a

Old question: [redacted] Construction of Morita context linking $\mathcal{V}$ and $\mathcal{W}$. 1st [redacted] part: the cat $\mathcal{U}$ of $(V, W, \alpha_1, \beta_1)$ where $\sum s\beta_1\alpha_1 s^{-1} = 1_W$. Generators $\left(\begin{array}{c|c} 1 & \alpha_1 \\ \hline \beta_1 & \Gamma \end{array}\right)\begin{pmatrix} V \\ W \end{pmatrix}$

Outline: What are the unclear points?

Main unclear point is the link between P_Γ and A.

There's a surjection $P_\Gamma \twoheadrightarrow A$ $\quad p(s) \mapsto \alpha_1 s \beta_1$ compatible with the grading.

$C = \left[\begin{array}{c|c} A & Y \\ \hline X & B \end{array}\right]$ $\quad A, C$ [redacted] are non unital; [redacted] B is unital

It should be clear that B and C are Morita equivalent, because? Look at dual pairs over B. [redacted]

B generated [redacted] by $t\beta_1\alpha_1 s^{-1}$, A generated by $\alpha_1 s^{-1} t\beta_1$

$X = \beta_1 A = ?$ You have [redacted] words $t\beta_1\alpha_1 s^{-1}$? The generators are $\left(\begin{array}{c|c} 0 & \alpha_1 s^{-1} \\ \hline t\beta_1 & 0 \end{array}\right) = \begin{pmatrix} 0 & y_s \\ x_t & 0 \end{pmatrix}$

$X = \sum_t x_t A =$ [redacted] $= \sum_t B x_t$

$Y = \sum y_s B = \sum A y_s$

$x_t(y_{s_1}x_{t_1})\cdots(y_{s_n}x_{t_n})$, $y_s(x_{t_1}y_{s_1})\cdots(x_{t_n}y_{s_n})$

$X = \sum_t t\beta_1 A$ [redacted] $=$ [redacted] $B\beta_1$, $Y =$ [redacted] $\alpha_1 B = \sum_s A\alpha_1 s^{-1}$

DL 4521 departure gate C21

Assembly construction, Serre's thm. go over to motivate retract of a free Γ-module $\Lambda \otimes V$

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b Apr 2, 2002

Assembly Construction. Consider infinite disc grp Γ, a principal Γ-bundle $\pi: P \longrightarrow X$, X compact.

(Γ acts on the left; locally over X $\exists$ continuous section $\sigma: K \longrightarrow P$ such that $\Gamma \times K \xrightarrow{\simeq} P_K$)

E = associated fibre bundle with fibre the group alg $\Lambda = \mathbb{C}[\Gamma] = \bigoplus_{s \in \Gamma} \mathbb{C}[s]$. Given $\sigma: K \longrightarrow P$ get

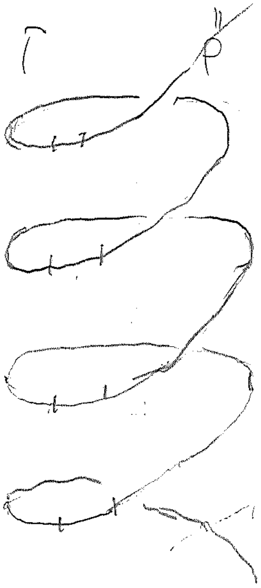
$$\Lambda \otimes K \xrightarrow{\sim} P_K, \qquad E_K = \Lambda \times_{\Gamma} P_K$$

Analog of line bundle where $\mathbb{C}$ replaced by Λ

Sections of E_K over K = $C(K) \otimes \Lambda$

Global sections $\Gamma(X, E)$ module over $\Lambda \otimes C(X)$.

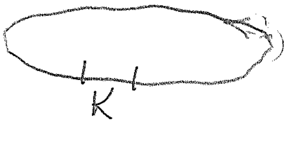
$$\Gamma = \mathbb{Z}, \quad \begin{tikzcd} \mathbb{Z} \arrow[r, tail] & \mathbb{R} \arrow[r, two heads, "\pi"] & \mathbb{R}/\mathbb{Z} \\ \Gamma & \overset{\shortparallel}{P} & X \end{tikzcd}$$



$\pi^{-1}(x) = y + \mathbb{Z}$ y: any coset rep.

$$E_x \cong \mathbb{C}[y+\mathbb{Z}] = \bigoplus_{n \in \mathbb{N}} \mathbb{C}[y+n]$$

$$C(\mathbb{R}/\mathbb{Z}) \otimes \mathbb{C}[\mathbb{Z}]$$



Next Serre's thm. Vector bundles over a compact space X are the same as finitely generated projective modules over the alg $C(X)$.

c   Trivial vector bundle $\begin{matrix} X \times V \\ \downarrow \\ X \end{matrix}$ space of sections is $C(X, V)$

Given $E = P \times_{\Gamma} \Lambda$ $\quad X = \bigcup U_i$

$$\begin{tikzcd} E_{U_i} & U_i \times (\Lambda \otimes V_i) \arrow[l, "\beta_i"'] & E_{U_i} \arrow[l, "\alpha_i"'] \end{tikzcd} \qquad \beta_i \alpha_i = 1$$

First do vector bundle case $\quad X = \bigcup U_i, \quad \sum_{\mathrm{Supp}} x_i^2 = 1,$

$$\begin{tikzcd} E_{U_i} & U_i \times V_i \arrow[l, "\beta_i"'] & E_{U_i} \arrow[l, "\alpha_i"'] \end{tikzcd} \qquad \beta_i \alpha_i = 1.$$

$$\begin{tikzcd} E & X \times \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \arrow[l, "(x_1\beta_1 \;\; x_2\beta_2)"'] & E \arrow[l, "\begin{pmatrix} x_1\alpha_1 \\ x_2\alpha_2 \end{pmatrix}"'] \end{tikzcd}$$

$P \xrightarrow{\pi} X$ principal Γ-bundle

$\pi^{-1}U \simeq U \times \Gamma$, $\quad E_U \simeq U \times \Lambda$

Assume
$X = \bigcup U_i$
$\mathrm{Supp}(x_i) \subset U_i$
$\sum' x_i^2 = 1.$

$$\begin{tikzcd} E & X \times \begin{bmatrix} \vdots \\ V_i \otimes \Lambda \\ \vdots \end{bmatrix} \arrow[l, "{[\cdots x_i\beta_i \cdots]}"'] & E \arrow[l, "\begin{bmatrix} \vdots \\ x_i\alpha_i \\ \vdots \end{bmatrix}"', "f"] \end{tikzcd}$$

$\Gamma = \mathbb{Z}$ $\qquad \mathbb{Z} \rightarrowtail \mathbb{R} \xrightarrow{\pi} \mathbb{R}/\mathbb{Z}$ $\qquad \pi(x) = x + \mathbb{Z}$

E is a retract of $X \times (V \otimes \Lambda)$

trivial bundle $X \times (\Lambda \otimes V)$ has space of sections

$C(X, \Lambda \otimes V) = C(X) \otimes \Lambda$

Output is a retract of the trivial bundle $X \times \overbrace{(\Lambda \otimes V)}^{\Lambda^n}$ which is the same as a retract of the free $C(X) \otimes \Lambda$ module c' Serre's thm E vector bundle over X compact

$$X = \bigcup_1^n U_i \ , \quad \sum \chi_i^2 = 1 \ , \quad \mathrm{Supp}(\chi_i) \subset U_i$$

$$\begin{tikzcd} E_{U_i} & U \times V_i \arrow[l, "\beta_i"'] & E_{U_i} \arrow[l, "\alpha_i"'] \end{tikzcd} \qquad \beta_i\alpha_i = 1_{E_{U_i}}$$

$$\begin{tikzcd} E & X \times \begin{pmatrix} V_1 \\ \vdots \\ V_n \end{pmatrix} \arrow[l, "(\chi_1\beta_1 \cdots \chi_n\beta_n)"'] & E \arrow[l, "\begin{pmatrix} \chi_1\alpha_1 \\ \chi_n\alpha_n \end{pmatrix}"'] \end{tikzcd}$$

$$(\chi_1\beta_1 \cdots \chi_n\beta_n)\begin{pmatrix} \chi_1\alpha_1 \\ \chi_n\alpha_n \end{pmatrix} = \sum_i \chi_i^2\beta_i\alpha_i = \sum_i \chi_i^2 = 1.$$

