Old question: Construction of Morita context linking $V$ and $W$. 1st part: the cat $U$ of $(V, W, \alpha, \beta)$ where $\sum \beta_i \alpha_i s^{-1} = 1_W$. Generators

$\begin{pmatrix} 1 & \alpha_i \\ \beta_i & \Gamma \end{pmatrix} (V) \quad (W)$

Outline: What are the unclear points?
Main unclear point is the link between $T_3$ and $A$.
There's a surjection $p : T_3 \rightarrow A$ $p(\delta) \rightarrow \delta_3 \beta_1$ compatible with the grading:

$C = \begin{bmatrix} A & Y \\ X & B \end{bmatrix}$  $A, C$ are non unital, $B$ is unital

It should be clear that $B$ and $C$ are Morita equivalent because? Look at dual pairs over $B$.

$B$ generated by $\delta_3 \alpha_i s^{-1}$, $A$ generated by $\alpha_i s^{-1} \beta_1$

$X = \beta_1 A = ?$ You have $\delta_3$ words $\delta_3 \alpha_i s^{-1}$? The generators are

$\begin{pmatrix} 0 & \alpha_i s^{-1} \\ t \beta_i & 0 \end{pmatrix} = \begin{pmatrix} 0 & \gamma_s \\ t \beta_i & 0 \end{pmatrix}$

$x_t (y_s x_t) \cdots (y_s x_t)$, $y_s (x_t y_s) \cdots (x_t y_s)$

$X = \sum_t t \beta_1 A = \bigoplus B \beta_1$, $Y = \bigoplus \alpha_1 B = \sum_s A \alpha_1 s^{-1}$

DL 4521 disappearance gate C21

Assembly construction, go over to motivate retract of a free $\Gamma$-module $\Lambda \otimes V$

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Construction. Consider infinite disc $\Gamma$, a principal $\Gamma$-bundle $\pi: P \to X \times X$ compact.

($\Gamma$ acts on the left, locally over $X$ and continues section $s: K \to P$ such that $\Gamma \times K \sim P$)

$E = \text{associated fibre bundle}$ with fibre the group $\mathbb{C}$

$\Lambda = \mathbb{C}[\Gamma] = \bigoplus_{\gamma \in \Gamma} \mathbb{C} s \gamma$. Given $s: K \to P$ get

$\Lambda \otimes K \to P / K$, $E_K = \Lambda \times K P_K$

Analog of line bundle where $\mathbb{C}$ replaced by $\Lambda$

Sections of $E_K$ over $K = C(K) \otimes \Lambda$

Global sections $\Gamma(X, E) \text{ module over } \Lambda \otimes C(X)$.

$\Gamma = \mathbb{Z}$:

$\mathbb{Z} \to \mathbb{R} \xrightarrow{\pi} \mathbb{R}/\mathbb{Z}$

$\tilde{\pi}(x) = y + \mathbb{Z}$ for any constant $y$.

$E_x = \mathbb{C}[y + \mathbb{Z}] = \bigoplus_{n \in \mathbb{N}} \mathbb{C}[y + n]$.

$C(\mathbb{R}/\mathbb{Z}) \otimes C[\mathbb{Z}]$

Next Serre's thesis. Vector bundles over a compact space $X$ are the same as finitely generated projective modules over the alg $C(X)$. 
Given \( E = P \times \Lambda \), \( X = U u_i \)

\[ E_{u_i} \xleftarrow{\beta_i} U_i \times (\Lambda \otimes V_i) \xrightarrow{\alpha_i} E_{u_i} \quad \beta_i \alpha_i = 1 \]

First do vector bundle case \( X = U u_i, \sum \alpha_i = 1 \),

\[ E_{u_i} \xleftarrow{\beta_i} U_i \times V_i \xrightarrow{\alpha_i} E_{u_i} \quad \beta_i \alpha_i = 1 \]

\[
\begin{align*}
E & \xleftarrow{\begin{pmatrix} \chi_1 & \beta_1 \\ \chi_2 & \beta_2 \end{pmatrix}} X \times \left( \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \right) \\
& \xrightarrow{\begin{pmatrix} \chi_1 & \alpha_1 \\ \chi_2 & \alpha_2 \end{pmatrix}} E
\end{align*}
\]

\[ P \xrightarrow{\pi} X \quad \text{principal } \Gamma \text{-bundle} \]

\[ \pi^{-1} U \simeq U \times \Gamma, \quad E_{u_i} = U \times \Lambda \]

Assume \( \Gamma = \mathbb{Z} \)

\[
\begin{align*}
E & \xleftarrow{\begin{pmatrix} \vdots & \chi_i \beta_i & \vdots \\ \vdots & \vdots & \vdots \end{pmatrix}} X \times \left( \begin{pmatrix} V_i \otimes \Lambda \\ \vdots \end{pmatrix} \right) \\
& \xrightarrow{\begin{pmatrix} \chi_i \alpha_i \\ \vdots \end{pmatrix}} E
\end{align*}
\]

\[ \pi(x) = x + \mathbb{Z} \]

\[ \Gamma = \mathbb{Z} \quad \mathbb{Z} \xrightarrow{\pi} \mathbb{R} \xrightarrow{\pi} \mathbb{R}/\mathbb{Z} \]

