_R \\ V_e \\ I_R \\ I_e \end{bmatrix} = \begin{bmatrix} V_R + V_e \\ I_R - I_e \end{bmatrix} = \begin{bmatrix} U \\ J \end{bmatrix}$$

Ω is the graph of $\begin{bmatrix} I_R \\ I_e \end{bmatrix} \longmapsto \begin{bmatrix} R & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I_R \\ I_e \end{bmatrix} = \begin{bmatrix} RI_R \\ 0 \end{bmatrix}$, so

$$\Omega = \left\{ \begin{bmatrix} R & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_R \\ I_e \end{bmatrix} ; \ \forall \begin{bmatrix} I_R \\ I_e \end{bmatrix} \right\}.$$

$\pi | \Omega$ is

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} R & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_R \\ I_e \end{bmatrix} = \begin{bmatrix} R & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} I_R \\ I_e \end{bmatrix} = \begin{bmatrix} U \\ J \end{bmatrix}$$

which has inverse $\begin{bmatrix} U \\ J \end{bmatrix} \longmapsto \frac{1}{+R} \begin{bmatrix} +1 & 0 \\ +1 & -R \end{bmatrix} \begin{bmatrix} I_R \\ I_e \end{bmatrix}$, so the projection of S onto Ω with kernel K is $\pi : S \to S/K$ followed by the inverse of $\Omega \xrightarrow{\sim} S/K$, i.e.

$$\begin{bmatrix} R & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{R} \begin{bmatrix} 1 & 0 \\ 1 & -R \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} = \frac{1}{R} \begin{bmatrix} R & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & -R & R \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ R^{-1} & R^{-1} & 0 & 0 \\ R^{-1} & R^{-1} & -1 & 1 \end{bmatrix}$$ call this e, then $$1 - e = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -R^{-1} & -R^{-1} & 1 & 0 \\ -R^{-1} & -R^{-1} & 1 & 0 \end{bmatrix}$$

On the other hand the product $e(1-e) = 0$, so you have the desired decomposition.

Next find the response to the forcing term given by $V_e = -\mathcal{E}$, $V_R = I_R = I_e = 0$.

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -R^{-1} & -R^{-1} & 1 & 0 \\ -R^{-1} & -R^{-1} & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -\mathcal{E} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \mathcal{E} \\ -\mathcal{E} \\ R^{-1}\mathcal{E} \\ R^{-1}\mathcal{E} \end{bmatrix} = \begin{bmatrix} V_R \\ V_e \\ I_R \\ I_e \end{bmatrix}$$

which is what you expected.

Idea that the hyperbolic functor might give rise to a kind (or variant) of a triple. The free quadratic spaces should be the hyperbolic spaces.

You know that an embedding of a quadratic space $(V, T: V \to V^*)$, T symmetric + nonsing.) into the hyperbolic space $H(W)$ is described by $\begin{bmatrix}\alpha\\ \beta\end{bmatrix}: V \to \begin{bmatrix} W \\ W^*\end{bmatrix}$ such that

$$\begin{tikzcd} V \arrow[r, "{\begin{bmatrix}\alpha\\ \beta\end{bmatrix}}"] \arrow[d, "T"'] & \begin{bmatrix} W \\ W^*\end{bmatrix} \arrow[d, "{\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}}"] \\ V^* & \begin{bmatrix} W^* \\ W\end{bmatrix} \arrow[l, "{[\alpha^t\ \beta^t]}"'] \end{tikzcd}$$

commutes i.e.

$$T = [\alpha^t, \beta^t]\begin{bmatrix}\beta\\ \alpha\end{bmatrix} = \alpha^t\beta + \beta^t\alpha$$

Thus the embedding yields a map $h = \beta^t\alpha : V \to V^*$ such that $h + h^t = T$. Conversely given such an h we can choose a factorization

$$\begin{tikzcd} V \arrow[r, "\alpha"] \arrow[d, "h"'] & W \arrow[dl, "\gamma"] \\ V^* & \end{tikzcd}$$

, then put $\beta = \gamma^t : V \to W^*$ and we have $h = \beta^t\alpha$, $h + h^t = T$.

An $h: V \to V^*$ s.t. $h + h^t = T$ has the form $h = \frac{1}{2}(T + X)$ where $X: V \to V^*$ is skewsymmetric. The simplest case is $X = 0$, $h = \frac{1}{2}T$, but non trivial X should yield bigger W's.

All the above reminds you of retracts of a free $\mathbb{Z}/2$-module. QUESTION: Can you do something

6p about the Cayley transform in the quadratic space situation that you couldn't handle before. "Before" refers to your attempt to link the theory of retracts W of a free $\mathbb{Z}/2$ module $\begin{bmatrix} V \\ V \end{bmatrix}$, where you encounted an odd operator X, to the Cayley transform.

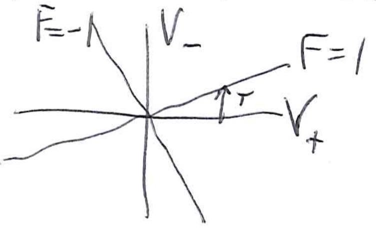
March 31, 03. IDEA about the Inverse Cayley Transform.

An Abstract LC network gives rise to a unitary representation of the infinite dihedral group $\langle F, \varepsilon \rangle$. You would like to express all of LC network theory in terms of these representations. An important ingredient of this theory is the dynamics, which is given by the resolvent $(s - X)^{-1}$ where X is the I.C.T. of $g = F\varepsilon$. X is skew symmetric + odd wrt ε.

May 2, 03: repn of $\langle F, \varepsilon \rangle \rightsquigarrow g = F\varepsilon = \dfrac{1+X}{1-X}$

$$A, H \rightsquigarrow X = A^{-1}H$$

Good case is where F, ε are "transversal" whence you have



$$\begin{bmatrix} 1 & -T^* \\ T & 1 \end{bmatrix} = 1 + X, \quad F(1+X) = (1+X)\varepsilon$$

$$\Longrightarrow F\varepsilon(1-X) = 1+X$$

Puzzle: How dynamics arise. Algebraically it seems you replace 1 by the Laplace transform variable s.