Same arg shows in the case of the assembly construction

$$E_{U_i} = P_{U_i} \times_\Gamma \mathbb{C}\Gamma \simeq (U_i \times \Gamma) \times_\Gamma \mathbb{C}\Gamma = U_i \times \mathbb{C}\Gamma$$

$\Longrightarrow$ E retract of the trivial bundle $X \times \Lambda^n$

This is the same as an idempotent $n \times n$ matrix over Λ. $\Longrightarrow$ you get class $\in K_0(C(X) \otimes \Lambda)$

d **First part.** The assembly constructions starts from a principal Γ-bundle $P \xrightarrow{\pi} X$, X compact and produces a retract of a $\overset{\text{fin. gen.}}{\text{a}}$ free module over $C(X)\otimes\Lambda$. Get class in $\overset{\chi(P)}{\in} K_0(C(X)\otimes\Lambda)$

$$C(X)\otimes\Lambda \subset C(X)\hat{\otimes} C^*_r\Gamma \qquad C^* \text{ alg completion}$$

$$\chi(P) \in K_0(C(X)\otimes\Lambda) \longrightarrow K_0(C(X)\hat{\otimes} C^*_r\Gamma)$$

ex. $\Gamma = \mathbb{Z}$, $C^*_r(\mathbb{Z}) = C(\mathbb{T})$ $\qquad \chi(P) \in K_0(C(\underbrace{\mathbb{R}/\mathbb{Z}\times\mathbb{T}}_{\text{2 torus.}}))$

2nd part. Retracts of a free Λ-module $\Lambda\otimes V$

Assume Γ finite group, to simplify. $\qquad \beta\alpha = 1$, $\alpha\beta = p$

$$\begin{tikzcd}
W & \Lambda\otimes V \arrow[l, two heads, "\beta"'] \arrow[d, shift left, "\eta_1"] & W \arrow[l, "\alpha"'] \arrow[dl, "\alpha_1 = \eta_1\alpha"] & \Lambda\otimes V \arrow[l, two heads, "\beta"'] \\
 & V \arrow[u, shift left, "\varepsilon_1"] \arrow[ul, two heads, "\beta_1 = \beta\varepsilon_1"] & &
\end{tikzcd}$$

$$1_{\Lambda\otimes V} = \sum_s s(\varepsilon_1\eta_1)s^{-1} \Rightarrow 1_W = \sum s\beta_1\alpha_1 s^{-1}$$

W retract of $\Lambda\otimes V$ $\Longrightarrow$ W Γ-module plus operator $h = \beta_1\alpha_1$ satisfying $\sum shs^{-1} = 1_W$ (equivariant partition of 1)

$$\Lambda\otimes V = \left\{\sum_t t\otimes f(t) \,\middle|\, f:\Gamma\to V\right\}$$

$$\boxed{\beta\sum_t t\otimes f(t) = \sum_t t\beta_1 f(t), \qquad \alpha w = \sum_s s\otimes \alpha_1 s^{-1} w}$$

$$\overset{\alpha\beta}{\overset{\|}{p}}\sum_t t\otimes f(t) = \sum_s s\otimes \sum_t \underbrace{\alpha_1 s^{-1} t \beta_1}_{p(s^{-1}t)} f(t)$$

$$\alpha_1 W = \left\{\alpha_1\left(\sum_s s\otimes\sum_t p(s^{-1}t)f(t)\right)\right\} = \left\{\sum_t p(t)f(t) \,\middle|\, f:\Gamma\to V\right\}$$

$$\alpha\beta_1 v = \alpha(1\otimes v) = \sum_s s\otimes p(s^{-1})v$$

e Equivalence of categories

$\mathcal{W}$ category of (W, h) where W is a Γ-module and h is a $\mathbb{C}$-linear operator satisf $\sum_s s h s^{-1} = 1_W$

$\mathcal{V}$ category of $(V, \{p(s)\})$ where V is a vector space with $\mathbb{C}$-linear operators $p(s)$, $s \in \Gamma$ satisf.

$$p(u) = \sum_{u=st} p(s)p(t) \quad ; \qquad V = \sum_t p(t)V \, , \quad \bigcap_s \mathrm{Ker}\{p(s) \text{ on } V\} = 0$$

Claim $\mathcal{W}$ and $\mathcal{V}$ are equivalent categories.

Pf. Introduce $\mathcal{U}$ cat of $(W, V, \beta_1, \alpha_1)$ where W is a Γ-module, V is a vector space, and $\beta_1 : W \hookleftarrow V$, $\alpha_1 : V \twoheadleftarrow W$ are $\mathbb{C}$ linear maps such that $\sum s\beta_1\alpha_1 s^{-1} = 1_W$, β_1 injective and α_1 surjective.

Forget V functor : $\mathcal{U} \longrightarrow \mathcal{W}$

$$(W, V, \beta_1, \alpha_1) \longmapsto (W, h = \beta_1\alpha_1)$$

Forget W functor: $(W, V, \beta_1, \alpha_1) \longmapsto (V, p(s) = \alpha_1 s \beta_1)$

$$\sum_t p(st^{-1})p(t) = \sum_t \alpha_1 s t^{-1} \beta_1 \alpha t \beta_1 = \alpha_1 s \beta_1 = p(s)$$

J Cuntz Non-Comm. Simp Complexes + the Baum-Connes conjecture.

f_1

$$\begin{tikzcd} A \arrow[r, "\Delta"] \arrow[d, "\Delta'"'] \arrow[dr, phantom, "\text{comm.}"] & \mathbb{C}\Gamma \otimes A \\ \mathbb{C}\Gamma_+ \otimes A \arrow[ur] & \end{tikzcd}$$

both Δ, Δ' send p_{ij} to $e_{ij} \otimes p_{ij}$

where $e_{ij} \in \Gamma \subset \Gamma_+$

You have to decide whether or not A is Γ_+-graded. Let G be a semigroup

$$A = \bigoplus_{s \in G} A_s$$ a G-graded algebra

i.e. $A_s A_t \subset A_{st}$. Then

You want A to be defined by generators and relns which are homog wrt G.

$$G = \Gamma_+ = \{e_{ij}\} \amalg \{*\}$$

gen p_{ij} of degree e_{ij}

rels $\sum_j p_{ij} p_{jk} = p_{ik}$ homog of degree ik

$p_{ij} p_{k\ell} = 0$ $j \neq k$ [homog of degree $*$.

$$\Delta : A \longrightarrow \mathbb{C}\Gamma_+ \otimes A$$

p_{ij} $e_{ij} \otimes p_{ij}$

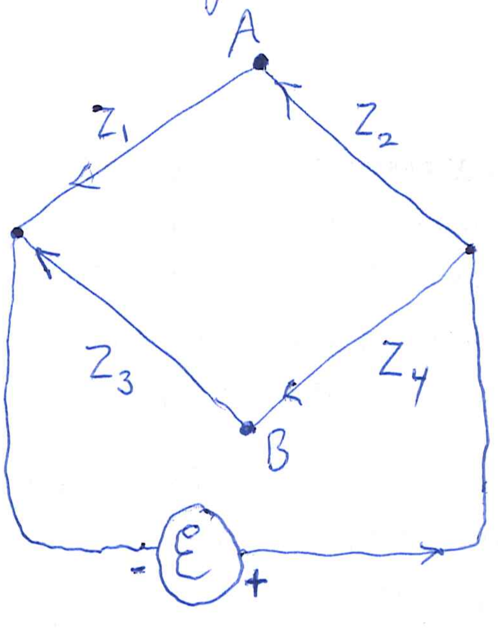
$(e_{ij} \otimes p_{ij})(e_{k\ell} \otimes p_{k\ell})$