\( E \) is a retract of \( X \times (\Lambda \otimes V) \)

Trivial bundle \( X \times (\Lambda \otimes V) \) has space of sections

\[ C(X, \Lambda \otimes V) = C(X) \otimes \Lambda \]

Output is a retract of the trivial bundle \( X \times (\Lambda \otimes V) \)

which is the same as a retract of the free \( C(X) \otimes \Lambda \) module
c) Since the $E$ vector bundles over $X$ compact $X = \bigcup U_i$, \( \sum x_i^2 = 1 \), $\text{Supp}(x_i) \subset U_i$:

\[
E_{U_i} \xleftarrow{\beta_i} U \times V_i \xrightarrow{\alpha_i} E_{U_i}, \quad \beta_i \alpha_i = 1_{E_{U_i}}.
\]

\[
E = \frac{\left( x_1 \beta_1 \cdots x_n \beta_n \right)}{\left( x_1 \alpha_1 \cdots x_n \alpha_n \right)} \quad x \times \left( \begin{array}{c} V_1 \\ \vdots \\ V_n \end{array} \right) \quad \frac{\left( x_1 \alpha_1 \cdots x_n \alpha_n \right)}{\left( x_1 \beta_1 \cdots x_n \beta_n \right)} \rightarrow
\]

\[
(x_1 \beta_1 \cdots x_n \beta_n)(x_1 \alpha_1 \cdots x_n \alpha_n) = \sum_i x_i^2 \beta_i \alpha_i = \sum_i x_i^2 = 1.
\]

Same arg shows in the case of the assembly construction:

\[
E_{U_i} = U_i \times \Gamma \times \left( U_i \times \overline{\Gamma} \right) = U_i \times \Gamma
\]

\[
\Rightarrow E \text{ retract of the trivial bundle } X \times \Lambda^n.
\]

This is the same as an idempotent $n \times n$ matrix over $\Lambda$. \( \Rightarrow \) you get class $\in \mathcal{K}_0(C(X) \otimes \Lambda)$.
1st part. The assembly constructions starts from a principal $\Gamma$-bundle $P \longrightarrow X$, $X$ compact and produces a retract of a free $\Lambda$-module $C(X) \otimes \Lambda$

Let class $\chi(p) \in K_0(C(X) \otimes \Lambda)$

\[ C(X) \otimes \Lambda \subset C(X) \otimes C^*_\Lambda \Gamma \]

$C^*_\Lambda \Gamma$ completion.

\[ \chi(p) \in K_0(C(R/Z \times \Pi)) \]

$\chi(p) \in K_0(C(R/Z \times \Pi))$

2nd part. Retractions of a free $\Lambda$-module $\Lambda \otimes \mathbb{V}$

Assume $\Gamma$ finite group to simplify.

$\beta = 1$, $\alpha \beta = p$

\[ W \overset{\beta}{\leftarrow} \Lambda \otimes \mathbb{V} \overset{x}{\leftarrow} W \overset{\beta}{\leftarrow} \Lambda \otimes \mathbb{V} \]

\[ \beta_1 = \beta \varepsilon_1 \]

\[ x_1 = \eta_1 x \]

\[ 1_{\Lambda \otimes \mathbb{V}} = \sum_s s(\varepsilon_1 \eta_1) s^{-1} \]

\[ 1_W = \sum s \beta_1 \alpha_1 s^{-1} \]

$W$ retract of $\Lambda \otimes \mathbb{V} \rightarrow W$ $\Gamma$-module plus operator $h = \beta_1 \alpha_1$, satisfying $\sum_s h s^{-1} = 1_W$ (equivariant partition of $\beta$)

\[ \Lambda \otimes \mathbb{V} = \left\{ \sum_t t \otimes f(t) \mid f : \Gamma \rightarrow \mathbb{V} \right\} \]

\[ \beta \sum_t t \otimes f(t) = \sum_t t \beta_1 f(t), \quad \alpha \omega = \sum_s s \otimes \alpha_1 s^{-1} \omega \]

\[ p \sum_t t \otimes f(t) = \sum_s s \otimes \sum_t \alpha_1 s^{-1} t \beta_1 f(t) \]

\[ \alpha_1 \omega = \left\{ \alpha_1 \left( \sum_s s \otimes p(s^{-1}) f(t) \right) \right\} = \left\{ \sum_t p(t) f(t) \mid f : \Gamma \rightarrow \mathbb{V} \right\} \]

\[ \alpha \beta_1 \omega = \alpha \left( \Lambda \otimes \mathbb{V} \right) = \sum_s s \otimes p(s^{-1}) \mathbb{V} \ni \omega \]
Equivalence of categories

W category of \((W, h)\) where \(W\) is a \(\Gamma\)-module and \(h\) is a \(\Gamma\)-linear operator satisfying \(\sum s_h s^{-1} = 1_W\).

