Nutor again after a gap of 7 days

My construction

\[ A, B, L \text{ algebras}, J \triangleleft L \text{ ideal} \]

\[ A \xrightarrow{\theta} L \otimes B \text{ cong mod } J \otimes B \]

\[ S = \bigoplus_{n > 0} t^n J^n \subset C[t] \otimes L \]

\[ K = \text{ ideal } (1-t^2)J^2 \text{ in } S \]

\[ S_h = \bigoplus_{n > 0} t^n J^n \]

\[ J_\#^n = \bigoplus_{i \geq n} [J^i, J^j] \]

\[ J_{\#}^n = \bigoplus_{i \geq n} [J^i, J^j] \]

J-adic trace

\[ \mu_m : S \rightarrow J^{2m+1} \]

\[ P_m(t) = \prod_{k=1}^{\#(J_{\#})/2} \left( 1 - \frac{t}{2^k} \right) \]

\[ \mu_m(\theta \cdot x) = \frac{P_m(\theta) (1 - (-1)^n)}{2^{m+1}} \left( \frac{\#(x)}{2^{m+1}} \right) \quad x \in J^{2m+1} \]

\[ \mu_m \text{ is the composition} \]

\[ \mu_m = \frac{1}{2^{m+1}} (d_1 - d_{-1}) P_m(t \cdot x) \]

\[ P_m(t \cdot x) \text{ carries } K^{m+1} \text{ into } K \]

\[ \mu_m \text{ is killed by } d_1, d_{-1} \]

\[ \mu_m \text{ is clearly defined on } S \]
Anyway the basic construction is
\[ p = \frac{1}{2}(\theta + \phi') : A \to L \otimes B \]
\[ q = \frac{1}{2}(\theta - \phi') : A \to L \otimes B \]
\[ p + tq : A \to S \otimes B \quad \text{linear map} \]
\[ \text{curvature} \quad \exists (1-t^2)q^2 : A [t^2] \to (1-t^2)T^2 \otimes B = K \otimes B \]
\[ p + tq \quad \text{induces} \]
\[ RA \quad \xrightarrow{u} \quad R(S \otimes B) \quad \xrightarrow{S} \quad S \otimes RB \]
\[ X(RA) \quad \xrightarrow{u^*} \quad X(S \otimes RB) \quad \xrightarrow{i} \quad S_p \otimes X(RB) \]
\[ \text{Basic map is} \]
\[ X(RA) \quad \xrightarrow{u^*} \quad X(S \otimes RB) \quad \xrightarrow{i} \quad S \otimes X(RB) \quad \xrightarrow{\varphi} \quad J_{2m+1} \otimes X_B[2m] \]
\[ F^p \quad \xrightarrow{\varphi} \quad K \otimes RB + S \otimes RB \quad \xrightarrow{\sum_i b_i(k_i)[p]} \quad J_{2m+1} \otimes F^{p-2m} \]

Thus get a map of towers
\[ \chi_A \quad \to \quad J_{2m+1} \otimes X_B[2m] \]

Claim \[ X_A \quad \to \quad J_{2m+1} \otimes X_B[2m+2] \]
\[ J_{2m+1} \otimes X_B[2m] \to J_{2m+1} \otimes X_B[2m+2] \quad \text{commutes} \]
Now I want to relate my construction to vectors. 

$$Q = QA = \bigoplus Q_n, \quad Q_n = L^n A$$

$RQ \times (RQ)$ inherit grading
degree of $D$, $L_D$
canonical $\phi$ and $h_D$

$$[L_D, h_D] = 0$$

My map

$$x(RA) \rightarrow S \otimes x(RB) \rightarrow J^{2m+1} \otimes x(RB)$$

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$$A \xrightarrow{\phi \otimes h} S \otimes B$$

induces

$$RA \rightarrow S \otimes RB$$

$$x(RA) \rightarrow S \otimes x(RB)$$

Introduce $Q = QA = \bigoplus Q_n$

$\theta, \theta'$ give rise to a homom.

$$Q \rightarrow L \otimes B$$

satisfy $Q_n \rightarrow J^n \otimes B$

when $Q \xrightarrow{\text{linear map}} S \otimes B$

induces

$$RQ \rightarrow S \otimes RB$$

$$x(RQ) \rightarrow S \otimes x(RB), \quad L_D \rightarrow \text{td}$$

compact w. grading
Way to say things I think is as follows.

Any map:

\[ X(\text{RA}) \xrightarrow{L_x} X(\text{RQ}) \xrightarrow{P_m(L_D)Y_{-}} S_{\beta} \otimes X(\text{RB}) \xrightarrow{P_m(t^D)T_{-}} J^2_{\#} \otimes X(\text{RB}) \]

coincides with:

\[ X(\text{RA}) \xrightarrow{L_x} X(\text{RQ}) \xrightarrow{P_m(L_D)Y_{-}} J^2_{\#} \otimes X(\text{RB}) \]

In order to write this out I just have to define both maps, then state they coincide.