$[*] \otimes \underbrace{p_{ij} p_{k\ell}}_{0}$

You don't understand properly the counit

A

Example. "Wheatstone" bridge with complex impedances



$$\varphi(A) = Z_1 \overbrace{\frac{\mathcal{E}}{Z_1+Z_2}}^{\text{current thru A}}$$

$$\varphi(B) = Z_3 \frac{\mathcal{E}}{Z_3+Z_4}$$

$$\mathcal{E}_0 = \left(\frac{Z_1}{Z_1+Z_2} - \frac{Z_3}{Z_3+Z_4}\right)\mathcal{E}$$

$$Z_0 = \frac{Z_1 Z_2}{Z_1+Z_2} + \frac{Z_3 Z_4}{Z_3+Z_4}$$

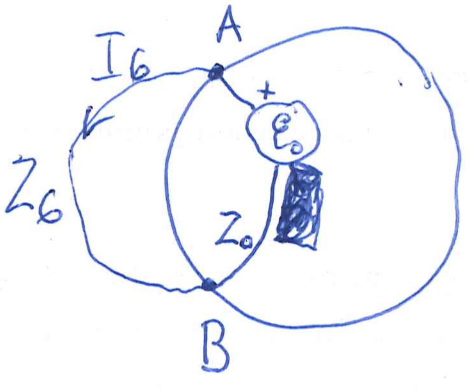
balance condition

$$0 = \mathcal{E}_0 = Z_1(\cancel{Z_3}+Z_4) - (\cancel{Z_1}+Z_2)Z_3$$

$$\therefore Z_1 Z_4 = Z_2 Z_3 \qquad \frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

Last step is to add Z_6 between A, B

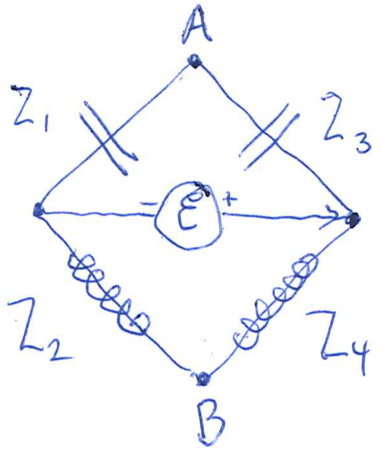
Yes.



$$(Z_6+Z_0)I_6 = \mathcal{E}_0$$

$$I_6 = \frac{\mathcal{E}_0}{Z_6+Z_0} = \frac{\left(\frac{Z_1}{Z_1+Z_2} - \frac{Z_3}{Z_3+Z_4}\right)\mathcal{E}}{\left(\frac{Z_1 Z_2}{Z_1+Z_2} + \frac{Z_3 Z_4}{Z_3+Z_4} + Z_6\right)}$$

B



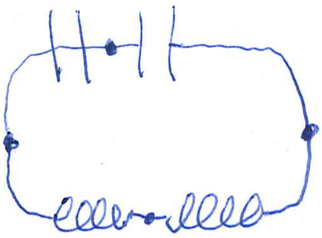
Balance condition

$$Z_1 Z_4 = Z_2 Z_3$$

i.e. when $\varphi_A - \varphi_B = 0$

$$\mathcal{E}_0 = Z_1 \frac{\mathcal{E}}{Z_1 + Z_3} - Z_2 \frac{\mathcal{E}}{Z_2 + Z_4} = \varphi_A - \varphi_B$$

$$Z_0 = \frac{Z_1 Z_3}{Z_1 + Z_3} + \frac{Z_2 Z_4}{Z_2 + Z_4}$$



$(4-1)+2 = 5$

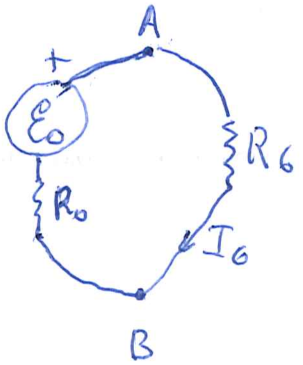
$$\frac{1}{C_1 s} + \frac{1}{C_3 s}$$

What is the problem of interest? The 2 port going from the $\mathcal{E}$ terminals to AB

$$\mathcal{E}_0 = \left(\frac{Z_1}{Z_1 + Z_2} - \frac{Z_3}{Z_3 + Z_4}\right)\mathcal{E}$$

$$Z_0 = \frac{Z_1 Z_2}{Z_1 + Z_2} + \frac{Z_3 Z_4}{Z_3 + Z_4}$$

$$I_{AB} = \frac{\mathcal{E}_0}{Z_0} = \frac{\{Z_1(Z_3 + Z_4) - Z_3(Z_1 + Z_2)\}\mathcal{E}}{Z_1 Z_2 (Z_3 + Z_4) + Z_3 Z_4 (Z_1 + Z_2)}$$

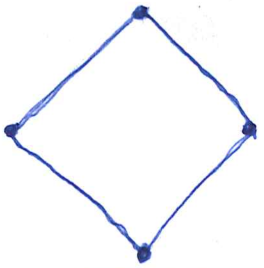


$$I_6 (R_0 + R_6) = \mathcal{E}_0$$

C Study bridge

$v=4$, $e=4$, $l=1$

$v-1+l = 3+1 = 4 = e$



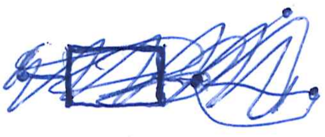
Program: Review the good case in which, where the dominant variables are linearly independent on the state space (Kirchhoff space).

Note that because the Kirchhoff space $\mathcal{K}$ is $\subset$ $\mathcal{C}^1 \oplus C_1$ there ~~exists a subspace~~ is some set of e edge variables which is independent on $\mathcal{K}$. In fact $\mathcal{K} = \bar{\mathcal{C}}^0 \oplus H_1 \subset \mathcal{C}^1 \oplus C_1$, so you can pick any $v-1$ voltage variables ind on $\bar{\mathcal{C}}^0$ and any l current " " " H_1.
(The good case is where $\bar{\mathcal{C}}^0 \xrightarrow{\sim} \mathcal{C}^1_{\mathbf{C}}$, $H_1 \xrightarrow{\sim} C_{1,L}$.)

Now the idea is to ignore $s=0, \infty$ i.e. localize to make $s=\partial_t$ invertible, Then the difference between dominant + recessive variables shouldn't matter.

Look at these vector spaces properly — replace by free modules over $\mathbb{C}[s, s^{-1}]$.

How to handle free edges? Kirchhoff says $I=0$



$L\dot{I} = V \implies V=0$

$C\dot{V} = I \implies V = \text{const}$

D The problem? Consider a connected LC network, examine free motion, exponential solution, system has (two real) coordinates (V_σ, I_σ) for each edge σ, a (Feynmann) path for the system is a function of time $(\vec{V}_t, \vec{I}_t)$ with values in $C^1 \oplus C_1$. You have Kirchhoff constraints: $\vec{V}_t \in \bar{C}^0$, $\vec{I}_t \in H_1$. Also have dynamical conditions

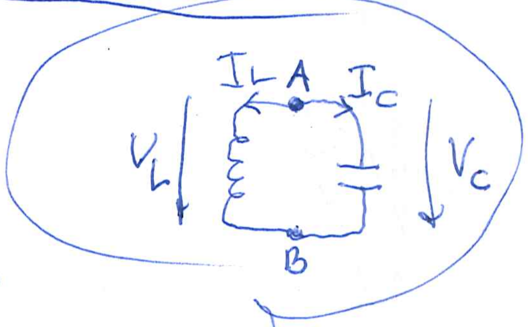
$$\begin{cases} L_\sigma \partial_t I_{t,\sigma} = V_{t,\sigma} & \sigma \text{ L type} \\ C_\sigma \partial_t V_{t,\sigma} = I_{t,\sigma} & \sigma \text{ C type} \end{cases}$$

(exp s) exp. solution has form $e^{st}(\vec{V}, \vec{I})$, where $(\vec{V}, \vec{I}) \in \bar{C}^0 \oplus H_1$

and

$$L_\sigma s I_\sigma = V_\sigma \quad \sigma \text{ L-type}$$
$$C_\sigma s V_\sigma = I_\sigma \quad \sigma \text{ C-type.}$$

$$\begin{tikzcd} \bar{C}^0 \arrow[r, hook] & C^1 \arrow[d, leftrightarrow, "Z_s"] & \\ & C_1 & H_1 \arrow[l, hook'] \end{tikzcd}$$



It should be obvious that an exponential solution with $s \neq 0, \infty$ should be a pair $(\vec{V}, \vec{I}) \in \bar{C}^0 \oplus H_1$ such that $Z_s \vec{I} = \vec{V}$

Continue with example

variables V_L, I_L, V_C, I_C coordinatize $C^1 \oplus C_1$

Kirchhoff $I_L + I_C = 0$ $(1, -1)$ gen H_1

$V_L = V_C$

E

H_1 gen. by $[L] - [C]$

$$Z_s([L]-[C]) = \left(Ls, \frac{-1}{Cs}\right)$$

$$\begin{tikzcd}
\bar{C}^0 \arrow[r, hook] & C^1 \\
V \arrow[r, mapsto] & (V_L, V_C) \arrow[d, mapsto] \\
~~([A]-[B])~~ & \left(Ls[L]I_L, \frac{1}{Cs}[C]I_C\right) \arrow[l, mapsto]
\end{tikzcd}$$

$$([A]-[B])\left(Ls I_L + \frac{1}{Cs} I_C\right).$$

Notation very confused.