\(\mathcal{U}\) category of \((V, \{p(s)\}_s)\) where \(V\) is a vector space with \(\Gamma\)-linear operators \(p(s), s \in \Gamma\) satisfying

\[ p(u) = \sum_{u = st} p(s) p(t), \quad V = \sum_{t} p(t) V, \quad \bigcap_{s} \text{Ker}\{p(s)_{|V}\} = 0 \]

Claim \(W\) and \(\mathcal{U}\) are equivalent categories.

Pf. Introduce \(\mathcal{U}\) cat of \((W, V, \beta, \alpha)\) where \(W\) is a \(\Gamma\)-module, \(V\) is a vector space, and \(\beta: W \rightarrow V, \alpha: V \leftarrow W\) are \(\Gamma\)-linear maps such that \(\beta, \alpha\) injective and \(\alpha\) surjective.

Forget \(V\) functor: \(\mathcal{U} \longrightarrow W\)

\((W, V, \beta, \alpha) \longmapsto (W, h = \beta \circ \alpha)\)

Forget \(W\) functor: \((W, V, \beta, \alpha) \longmapsto (\bigvee, \sum p(s) = \alpha \circ \beta)\)

\[ \sum_{t} p(st^{-1}) p(t) = \sum_{t} \alpha_i (st^{-1}) \beta_j \alpha t \beta_j = \alpha_i \beta_j = p(s) \]

\(\mathcal{U}\) cat is the \(\Gamma\)-category of \(\mathcal{U}\) cat.

Forget the \(\mathcal{U}\) cat, the \(\mathcal{U}\) cat is the \(\Gamma\)-category of \(\mathcal{U}\) cat.

Forget the \(\mathcal{U}\) cat, the \(\mathcal{U}\) cat is the \(\Gamma\)-category of \(\mathcal{U}\) cat.

Forget the \(\mathcal{U}\) cat, the \(\mathcal{U}\) cat is the \(\Gamma\)-category of \(\mathcal{U}\) cat.
$A \xrightarrow{\Delta} \mathbb{C} \mathbb{F} \otimes A$

both $\Delta, \Delta'$ send $\pi_{ij}$ to $e_{ij} \otimes \pi_{ij}$

and the set where $e_{ij} \in \mathbb{C} \mathbb{F}$

you have to decide whether or not $A$

is $\mathbb{C} \mathbb{F}$-graded. Let $\mathbb{C}$ be a semi-group

\[ A = \bigoplus_{s \in S} A_s \quad \text{a } \mathbb{C}\text{-graded algebra} \]

i.e. $A_s A_t \subseteq A_{st}$. Then

You want $A$ to be defined by generators

and rels which are homog wrt $G$.

$G = \mathbb{C} \mathbb{F} = \{e_{ij}\} \cup \{x\}$

$\text{gen } \pi_{ij} \text{ of degree } e_{ij}$

$\text{rels } \sum_j \pi_{ij} \pi_{jk} = \pi_{ik}$

$\pi_{ij} \pi_{jk} = 0$ for $j \neq k$

$\text{homog of degree } i k$

$\Delta : A \xrightarrow{} \mathbb{C} \mathbb{F} \otimes A$

$\pi_{ij} \otimes e_{ij} \otimes \pi_{ij}$

$\pi_{ij} \pi_{jk} = 0$

You don't understand properly

the counit
Example. "Wheatstone" bridge with complex impedances

\[ \psi(A) = Z_1 \frac{\varepsilon}{Z_1 + Z_2} \]

\[ \psi(B) = Z_3 \frac{\varepsilon}{Z_3 + Z_4} \]

\[ \varepsilon_0 = \left( \frac{Z_1}{Z_1 + Z_2} - \frac{Z_3}{Z_3 + Z_4} \right) \varepsilon \]

\[ Z_0 = \frac{Z_1 Z_2}{Z_1 + Z_2} + \frac{Z_3 Z_4}{Z_3 + Z_4} \]

**Balance Condition**

\[ 0 = \varepsilon_0 = Z_1 (\varepsilon_0 + Z_4) - (\varepsilon_0 + Z_2) Z_3 \]

\[ \therefore Z_1 Z_4 = Z_2 Z_3 \]

\[ \frac{Z_1}{Z_2} = \frac{Z_3}{Z_4} \]

Last step is to add \( Z_6 \) between \( A \) and \( B \).

\[ (Z_6 + Z_0) I_0 = \varepsilon_0 \]

\[ I_0 = \frac{\varepsilon_0}{Z_6 + Z_0} = \frac{\left( \frac{Z_1}{Z_1 + Z_2} - \frac{Z_3}{Z_3 + Z_4} \right) \varepsilon}{\left( \frac{Z_1 Z_2}{Z_1 + Z_2} + \frac{Z_3 Z_4}{Z_3 + Z_4} + Z_6 \right)} \]
Balance condition
\[ Z_1 Z_4 = Z_2 Z_3 \]