But you do learn that an exponential solution frequency s is a null vector for the quadratic form

$$\bar{C}^0 \hookrightarrow C^1 \longrightarrow C_1 \longrightarrow \bar{C}_0$$

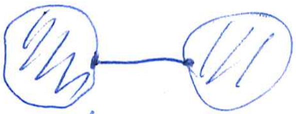
same as 0 power?

Better would be

$$\begin{tikzcd}
\bar{C}^0 \arrow[r, hook] \arrow[d, dashed] & C^1 \arrow[d] & \\
\bar{C}_0 & C_1 \arrow[l, two heads] & H_1 \arrow[l, hook']
\end{tikzcd}$$

Idea. Bring in Grassmannian stuff, need to be careful about signs.

F [redacted] What about



You want to write up [redacted] what you've learned. Let's start with an LC network (conn.). You want to keep track of symplectic structure, power.

Obvious question: When is [redacted] the time evolution for a closed LC network [redacted] given the Hamiltonian flow arising from the symplectic structure and the power quadratic form? You believe this is true for a ladder network.

Another question: free edges in a ladder network

_______________: suppose you have the good case where the dominant variables are independent on the Kirchhoff spaces. Is the question above about Hamiltonian flow true?

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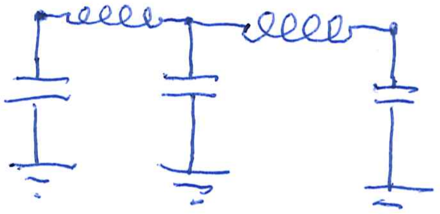
dominant variables indep means that the composites

$$\bar{C}^0 \hookrightarrow C^1 \twoheadrightarrow C^1_{\mathbf{C}} \qquad L_\sigma \dot{I}_\sigma = V_\sigma$$

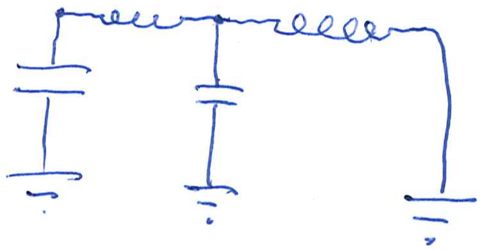
$$H_1 \hookrightarrow C_1 \twoheadrightarrow C_{1,\mathbf{L}} \qquad C_\sigma \dot{V}_\sigma = I_\sigma$$

are isos. This implies that $l = \dim(H_1) =$ no. of L_σ's

$V-1 = \dim(\bar{C}^0) =$ —— C_σ's



$V-1=3$



$V-1$ ~~2~~ $= 3-1$ [redacted] 2

$l=2$

6 Analyze $\bar{\mathcal{C}}^0 \xrightarrow{\sim} \mathcal{C}^1_{C}$

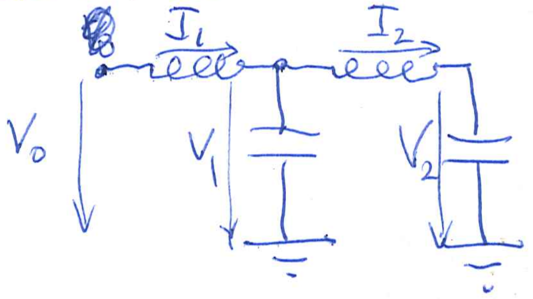
You have a graph with v nodes and $v-1$ edges, ~~no loops~~.

Maybe better to analyze $\bar{\mathcal{C}}^0 \xrightarrow{\sim} \mathcal{C}^1_{C}$

$H_1 \xrightarrow{\sim} \mathcal{C}_{1,L}$

seems too hard

Look at symplectic picture, try to see whether the free motion of a connected LC network is a Hamiltonian flow with Hamiltonian the power quadratic form. Ex ladder network.



$$\overset{0}{\overset{\shortparallel}{V_0}} - V_1 = L_1 s I_1 \qquad I_2 - \overset{0}{\overset{\shortparallel}{I_3}} = C_2 s V_2$$

$$I_1 - I_2 = C_1 s V_1$$

$$V_1 - V_2 = L_2 s I_2$$

$$s\begin{bmatrix} L_1 & & & \\ & C_1 & & \\ & & L_2 & \\ & & & C_2 \end{bmatrix}\begin{bmatrix} I_1 \\ V_1 \\ I_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} & -1 & & \\ 1 & & -1 & \\ & 1 & & -1 \\ & & 1 & \end{bmatrix}\begin{bmatrix} I_1 \\ V_1 \\ I_2 \\ V_2 \end{bmatrix}$$

$$\begin{vmatrix} 1 & -1 & 0 \\ & & -1 \\ & 1 & \end{vmatrix} = +1$$

H Scratch for Sept 11. At some point what's important is the symplectic picture, which should be a linear algebra translation of the physics.

Kinematics should correspond to symplectic structure

Dynamics ———————————————— Lagrangian subspaces

You conjecture that LC networks form a special case of "linear phase space + quadratic Hamiltonian" physics.

You would like a good theory of "glueing" that is attaching networks together. This means you need a notion of [redacted] $\partial M \subset N$. Can you adapt cobordism ideas?

Recall where you left forced **harmonic** oscillators. Problem of subspace of phase space, in which the forcing term is supported, generated by the forcing term.

Perhaps you can use the Grassmannian picture

I Go back to a connected LC network where the dominant coords are (currents I_τ τ L type
voltage drops V_σ σ C type

$$L_\tau s \dot{I}_\tau = V_\tau, \quad C_\sigma s I_\sigma \ V_\sigma$$

Assume the dom. coords form a basis for linear fnls on the Kirchhoff space $\bar{\mathcal{C}}^0 \oplus H_1$.

Means $\begin{pmatrix}\bar{\mathcal{C}}^0 \\ H_1\end{pmatrix} \hookrightarrow \begin{pmatrix}\mathcal{C}^1 \\ \mathcal{C}_1\end{pmatrix} \twoheadrightarrow \begin{pmatrix}\mathcal{C}^1_C \\ \mathcal{C}_{1L}\end{pmatrix}$ is an isom.

i.e. $\bar{\mathcal{C}}^0 \xrightarrow{\sim} \mathcal{C}^1_C$ and $H_1 \xrightarrow{\sim} \mathcal{C}_{1L}$

Look at the condition. Depends on the subnetwork containing all nodes and C-type edges.

Try removing one edge at a time.

$$\begin{tikzcd}
 & \mathcal{C}^1_L \arrow[d] \arrow[dr, "\sim"] & \\
\bar{\mathcal{C}}^0 \arrow[r, hook] \arrow[dr, "\sim"'] & \mathcal{C}^1 \arrow[r, two heads] \arrow[d] & H^1 \\
 & \mathcal{C}^1_C &
\end{tikzcd}$$

Maybe the good viewpt is to look at

$$\begin{tikzcd}
\bar{\mathcal{C}}^0 \arrow[r, hook] \arrow[dr, "\sim"'] & \mathcal{C}^1 \arrow[r, two heads] \arrow[d] & H^1 \\
 & \mathcal{C}^1_C &
\end{tikzcd}$$

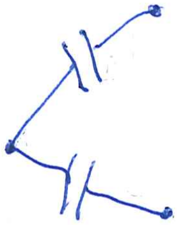
as functors with respect to inclusion seems clearer.