I.e. when \( \psi_A - \psi_B = 0 \)

\[ E_0 = \frac{\sum Z_1 \frac{\psi}{Z_1 + Z_2} - Z_2 \frac{\psi}{Z_2 + Z_4}}{Z_0} = \frac{\psi_A - \psi_B}{Z_0} \]

\[ Z_0 = \frac{Z_1 Z_2}{Z_1 + Z_2} + \frac{Z_3 Z_4}{Z_3 + Z_4} \]

\[ (4 - 1) + 2 = 5 \]

\[ \frac{1}{C_1 s} + \frac{1}{C_3 s} \]

What is the problem of interest? The 2 port going from the \( E \) terminals to \( AB \)

\[ E_0 = \left( \frac{Z_1}{Z_1 + Z_2} - \frac{Z_3}{Z_3 + Z_4} \right) E \]

\[ Z_0 = \frac{Z_1 Z_2}{Z_1 + Z_2} + \frac{Z_3 Z_4}{Z_3 + Z_4} \]

\[ I_{AB} = \frac{E_0}{Z_0} = \frac{\left\{ Z_1 (\xi_2 + Z_4) - Z_3 (\xi_1 + Z_2) \right\} E}{Z_1 Z_2 (Z_3 + Z_4) + Z_3 Z_4 (Z_1 + Z_2)} \]

\[ I_6 (R_0 + R_6) = E_0 \]
v = 4, e = 4, \( b = 1 \)
\[ v - 1 + b = 3 + 1 = 4 = e \]

Program: Regroup the good case in which, where the dominant variables are linearly independent on the state space (Kirchhoff space).

Note that because the Kirchhoff space is \( C \subset C' + C_1 \), there \( \exists \) some set of edge variables which is independent on \( H \).

In fact \( K = C' + H_1 \subset C' + C_1 \), so you can pick any \( v - 1 \) voltage variables and on \( C' \), and any \( b \) current on \( H_1 \).

(The good case is where \( C' \xrightarrow{} C \), \( H_1 \xrightarrow{} C_1 \).)

Now the idea is to ignore \( s = 0, \infty \), i.e. localize to make \( s = 0 \) invertible. Then the difference between dominant and recessive variables shouldn't matter.

Look at these vector spaces properly—replace by free modules over \( \mathbb{C}[s, s^{-1}] \).

**How to handle free edges?**

Kirchhoff says

\[ I = 0 \]
\[ V = 0 \]
\[ C \cdot V = I \rightarrow V = \text{const} \]
The problem? Consider a connected LC network, examine free motion, exponential solution, system has coordinates \((V_0, I_0)\) for each edge \(\sigma\), path for the system is a function of time \((\tilde{V}_t, \tilde{I}_t)\) with values in \(C^0 \oplus C_1\). You have Kirchhoff constraints: \(\tilde{V}_t \in C^0, \tilde{I}_t \in H_1\). Also have dynamical conditions:

\[
\begin{align*}
L_\sigma \frac{\partial}{\partial t} \tilde{I}_t,\sigma &= V_t, \sigma \text{ L-type} \\
C_\sigma \frac{\partial}{\partial t} \tilde{V}_t,\sigma &= \tilde{V}_t, \sigma \text{ C-type}
\end{align*}
\]

where \((\tilde{V}_t, \tilde{I}_t) \in C^0 \oplus H_1\) and

\[
\begin{align*}
L_\sigma s \tilde{I}_t,\sigma &= \tilde{V}_t, \sigma \text{ L-type} \\
C_\sigma s \tilde{V}_t,\sigma &= \tilde{I}_t, \sigma \text{ C-type}
\end{align*}
\]

It should be obvious that an exponential solution with \(s \neq 0, \infty\) should be a pair \((\tilde{V}, \tilde{I}) \in C^0 \oplus H_1\) such that \(Z_s \tilde{I} = \tilde{V}\)

Continue with example
 variables \(V_L, I_L, V_C, I_C\) coordinateize \(C^0 \oplus C_1\)

Kirchhoff \(I_L + I_C = 0\) \(\text{or} (1, -1)\) gen \(H_1\),

\(V_C = V_C\)
$E$

$H_1$ gen. by $[L] \otimes [C]$

$Z_s([L] - [C]) = (Ls, \frac{1}{Cs})$

$\xi \rightarrow \xi'$

$V \rightarrow (V_L, V_C)$

$[A] - [B](Ls I_L + \frac{1}{Cs} [C] C)$

Notation very confused.

But you do learn that an exponential solution frequency $s$ is a null vector for the quadratic form.

Better would be

$Idea. Bring in Hassmannian stuff, need to be careful about signs.
What about \[ \frac{\partial}{\partial t} \mathcal{H} \]?

You want to write up what you've learned. Let's start with an LC network (comm). You want to keep track of symplectic structure, power.

Obvious question: the time evolution for a closed LC network given the Hamiltonian flow arising from the symplectic structure and the power quadratic form? You believe this is true for a ladder network.