[redacted]

[redacted]

~~Certificate of insurance~~
~~Registration document~~

J So it seems that when the domin. coords ~~[illegible]~~ form a basis for $\mathcal{K}$, that then the network is a tree of ~~[illegible]~~ C-edges, with L' edges adjoined.



~~[illegible]~~

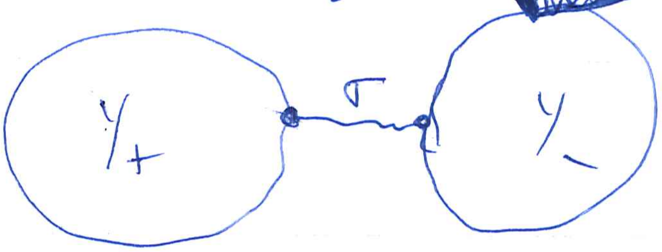
Start with a connected graph X. You would like remove an edge σ without changing $\bar{C}^0(X)$. ~~It's probably better to~~

~~Probably $H^0(X)$~~ For X connected $C^0(X) = H^0(X)$ = functions E: {nodes} → $\mathbb{R}$ and $\bar{C}^0(X) = H^0(X)/$ constant functions on the nodes. If X conn then

$$0 \to \mathbb{R} \to C^0(X) \xrightarrow{\delta} C^1(X) \to H^1(X) \to 0$$

leading to $-1 + v - e + l = 0$ or $e = (v-1) + l$

Now you want to remove an edge σ in X. Let $Y = X - \mathrm{Int}(\sigma)$. Picture



$Y = Y_+ \amalg Y_-$

K You want to carefully analyze the ~~what happens~~ ~~start~~ restriction maps from X to Y.

$$\begin{tikzcd}
0 \arrow[r] & \mathbb{R} \arrow[r] \arrow[d] & C^0(X) \arrow[r, "\delta"] \arrow[d, "S"] & C^1(X) \arrow[r] \arrow[d] & H^1(X) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & H^0(Y) \arrow[r] \arrow[d, equal] & C^0(Y) \arrow[r, "\delta"] \arrow[d, equal] & C^1(Y) \arrow[r] \arrow[d, equal] & H^1(Y) \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & \begin{pmatrix} H^0(Y_+) \\ H^0(Y_-) \end{pmatrix} \arrow[r] & \begin{pmatrix} C^0(Y_+) \\ C^0(Y_-) \end{pmatrix} \arrow[r, "\delta"] & \begin{pmatrix} C^1(Y_+) \\ C^1(Y_-) \end{pmatrix} \arrow[r] & \begin{pmatrix} H^1(Y_+) \\ H^1(Y_-) \end{pmatrix} \arrow[r] & 0
\end{tikzcd}$$

Clear that X conn. ⟹ Y_+ and Y_- conn.

because if Y_+ ~~$= A \sqcup B$ (conn, $\neq \phi$) then σ has to join Y_- to A or B, so either $A \cup Y_-$~~ is not conn, then σ joins a conn. comp of A to one of B which is a component of X.

$$\begin{tikzcd}
 & & 0 \arrow[d] \\
 & & \mathbb{R}\sigma \arrow[d] \\
0 \arrow[r] & \bar{C}^0(X) \arrow[r, "\delta"] \arrow[d, "S"] & C^1(X) \arrow[d] \\
0 \arrow[r] & \begin{pmatrix} \bar{C}^0(Y_+) \\ \bar{C}^0(Y_-) \end{pmatrix} \arrow[r] & \begin{pmatrix} C^1(Y_+) \\ C^1(Y_-) \end{pmatrix} \arrow[d] \\
 & & 0
\end{tikzcd}$$

$$\begin{tikzcd}
 & 0 & 0 \arrow[d] & 0 \arrow[d] & 0 & \\
 & 0 \arrow[d] & 0 \arrow[r] \arrow[d] & \mathbb{R}\sigma \arrow[d] & \cancel{\mathbb{R}\sigma} & \\
0 \arrow[r] & \mathbb{R} \arrow[r] \arrow[d, "\binom{1}{1}"] & C^0(X) \arrow[r, "\delta"] \arrow[d, "s"] & C^1(X) \arrow[r] \arrow[d] & H^1(X) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & \begin{pmatrix} \mathbb{R} \\ \mathbb{R} \end{pmatrix} \arrow[r] \arrow[d] & \begin{pmatrix} C^0(Y_+) \\ C^0(Y_-) \end{pmatrix} \arrow[r, "\delta"] \arrow[d] & \begin{pmatrix} C^1(Y_+) \\ C^1(Y_-) \end{pmatrix} \arrow[r] \arrow[d] & \begin{pmatrix} H^1(Y_+) \\ H^1(Y_-) \end{pmatrix} \arrow[r] & 0 \\
 & 0 & 0 & 0 & 0 &
\end{tikzcd}$$

$$\begin{tikzcd}
 & 0 \arrow[d] & 0 \arrow[d] & 0 \arrow[d] & \\
 & 0 \arrow[r] \arrow[d] & \mathbb{R} \arrow[r] \arrow[d] & \begin{pmatrix} \mathbb{R} \\ \mathbb{R} \end{pmatrix} \arrow[d] \arrow[dddll, bend left=60] & \\
0 \arrow[r] & 0 \arrow[r] \arrow[d] & C^0(X) \arrow[r, "\sim"] \arrow[d] & \begin{pmatrix} C^0(Y_+) \\ C^0(Y_-) \end{pmatrix} \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & \mathbb{R}\sigma \arrow[r] \arrow[d] & C^1(X) \arrow[r] \arrow[d] & \begin{pmatrix} C^1(Y_+) \\ C^1(Y_-) \end{pmatrix} \arrow[r] \arrow[d] & 0 \\
 & \mathbb{R}\sigma \arrow[r] \arrow[d] & H^1(X) \arrow[r] \arrow[d] & \begin{pmatrix} H^1(Y_+) \\ H^1(Y_-) \end{pmatrix} \arrow[r] \arrow[d] & 0 \\
 & 0 & 0 & 0 &
\end{tikzcd}$$

M What do you want? ~~Key You~~ ~~start with $\bar{\mathcal{C}}^0(X) \rightarrow$~~ Go back to the assumption that

~~$\bar{\mathcal{C}}^0(X) \oplus H_1(X)$~~

$$\bar{\mathcal{C}}^0(X) \xrightarrow{\sim} \mathcal{C}^1_c(X), \quad H_1(X) \xrightarrow{\sim} C_{1L}(X)$$

be careful

$$\begin{tikzcd}
\begin{pmatrix} \bar{\mathcal{C}}^0(X) \\ H_1(X) \end{pmatrix} \arrow[r, hook] \arrow[rr, bend right, "\sim"'] & \begin{pmatrix} \mathcal{C}^1(X) \\ \mathcal{C}_1(X) \end{pmatrix} \arrow[r, two heads] & \begin{pmatrix} \mathcal{C}^1_c(X) \\ \mathcal{C}_{1L}(X) \end{pmatrix}
\end{tikzcd}$$

You do get that $\bar{\mathcal{C}}^0(X) \xrightarrow{\sim} \mathcal{C}^1_c(X)$

Suppose you remove an L edge?

~~What~~ Firm up what you believe, namely, that in the "good" case, you have a maximal tree of C-edges and the remaining edges have L-type, and there's an obvious ~~correspondence between L edges and~~ basis for H_1 corresponding to the L edges, namely you ~~complete~~ close the L edge by the path joining its boundary in the tree.