Another question: free edges in a ladder network: suppose you have the good case where the dominant variables are independent on the Kirchhoff spaces. Is the above about Hamiltonian flow true?

Dominant variables independent means that the compositions

\[ L \overset{\dot{L}}{\longrightarrow} V \]

\[ C_s \overset{\dot{V}}{\longrightarrow} T_s \]

are isos. This implies that

\[ l = \dim (H_1) = \text{no. of } L_i \]

\[ v-1 = \dim (\mathcal{E}_0) = \text{no. of } C_i \]

\[ V-1 = 3 \]

\[ \mathcal{E} = 2 \]
Analyze $C \overset{\cdot}{\rightarrow} C$.

You have a graph with $v$ nodes and $v-1$ edges. Maybe better to analyze $C \overset{\cdot}{\rightarrow} C'$.

$H_1 \sim C_{1,1}$ seems too hard.

Look at symplectic picture, try to see whether the free motion of a connected LC network is a Hamiltonian flow with Hamiltonian the power quadratic form. Ex ladder network.

$$V_0 - V_1 = L_1 s I_1, \quad I_2 - I_3 = C_2 s V_2$$

$$V_0 \downarrow V_1 \quad I_1 - I_2 = C_1 s V_1$$

$$V_1 \downarrow V_2 \quad V_1 - V_2 = L_2 s I_2$$

$$\begin{bmatrix}
  L_1 & C_1 \\
  C_1 & L_2
\end{bmatrix}
\begin{bmatrix}
  I_1 \\
  V_1
\end{bmatrix}
= 
\begin{bmatrix}
  -1 & 0 \\
  0 & 1
\end{bmatrix}
\begin{bmatrix}
  I_1 \\
  V_1
\end{bmatrix}$$

$$\begin{bmatrix}
  1 & -1 & 0 \\
  -1 & 1 & -1
\end{bmatrix}
= +1$$
scratch for Sp’tt. At some point what’s important is the symplectic picture, which should be a linear algebra translation of the physics. Kinematics should correspond to symplectic structure, Hamiltonians to Lagrangian subspaces.

You conjecture that LC networks form a special case of “linear phase space + quadratic Hamiltonian” physics.

You would like a good theory of “glueing” that is attaching networks together. This means you need a notion of $\exists N < N$. Can you adapt bordism ideas?

Recall where you left forced harmonic oscillators. Problem of subspace of phase space, in which the forcing term is supported, generated by the forcing terms.

Perhaps you can use the Grassmannian picture...
Go back to a connected IC network where the dominant modes are current $I_T$ $T$-type voltage drops $V_T$ or $C$-type:

$$L_s I_s = V_s, \quad C_s T = V_s$$

Assume the dom. modes form a basis for linear fields in the Kirchhoff space $\mathbb{C}^0 \oplus H_1$.

Means $$(\mathbb{C}^0) \leftrightarrow (C^1) \rightarrow \left(\frac{e_1^c}{e_1^c}\right)_\mathbb{C}$$ is an isom.

i.e. $\mathbb{C}^0 \rightarrow C^1$ and $H_1 \rightarrow C_1^c$.

Look at the condition $H_1$. Depends on the subnetwork containing all nodes and C-type edges.

Try removing one edge at a time.

Maybe the good viewpoint is to look at

$$\mathbb{C}^0 \rightarrow C^1 \rightarrow H_1$$

as functions with respect to inclusion seems clear.
It seems that when the domain words form a basis for $X$, then the network is a tree of $\mathcal{C}$-edges, with $L'$ edges adjoined.

Start with a connected graph $X$. You would like to remove an edge $\sigma$ without changing $\mathcal{C}^0(X)$.

For $X$ connected, $\mathcal{C}^0(X) = H^0(X)$ = functions $E : \text{nodes} \to \mathbb{R}$ and $\mathcal{C}^0(X) = H^0(X)/\text{constant functions on the nodes}$. If $X$ conn then $0 \to \mathbb{R} \to \mathcal{E}^0(X) \xrightarrow{\mathcal{S}} \mathcal{C}^1(X) \to H^1(X) \to 0$

leading to $-1 + v - e + l = 0$ or $e = (v-1)+l$

Now you want to remove an edge $\sigma$ in $X$. Let $Y = X - \text{Int}(\sigma)$.

Picture

\[ Y = Y_+ \cup Y_- \]
You want to carefully analyze the restriction maps from $X$ to $Y$.

$$0 \to \mathbb{R} \to C^0(X) \xrightarrow{S} C^1(X) \to H^1(X) \to 0$$

$$0 \to H^0(Y) \to C^0(Y) \xrightarrow{S} C^1(Y) \to H^1(Y) \to 0$$

$$0 \to (\mathbb{H}^0(Y_+), \mathbb{H}^0(Y_-)) \to (C^0(Y_+), C^0(Y_-)) \xrightarrow{S} (C^1(Y_+), C^1(Y_-)) \to (H^1(Y_+), H^1(Y_-)) \to 0$$

Clearly, $X$ connected $\Rightarrow$ $Y_+$ and $Y_-$ connected.

because if $Y_+$ connected, $Y_-$ connected, then $Y_+ \cup Y_-$ is connected.