N You learn that $\mathcal{C}^0(X) \xrightarrow{\sim} \mathcal{C}^0(Y)$

$H^1(X) \xrightarrow{\sim} H^1(Y)$

but $\mathcal{C}^1(X) \overset{\text{maps onto}}{\twoheadrightarrow} \mathcal{C}^1(Y)$ with kernel $\mathbb{R}$

$$\begin{tikzcd}
 & & & 0 \arrow[d] & & \\
 & & & \mathbb{R}\sigma \arrow[d] & & \\
0 \arrow[r] & \mathbb{R} \arrow[r] \arrow[d] & \mathcal{C}^0(X) \arrow[r, "\delta"] \arrow[d, "S", "\wr"'] & \mathcal{C}^1(X) \arrow[r] \arrow[d] & H^1(X) \arrow[r] \arrow[d, "\wr"] & 0 \\
0 \arrow[r] & \mathbb{R}\times\mathbb{R} \arrow[r] & \mathcal{C}^0(Y) \arrow[r, "\delta"] & \mathcal{C}^1(Y) \arrow[r] \arrow[d] & H^1(Y) \arrow[r] & 0 \\
 & & & 0 & &
\end{tikzcd}$$

What do you want to happen? Look at the case where the graph remains connected.

$$\begin{tikzcd}
 & & & \mathbb{R}\sigma \arrow[d] \arrow[r, phantom, "\cong"] & \mathbb{R}\sigma \arrow[d] & \\
0 \arrow[r] & \mathbb{R} \arrow[r] \arrow[d, equal] & \mathcal{C}^0(X) \arrow[r] \arrow[d, "S"] & \mathcal{C}^1(X) \arrow[r] \arrow[d] & H^1(X) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & \mathbb{R} \arrow[r] & \mathcal{C}^0(Y) \arrow[r] & \mathcal{C}^1(Y) \arrow[r] & H^1(Y) \arrow[r] & 0
\end{tikzcd}$$

[redacted] You want to remove the L edges, and you need the graph to stay connected. This maybe follows from ~~[illegible]~~ the fact that the L-edges yield a basis in $H_1(X)$.

N′ What can you do with

$$\bar{C}^0(X) \xrightarrow{\sim} C^1_c(X)$$

$\bar{C}^0$ rank $V-1$

∴ $V-1$ C edges

ℓ L edges

Hope: That extra components yield too many vertices.

Suppose then you have a graph X

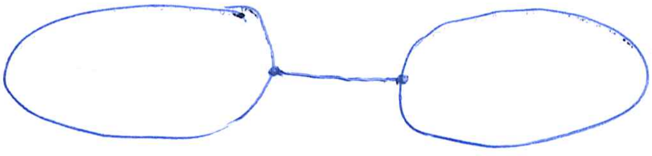
Consider X a conn. graph, σ an edge such that $X - \sigma = Y$ is disconnected. Put another way, Y is disconnected but $X \cup \sigma$ is connected. Picture

Y 

possible attaching 

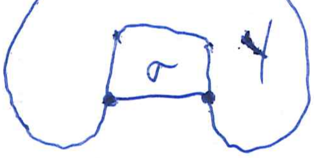
Clearly Y has 2 components.

Y : 

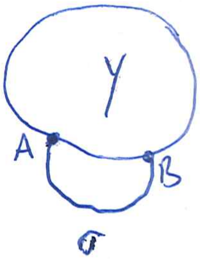
X : 

Kinematics again. X connected graph σ edge of X, $Y = X - \sigma$. Two cases

Y connected and σ extends to a loop (circuit) in X.

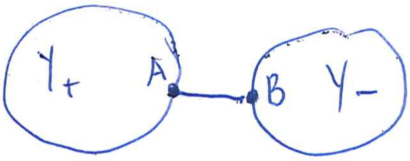


better



includes $A=B$

Y has 2 components



$A \in Y_+$
$B \in Y_-$ joined by σ.

Maximal tree picture: Choose a maximal tree T in X, Maximal $\Rightarrow$ $T \subset X$ same vertices. You can collapse T to a point to get X/T = wedge of l S^1's, each S^1 coming from an open edge in $X - T$.

This picture shows X as a tree with l extra edges attached.

Loop picture. Given X connected choose a maximal set of edges $\sigma_1, \dots, \sigma_l$ such that the graph remains connected upon their removal. Let $T = X - \{\sigma_1, \dots, \sigma_l\}$. Then T is connected and if τ is any edge in T, then $T - \{\tau\}$ has 2 components.

P T is a tree because let λ be a circuit in T,

Change notation. Let X be a conn graph such that $X - \sigma$ is disconnected for every edge σ.

So you have a problem with 

X connected graph. Usual stuff 

$$0 \longrightarrow \bar{\mathcal{C}}^0(X) \longrightarrow \mathcal{C}^1(X) \longrightarrow H^1(X) \longrightarrow 0$$

First remove those edges whose endpoints coincide ending. You lose the same thing from both $\mathcal{C}^1(X)$ and $H^1(X)$. Next is to remove edges without disconnecting the graph. Notice that you are not affecting the vertices. When σ is an edge of X such that $X - \mathrm{Int}(\sigma) = Y$ is disconnected. You should know that Y has two components, and also that ~~the generator~~ again you loose the same amount from $\mathcal{C}^1(X)$ and $H^1($?

Need to review. Chain picture? $X = Y_+ \sigma Y_-$

$$\begin{tikzcd} {} \arrow[d] \\ C_0(Y) \arrow[d] \\ C_0(X) \arrow[d] \\ \mathbb{R} \end{tikzcd}$$

$$Y = Y_+ \amalg Y_-$$

$$X = Y_+ \sigma Y_-$$

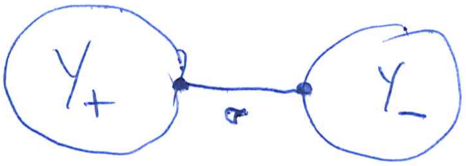
Q

$$\begin{tikzcd}
 & 0 \arrow[d] & 0 \arrow[d] & 0 \arrow[d] & & \\
0 \arrow[r] & H_1(Y) \arrow[r] \arrow[d] & C_1(Y) \arrow[r] \arrow[d] & C_0(Y) \arrow[r] \arrow[d, "\wr"] & H_0(Y) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & H_1(X) \arrow[r] \arrow[d] & C_1(X) \arrow[r] \arrow[d] & C_0(X) \arrow[r] \arrow[d] & H_0(X) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & C_1(X,Y) \arrow[r] \arrow[uurrr, bend right] & C_1(X,Y) \arrow[r] \arrow[d] & 0 \arrow[r] \arrow[d] & 0 & \\
 & & 0 & 0 & &
\end{tikzcd}$$

$$0 \to H_1(Y) \xrightarrow{\sim} H_1(X) \xrightarrow{0} \mathbb{R} \longrightarrow H_0(Y) \longrightarrow H_0(X) \longrightarrow 0$$

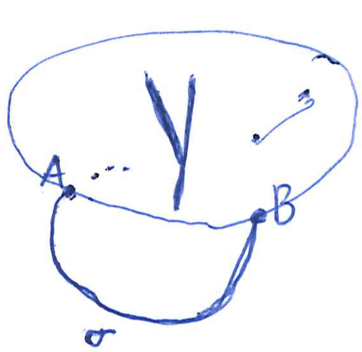
You [illegible] are assuming Y is disconnected

$\therefore H_0(Y) = \mathbb{R} \oplus \mathbb{R}$.



This is not the interesting case. The interesting case is when Y is connected

$$0 \to H_1(Y) \to H_1(X) \longrightarrow \mathbb{R} \xrightarrow{0} H_0(Y) \xrightarrow{\sim} H_0(X) \to 0$$



You want to join A, B by a path in Y to get the extra loop in X.

When you take ~~a~~

so you end up with a conn. graph T [illegible] each edge dist end pts. removal disconnects T

R

Repeat what you've learned.