$\bigcirc$ joins a conn. comp. of $A$ to one of $B$.

which is a component of $X$. 

$$0 \to C^0(X) \xrightarrow{S} C^1(X) \to (C^0(Y_+), C^0(Y_-)) \to (C^1(Y_+), C^1(Y_-)) \xrightarrow{S} (\mathbb{H}^0(Y_+), \mathbb{H}^0(Y_-)) \to 0$$
M: What do you want? Start with $C^0(X)$. Go back to the assumption that $C^0(X) \sim C^1_c(X)$, $H_1(X) \sim C^1_{bc}(X)$.

Be careful:

\[
\begin{pmatrix}
C^0(X) \\
H_1(X)
\end{pmatrix} \sim \begin{pmatrix}
C^1_c(X) \\
C^1_b(X)
\end{pmatrix} \sim \begin{pmatrix}
C^1_c(X) \\
C^1_{bc}(X)
\end{pmatrix}
\]

You do get that $C^0(X) \sim C^1_c(X)$.

Suppose you remove an L edge?

Fill up what you believe, namely, that in the "good" case, you have a tree of C-edges and the remaining edges have L-type, and there's an obvious correspondence between L-edges and a basis for $H_1$, corresponding to the L edges, namely you close the L edges by the path joining its boundary in the tree.
You learn that \( C^0(X) \xrightarrow{\sim} \mathbb{R} \) and \( \mathbb{R} \xrightarrow{\sim} \mathbb{R} \) with kernel \( \mathbb{R} \).

\[
\begin{align*}
0 & \rightarrow \mathbb{R} \rightarrow C^0(X) \xrightarrow{S} C^1(X) \rightarrow H^1(X) \rightarrow 0 \\
0 & \rightarrow \mathbb{R} \times \mathbb{R} \rightarrow C^0(Y) \xrightarrow{S} C^1(Y) \rightarrow H^1(Y) \rightarrow 0
\end{align*}
\]

What do you want to happen? Look at the case where the graph remains connected.

\[
\begin{align*}
0 & \rightarrow \mathbb{R} \rightarrow C^0(X) \rightarrow C^1(X) \rightarrow H^1(X) \rightarrow 0 \\
0 & \rightarrow \mathbb{R} \rightarrow C^0(Y) \rightarrow C^1(Y) \rightarrow H^1(Y) \rightarrow 0
\end{align*}
\]

You want to remove the L edges, and you need the graph to stay connected. This maybe follows from understanding the fact that the L-edges yield a basis in \( H_1(X) \).
What can you do with $\tilde{c}^o(X) \rightarrow \tilde{c}^i_c(X)$?

$\tilde{c}^o$ rank $v-1$
$\tilde{c}^i_c$ $v-1$ C edges
$\tilde{c}^i_c$ $v$ L edges

Hope: That extra components yield too many vertices.

Suppose then you have a graph $X$.

Consider $X$ a conn. graph, $\sigma$ an edge such that $X-\sigma = Y$ is disconnected.

Put another way, $Y$ is disconnected but $X \cup \sigma$ is connected. Picture

Possible attaching

Clearly $Y$ has 2 components.

$Y$:

$X$:
Kinematics again. \( X \) connected graph with edge of \( X \), \( Y = X - \sigma \). Two cases:

1. \( Y \) connected and \( \sigma \) extends to a loop (circuit) in \( X \).

\[ \begin{array}{c}
\text{better} \\
\text{includes} \\
A = B \\
\end{array} \]

\( Y \) has 2 components:

\[ \begin{array}{c}
Y_+ \\
A \rightarrow B \\
Y_- \\
\end{array} \quad A \in Y_+ \\
B \in Y_- \quad \text{joined by } \sigma \\
\]

Maximal tree picture: Choose a maximal tree \( T \) in \( X \), Maximal \( \Rightarrow T \subseteq X \) same vertices. You can collapse \( T \) to a point to get \( X/T = \text{wedge of } l \cdot s' \)'s, each \( s' \) coming from an open edge in \( X - T \).

This picture shows \( X \) as a tree with \( l \) extra edges attached.

Loop picture. Given \( X \) connected choose a maximal set of edges such that the graph remains connected upon their removal. Let \( T = X - \{ \sigma_1, \ldots, \sigma_l \} \). Then \( T \) is connected and if \( T \) is any edge in \( T \), then \( T - \{ \sigma \} \) has 2 components.
T is a tree because let $X$ be a circuit in $T$.

Change notation. Let $X$ be a graph such that $X - e$ is disconnected for every edge $e$.

So you have a problem with

$X$ connected graph. Usually stuff

$0 \rightarrow \mathcal{C}(X) \rightarrow \mathcal{C}'(X) \rightarrow H'(X) \rightarrow 0$

First remove those edges whose endpoints coincide ending. You lose the same thing from both $\mathcal{C}'(X)$ and $H'(X)$. Next is to remove edges without disconnecting the graph. Notice that you are not affecting the vertices. When $e$ is an edge of $X$ such that $X - Int(e) = Y$ is disconnected, you should know that $Y$ has two components, and also that the generator again you lose the same amount from $\mathcal{C}'(X)$ and $H'(X)$?

Need to reverse. Chain picture?

$X = Y_+ \cup Y_-$

$Y = Y_+ \cup Y_-$

$X = Y_+ \cup Y_-$
You are assuming $Y$ is disconnected.

\[ H_0(Y) = \mathbb{R} \oplus \mathbb{R}. \]

This is not the interesting case. The interesting case is when $Y$ is connected.

\[ 0 \to H_1(Y) \to H_1(X) \to \mathbb{R} \to H_0(Y) \to H_0(X) \to 0 \]

You want to join $A, B$ by a path in $Y$ to get the extra loop in $X$.

When you take a

so you end up with a graph $T$ and disconnects $T$
Repeat what you've learned.

\( X \) connected graph, or an edge, \( Y \equiv X - (\text{Int}) \)

\[ C_0(Y) \rightarrow C_0(X) \rightarrow C_0(X,Y) \]

Two cases: \( Y \) connected. \( H_0Y = 1R \) \( H_1X = H_1Y \oplus R_0 \) where \( \hat{o} \) is a loop current obtained by forming a path in \( Y \) joining the ends of \( \hat{o} \). \( Y \) disconnected = \( H_0Y = R \oplus R \) and \( H_1Y = H_X \)
Now that you understand more about connected graphs, namely, that a connected graph is a tree iff removing any edge disconnects the graph, you should examine the good LC circuit situation where the dominant form coordinates for the Kirchhoff space.

You know that in this case

\[ H_1(X) \xrightarrow{\sim} C_{1,2} \]

\[ \mathbf{C}(X) \xrightarrow{\sim} C'_2 \]

\[ C(X)/C_{10}(X) \]

So you should be able to remove one L edge to obtain a connected \( Y = X - e \), etc.

You end up with

Start again. The arguments you've used in the "good" case (where dominant variables form a coordinate system for the Kirchhoff space)

\[ (C^0) \xrightarrow{H_1} (C'_2) \]

Think of \( C'_1 \) as \( L \) as a partition of the edges in the graph.

Consider a tree:

\[ \mathbf{C}(X) \xrightarrow{\sim} C'_1 \]

\[ C_{10} = V_0 \]

\[ \mathbf{C}(X) \xrightarrow{\sim} C'_1 \]

\[ C_{10} = V_0 \]

\[ 0 \xleftarrow{\mathbf{C}(X)} C'_1 \xleftarrow{\sim} C_{10} \xleftarrow{\sim} H_1 \xleftarrow{\sim} 0 \]
The Kirchhoff space is just $C^0$, the space of node potentials, same as $C^1$, the space of edge voltage drops. $C_1$ is the space of currents (1-chains); there are no closed 1-chains. So if $C$-edges are used, you get $V_0 = C_0 I_0 = 0$.

So it's clear that the flow is 0 in the Kirchhoff space. A point of K.S. is any set of edge voltage drops.

So where are you now? Review!

Consider LC network. Good case: Where dominant variables $V_0$ or C-type are independent on $I_0$ or L-type $R = C^0 \times H_1$.

In the good case you are able to see the flow on $R$, because the dynamic relations

$$\dot{I}_0 = L_0^{-1} V_0$$

$$V_0 = C_0 I_0$$

can be expressed in terms of the dominant variables.
So what next??? Since $X$ has dimension $e$ there is always some set of $e$ edge variables which restricts to a coordinate system on $X$.

Recall problem: Flow on the Kirchhoff connected space $X = C_0 \otimes H_1$ for a LC network.

Possible approach: The flow should satisfy

\[ L_0 I_0 = V_0 \quad \text{for \ type L} \]
\[ C_0 V_0 = I_0 \quad \text{for \ type C} \]

You have total of $e$ dynamical equations, and $X$ has dim $e$. So you need some kind of non-degeneracy.

You understand the good case where the dominant variables form a coordinate system on $X$, equivalently $C_0 \sim C^1$ and $H_1 \sim C_{1L}$

Where next? Something involving $s = 0$, $\infty$ has to intervene.

Why? Use L.T. notation

\[ L_0 s I_0 = V_0 \quad \text{type L} \]
\[ C_0 s V_0 = I_0 \quad \text{type C} \]

It seems that interchanging $L, C$
V is the same as interchanging s and s⁻¹! Suppose you start with a connected LC network where the L number = l C number = v−l.

Now change the type of some edge.

Today Sept 15, 02 you want to study the symplectic structure at least in the good case. You want to understand linear symplectic picture. Symplectic vs. polarization into complementary Lagrangian subspaces. Obvious question: Is the flow Hamiltonian?