X connected graph, σ an edge, $Y = X - (\text{Int})\sigma$

$$C_\bullet(Y) \longrightarrow C_\bullet(X) \longrightarrow C_\bullet(X,Y]$$

~~$0 \to H_1(Y) \to C_1(Y) \to C_0(X) \to H_0(Y) \to 0$~~

~~$0 \to H$~~

$$\begin{tikzcd}
 & 0 \arrow[d] & 0 \arrow[d] & 0 \arrow[d] & \\
0 \arrow[r] & H_1Y \arrow[r] \arrow[d] & H_1X \arrow[r] \arrow[d] & \mathbb{R}\sigma \arrow[d, equal] \arrow[dddll, bend left] & \\
0 \arrow[r] & C_1Y \arrow[r] \arrow[d] & C_1X \arrow[r] \arrow[d] & \mathbb{R}\sigma \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & C_0Y \arrow[r, "\sim"] \arrow[d] & C_0X \arrow[r] \arrow[d] & 0 \arrow[r] \arrow[d, equal] & 0 \\
 & H_0Y \arrow[r] \arrow[d] & H_0X \arrow[r] \arrow[d] & 0 \arrow[r] & 0 \\
 & 0 & 0 & &
\end{tikzcd}$$

$$\boxed{0 \to H_1Y \to H_1X \to \mathbb{R}\sigma \longrightarrow H_0Y \longrightarrow H_0X \to 0}$$

Two cases: Y connected $H_0Y = \mathbb{R}$ $H_1X = H_1Y \oplus \mathbb{R}\tilde{\sigma}$ where $\tilde{\sigma}$ is a loop current obtained from a path in Y joining the ends of σ. Y disconnected $\Rightarrow$ $H_0Y = \mathbb{R}\oplus\mathbb{R}$ and $H_1Y = H_1X$

S Now that you understand more about connected graphs, namely, that a connected graph is a tree iff removing any edge disconnects the graph, you should examine the good LC circuit situation where the dominant variables form coordinates for the Kirchhoff space.

You know that in this case

$$\begin{tikzcd} H_1(X) \arrow[r, "\sim"] & C_{1,L} \arrow[d, phantom, "\shortparallel"] & & \bar{C}^0(X) \arrow[r, "\sim"] & C^1_C \\ & C_1(X)/C_{1C}(X) & & & \end{tikzcd}$$

So you should be able to remove one L edge σ ~~with~~ to obtain a connected $Y = X \sim \sigma$, etc.

You end up with

Start again. The arguments you've used in the "good" case (where the dominant variables form a coord system for the Kirchhoff space:

$$\begin{pmatrix} \bar{C}^0 \\ H_1 \end{pmatrix} \xrightarrow{\sim} \begin{pmatrix} C^1_C \\ C_{1L} \end{pmatrix}$$

You want to think of C, L as a partition of the edges in the graph.

Consider a tree: $\bar{C}^0 \xrightarrow{\sim} C^1$ with capacitance edges. $C_\sigma I_\sigma = \dot{V}_\sigma$

$\bar{C}^0 \xrightarrow{\sim} C^1$,

$$\begin{tikzcd} & & & 0 \arrow[d, phantom, "\shortparallel"] & \\ 0 & \bar{C}_0 \arrow[l] & C_1 \arrow[l, "\sim"'] & H_1 \arrow[l] & 0 \arrow[l] \end{tikzcd}$$

T   Considering a tree.

$$\begin{tikzcd} \bar{C}^0 \arrow[r, "\sim"] & C^1 & \overset{H^1}{\underset{0}{\shortparallel}} \\ \bar{C}_0 & C_1 \arrow[l, "\sim"'] & \overset{H_1}{\underset{0}{\shortparallel}} \end{tikzcd}$$

The Kirchhoff space is just $\bar{C}^0$, the space of node potentials, same as C^1 = the space of edge voltage drops. C_1 is the space of currents (1-chains); there are no closed 1-chains. So if C-edges are used, you get $\dot{V}_\sigma = C_\sigma \underset{\substack{\shortparallel \\ 0}}{I_\sigma} = 0$.

So it's clear that the flow is 0 on the Kirchhof space. A point of K.S. is any set of edge voltage drops.

So where are you now? Review!

Consider connected LC network. Good case: Where dominant variables $\left\{ \begin{array}{ll} V_\sigma & \sigma \text{ C-type} \\ I_\sigma & \sigma \text{ L-type} \end{array} \right.$ are independent on $\mathcal{K} = \bar{C}^0 \times H_1$

In the good case you are able to see the flow on $\mathcal{K}$, because the dynamic relations

$$\dot{I}_\sigma = L_\sigma^{-1} V_\sigma \qquad \sigma \text{ L-type}$$
$$\dot{V}_\sigma = C_\sigma I_\sigma \qquad \sigma \text{ C-type}$$

can be expressed in terms of the dominant variables.

4 So what next??? Since K has dimension e there is always some set of [redacted] e edge variables which restricts to a coordinate system in K.

[redacted] Recall problem: Flow on the Kirchhoff space $K = \bar{C}^0 \oplus H_1$ for a [redacted] connected LC network.

Possible approach: The flow should satisfy

$L_\sigma \dot{I}_\sigma = V_\sigma$ for σ [redacted] type L

$C_\sigma \dot{V}_\sigma = I_\sigma$ —— type C

You have total of e dynamical equations, and K has dim e. So you [redacted] need some kind of non degeneracy.

You understand the good case where the dominant variables form a [redacted] coord system on K, equivalently [redacted] $\underset{(v-1)}{\bar{C}^0} \xrightarrow{\sim} C^1_C$ and $\underset{(e)}{H_1} \xrightarrow{\sim} C_{1L}$

Where next? Something [redacted] involving $s = 0, \infty$ has to intervene.

Why? Use L.T. notation

$L_\sigma s I_\sigma = V_\sigma$ type [redacted] L

$C_\sigma s V_\sigma = I_\sigma$ type C

[redacted] It seems that interchanging L, C

V is the same as interchanging s and s^{-1}! Suppose you start with a connected LC network where the L number $= l$
C number $= v-1$.

Now change the type of some edge.

Today Sept 15, 02 you want to study the symplectic structure at least in the good case. You want to understand the linear symplectic picture. Symplectic v.s., polarization into complementary Lagrangian subspaces. Obvious question: Is the flow Hamiltonian?

Exact sequence

$$\begin{tikzcd}
 & 0 \arrow[d] & 0 \arrow[d] & 0 \arrow[d] & & \\
0 \arrow[r] & H_1(Y) \arrow[r] \arrow[d] & C_1(Y) \arrow[r] \arrow[d] & C_0(Y) \arrow[r] \arrow[d, "S"] & H_0(Y) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & H_1(X) \arrow[r] \arrow[d] & C_1(X) \arrow[r] \arrow[d] & C_0(X) \arrow[r] \arrow[d] & H_0(X) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & R \arrow[r, equal] \arrow[uurrr, bend right] & R \arrow[r] \arrow[d] & 0 \arrow[r] \arrow[d] & 0 \arrow[r] \arrow[d] & 0 \\
 & & 0 & 0 & 0 &
\end{tikzcd}$$

So what seems clear is that you get linear versions of maximal trees

W

Aim: To understand the dynamics.

Let's begin with $\mathbb{R}$-linearization of the $\mathbb{Z}$-kinematics for the graph. Review deleting an edge.

$Y = X - \sigma$ cocart diag square

$$\begin{tikzcd}
\partial\sigma \arrow[r, phantom, "\subset"] \arrow[d] & \bar{\sigma} \arrow[d] \\
Y \arrow[r, phantom, "\subset"] & Y \cup \sigma
\end{tikzcd}$$

$$0 \longrightarrow C_\cdot(Y) \longrightarrow C_\cdot(X) \longrightarrow \mathbb{Z}\sigma \longrightarrow 0$$

where $C_\cdot(X)$: $C_1(X) \xrightarrow{\partial} C_0(X)$

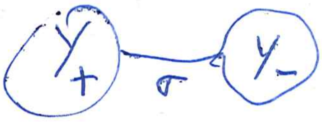
~~$0 \to H_1(Y) \to C_1(Y) \to C_1(X) \to H_0(Y) \to 0$ $C_0(Y)$ $C_0(X)$~~

$$\begin{tikzcd}
 & & 0 \arrow[d] & 0 \arrow[d] & & \\
0 \arrow[r] & H_1(Y) \arrow[r] \arrow[d] & C_1(Y) \arrow[r] \arrow[d] & C_0(Y) \arrow[r] \arrow[d, "\wr"] & H_0(Y) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & H_1(X) \arrow[r] \arrow[d] & C_1(X) \arrow[r] \arrow[d] & C_0(X) \arrow[r] \arrow[d] & H_0(X) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & \mathbb{Z}\sigma \arrow[r, equal] \arrow[uurrr, bend right] & \mathbb{Z}\sigma \arrow[r] \arrow[d] & 0 \arrow[r] \arrow[d] & 0 \arrow[r] & 0 \\
 & & 0 & 0 & &
\end{tikzcd}$$