So what seems clear is that you get linear versions of maximal trees.
Aim: To understand the dynamics.

Let's begin with the linearization of the system for the graph. Review deleting an edge.

\[ Y = X - \sigma \]

\[ Y = Y \cup \sigma \]

\[ 0 \rightarrow C_1(Y) \rightarrow C_0(X) \rightarrow H_0(Y) \rightarrow 0 \]

where \[ C_0(X) : C^1(X) \rightarrow C^0(X) \rightarrow \]

\[ 0 \rightarrow H_1(Y) \rightarrow C_1(Y) \rightarrow C_0(Y) \rightarrow H_0(Y) \rightarrow 0 \]

\[ 0 \rightarrow H_1(X) \rightarrow C_1(X) \rightarrow C_0(X) \rightarrow H_0(X) \rightarrow 0 \]

\[ 0 \rightarrow Z_\sigma = Z_{\sigma} \rightarrow 0 \rightarrow 0 \rightarrow 0 \]
Let

\[ 0 \to H_1(Y) \to H_1(X) \to \mathbb{Z} \to H_0(Y) \to H_0(X) \to 0 \]

Two cases: 
1. \( Y \) disconnected: \( Y \) has 2 components \( Y = Y_+ \cup Y_- \) and \( X = Y_+ \cup Y_- \) and \( H_1(X) \to H_1(Y) \to H_1(Y) \to H_1(X) \to 2 \sigma \to 0 \) exact

Next you want a real version which should generalize at least the good LC network case. You real version of \( C \):

\[ 0 \to H_1(X) \to C_1(X) \to C_0(X) \to \mathbb{R} \to 0 \]

So you have a chain complex of chain 1 consisting of \( \mathbb{R} \)-vector spaces, such that \( H_0 = \mathbb{R} \).

Consider "removing an edge \( \sigma \)." This means I guess that \( \sigma : C_1(X) \to \mathbb{R} \) is a 1-cochain.

Again you have the two cases:
Either \( I \) stays same and \( Y \) disconnects, or \( Y \) conn. \( I \) decrease by 1.
Our next step is to polarize $C'$, dividing the edges into $L_L$ types.
What do you have?

$$0 \rightarrow H_1 \rightarrow C_1 \rightarrow C_0 \rightarrow R \rightarrow 0$$

$$\begin{array}{c}
C_{1,L} \oplus C_{1,R} \\
\end{array}$$

Simplify a bit

$$0 \rightarrow H_1 \rightarrow C_1 \rightarrow \bar{C}_0 \rightarrow 0 \text{ exact}$$

$$\begin{array}{c}
C^+ \oplus C^- \\
\end{array}$$

So you definitely have a Grass situation. Stick to the "good" case

$$\begin{array}{c}
\bar{C}_0 \leftrightarrow C_1 \rightarrow H_1 \\
\end{array}$$

What is the good case exactly?

$$\begin{pmatrix}
C' \\
C_1 \\
\end{pmatrix} = \begin{pmatrix}
C_{1,L} & C_{1,R} \\
C_{1,L} & C_{1,R} \\
\end{pmatrix}$$

You want natural maps from $\bar{C}_0$ to $C'_{1,1}$ and from $H_1$ to $C_{1,L}$ to be isos.
$$CV_0^I = V_0$$
$$L_0 V_0 = V_0$$

Idea yesterday. If the kinematics of an abstract graph amounts to

$$0 \rightarrow H_1 \rightarrow C_1 \rightarrow C_0 \rightarrow R \rightarrow O$$
or dually

$$0 \rightarrow \bar{C}_0 \rightarrow C' \rightarrow H' \rightarrow O$$, then the abstract LC network is

$$0 \rightarrow H_1 \rightarrow C_1 \rightarrow \bar{C}_0 \rightarrow O$$

So you probably have

Q: Where is "the dynamics" in this situation?
The "kinematics" $$\bar{C}_0 \rightarrow C' \rightarrow H'$$ is fixed.
The "dynamics" should be given by a quadratic form $$Q_\pm = s_+ Q_+ + s_- Q_-$$ on $$C'$$.

abstract LC network consists of f.d. $$R$$ vs.

$$0 \rightarrow H_1 \rightarrow C_1 \rightarrow \bar{C}_0 \rightarrow O$$

Together $$C_1 = C_{1,L} \oplus C_{1,C}$$ is a polarized Euclidean space.
Strange \( 0 \rightarrow H_i \rightarrow C_i \rightarrow C_0 \rightarrow 0 \)

This seems to be a typical (polarized) Euclidean Grassmannian situation.

\[\overline{C}_0 \rightarrow C' \rightarrow H' \]

\[\text{where } T : C'_C \rightarrow C'_L\]

\[\overline{C}_0 = (1)C'_C \subset C'_L\]

You want to get a flow on \(\overline{C}_0 \oplus H'\), which should be canonically isomorphic to \(C'_L\). The flow should be a skew-symmetric operator. Obvious candidate is \((0, T^*)\).

Get s into the picture