X Get

$$0 \to H_1(Y) \to H_1(X) \to \mathbb{Z}\sigma \to H_0(Y) \to \overset{\overset{\mathbb{Z}}{\wr\wr}}{H_0(X)} \to 0$$

Two cases: ① Y disconnected: Y has 2 components $Y = Y_+ \sqcup Y_-$ and $X =$



and $H_1(Y) \cong H_1(X)$

② Y conn: $0 \to H_1(Y) \to H_1(X) \to \mathbb{Z}\sigma \to 0$ exact

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Next you want a real version which should generalize at least the good LC network cases. ~~So what You~~ Real version of C.

$$0 \to H_1(X) \to C_1(X) \xrightarrow{\partial} C_0(X) \to \mathbb{R} \to 0$$

So you have a chain complex of dim 1 consisting of R-vector spaces, such that $H_0 \cong \mathbb{R}$. Consider "removing an edge σ". This means I guess that $\sigma : C_1(X) \to \mathbb{R}$ is a 1-cochain.

$$\begin{tikzcd}
& & 0 \arrow[d] & & & \\
0 \arrow[r] & H_1(Y) \arrow[r] \arrow[d] & C_1(Y) \arrow[r] \arrow[d] & C_0(Y) \arrow[r] \arrow[d, "\wr S"] & H_0(Y) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & H_1(X) \arrow[r] \arrow[d] & C_1(X) \arrow[r] \arrow[d] & C_0(X) \arrow[r] & H_0(X) \arrow[r] & 0 \\
& \mathbb{R} \arrow[uurrr, "0"] & \mathbb{R} \arrow[r] \arrow[d] & 0 & & \\
& & 0 & & &
\end{tikzcd}$$

Again ~~the~~ you have the two cases. (Either l stays same and Y disconns, or Y conn. + l decreases by 1.

Y

Our next step is to polarize C^1, dividing the edges into L, C types.

What do you have?

$$\begin{tikzcd}
0 \arrow[r] & H_1 \arrow[r] & C_1 \arrow[r] \arrow[d, equal] & C_0 \arrow[r] & R \arrow[r] & 0 \\
 & & C_{1,C} \oplus C_{1,L} & & &
\end{tikzcd}$$

simplify a bit

$$\begin{tikzcd}
0 \arrow[r] & H_1 \arrow[r] & C_1 \arrow[r] \arrow[d, equal] & \bar{C}_0 \arrow[r] & 0 \\
 & & C_1^+ \oplus C_1^- & &
\end{tikzcd} \quad \text{exact}$$

So you definitely have a Grass situation. Stick to the "good" case

$$\begin{tikzcd}
\bar{C}^0 \arrow[r, hook] & C^1 \arrow[r, two heads] \arrow[d, leftrightarrow, "\mathbb{Z}_5"] & H^1 \\
\bar{C}_0 & C_1 \arrow[l, two heads] & H_1 \arrow[l, hook']
\end{tikzcd}$$

What is the good case exactly?

$$\begin{pmatrix} C^1 \\ C_1 \end{pmatrix} = \begin{pmatrix} C^1_C & C^1_L \\ C_{1,C} & C_{1,L} \end{pmatrix}$$

You want natural maps from $\bar{C}^0$ to C^1_C and from H_1 to C_{1L} to be isos.

Z_1

σ C type $\quad C_\sigma \dot{V}_\sigma = I_\sigma$

σ L type $\quad L_\sigma \dot{I}_\sigma = V_\sigma$

naturality should be important

Idea yesterday. If the kinematics of an abstract conn graph amounts to

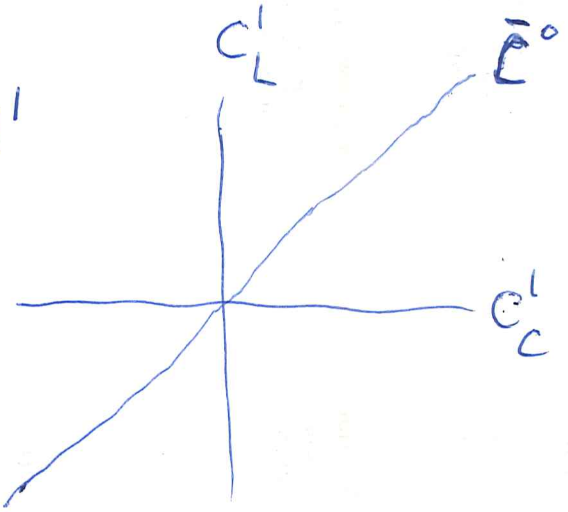
$$0 \to H_1 \to C_1 \to C_0 \to R \to 0$$

or dually

$$0 \to \bar{C}^0 \to C^1 \to H^1 \to 0,$$

then the abstract good LC network is

$$\begin{tikzcd}
 & C^1_L \arrow[d, hook] \arrow[dr, "\sim"] & \\
\bar{C}^0 \arrow[r, tail] \arrow[dr, "\sim"'] & C^1 \arrow[r, two heads] \arrow[d, two heads] & H^1 \\
 & C^1_C &
\end{tikzcd}$$



~~As you probably have~~

Q: ~~What~~ Where is "the dynamics" in this situation?

The "kinematics" $\bar{C}^0 \rightarrowtail C^1 \twoheadrightarrow H^1$ is fixed.

The "dynamics" should be given by a quadratic form $Q_s = sQ_+ + s^{-1}Q_-$ on C^1

---

abstract conn LC network consists of f.d. R v.s.

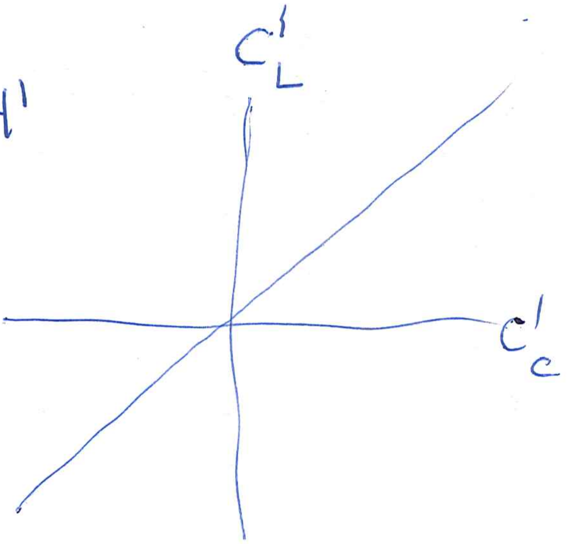
$$0 \longrightarrow H_1 \longrightarrow C_1 \longrightarrow \bar{C}_0 \longrightarrow 0$$

$C_1 = C_{1,L} \oplus C_{1,C}$ is a polarized Euclidean space a Strange

$$\begin{tikzcd}
0 \arrow[r] & H_1 \arrow[r] & C_1 \arrow[r] & \bar{C}_0 \arrow[r] & 0 \\
 & & \text{polarized Euclidean space} \arrow[u] & &
\end{tikzcd}$$

This seems to be a typical Grassmannian situation.

$$\begin{tikzcd}
 & C^1_h \arrow[d] & \\
\bar{C}^0 \arrow[r] \arrow[dr, "s"'] & C^1 \arrow[r] \arrow[d, two heads] & H^1 \\
 & C^1_c &
\end{tikzcd}$$



$$\boxed{\bar{C}^0 = \begin{pmatrix} 1 \\ T \end{pmatrix} C^1_c \subset C^1}$$

where $T : C^1_c \to C^1_L$

You want to get a natural flow on $\bar{C}^0 \oplus H_1$ which should be canonically isom to C^1. The flow should be a skew-symmetric operator. Obvious candidate is $\begin{pmatrix} 0 & -T^* \\ T & 0 \end{pmatrix}$.

Get s into the picture