Steps are:

\[ \text{RA} \xrightarrow{(p+q)x} \text{R}(S \otimes B) \xrightarrow{S \otimes \text{RB}} \]

\[ \text{RQ} \xrightarrow{(s)x} \text{R}(S \otimes B) \]

Yields:

\[ X(\text{RA}) \xrightarrow{S_{\beta} \otimes X(\text{RB})} \]

\[ X(\text{RQ}) \xrightarrow{(s)x} \]
Important objects are $p + t g$, $v$ so far.

Continue the analysis. So where are we? I have to explain the map

$$X( RA ) \xrightarrow{b_x} X( RQ ) \xrightarrow{g} \#_{X \geq 0} = \#_{X \geq 1}$$

$$\xrightarrow{s_1} \#_{X \geq 2} = \#_{X \geq 3}$$

$$\xrightarrow{s_m} \#_{X \geq 2m} = \#_{X \geq 2m+1}$$

Only the last part has not been defined.

$$X_{\geq k} = \bigoplus_{n \geq k} X(RQ)_n$$

Here $L_0 = n$.

Explanation:

$$Q \xrightarrow{v} S \otimes B \quad \text{lin rep. 1, comp } D^\otimes \chi^2$$

$$X( RQ ) \xrightarrow{\psi} S_q \otimes X( RB ) \quad \text{comp } L_0 \rightarrow t^0$$

Then the map is

$$X( RQ )_{\geq k} \xrightarrow{\psi_{\geq k}} S_q \geq k \otimes X( RB ) \xrightarrow{s_1} \#_{\#} \otimes X( RB )$$

But this is not the way to think. Go back to definition of $v$ and factor it

$$Q \xrightarrow{t^0} \bigoplus t^n Q_n$$
My construction

\[ A \xrightarrow{\theta} \bigotimes B \quad \text{ang. mod} \bigotimes B \]

\[ p = \frac{1}{2} (\theta + \theta') : A \rightarrow \bigotimes B \]

\[ q = \frac{1}{2} (\theta - \theta') : A \rightarrow \bigotimes B \]

\[ p + tq : A \rightarrow (L + tJ) \otimes B \subset S \otimes B \]

induces

\[ RA \rightarrow R(S \otimes B) \rightarrow S \otimes RB \]

\[ X(RA) \rightarrow X(S \otimes RB) \rightarrow S \otimes X(RB) \xrightarrow{\mu_2} J^{2m+2} X \]

Claim

\[ F^\circ_{IA} \rightarrow F^\circ_{K \otimes RB \otimes DB} \xrightarrow{\phi} \sum_{i \leq 0} F^\circ_{K^i \otimes P_{cm-2}^{cm-2}} \rightarrow J^{2m+1} \otimes P_{cm}^{cm} \]

Yet

\[ X_A \rightarrow J^{2m+1} \otimes X_P^{2m} \]

call this \( \mu^{2m} \)

\[ \mu^{2m}(\theta, \theta', \tau) \in H^2_{cm} (A, B) \]

Jachin's version of Mistor

Introduce \( Q = QA = \Omega A \) with \( e \)

graded vector space \( Q = \bigoplus_{n} Q_n \) where \( Q_n \)

canonical iden. \( Q = A \otimes A \quad \text{two canonical embeddings are} \quad \xi(a) = a + da, \quad \eta(a) = a - da \quad \text{autmor.} \)

order 2 \( \xi = (-1)^n \)

\( \theta, \theta' \) induces a homom.

\[ Q \xrightarrow{\otimes} L \otimes B \quad Q_n \rightarrow \bigotimes B \]
get factorization
\[ A \xrightarrow{i} Q \xrightarrow{\pi} S \otimes B \quad \text{cusp. grading} \]

get
\[ RQ \xrightarrow{?} R(S \otimes B) \xrightarrow{} S \otimes RB \quad \text{cusp. grading} \]

my map is therefore
\[ X(RQ) \xrightarrow{P_m(LD)} X(RQ) \xrightarrow{} X(RQ)^{2m+1} \xrightarrow{J^2m+1} \quad \text{cusp. grading} \]

What points to emphasize?
- A graded as a vector space
- \( RQ \) and \( X(RQ) \) inherit gradings.

There will be a problem linking the grading and the filtration. Maybe I should go over the link.

The alg \( Q \) is graded as a vector space:
\[ Q = \bigoplus Q_n \quad 1 \in Q \]

Although this grading is not compatible with the alg of the decreasing filtration
\[ Q_{\geq k} = \bigoplus_{n \geq k} Q_n \]

arising from this grading is compatible with the alg. structure:
\[ Q_{\geq i} Q_{\geq j} < Q_{\geq i+j} \quad 1 \in Q_{\geq 0} \]
Actually this writing project is rather challenging because there is so much to organize. There are too many ideas for me to handle all at once, in practice, too many maps to label and organize.

At the moment I am thinking about the end of the argument, the end map. So what do we do?

We have a homomorphism

\[ Q \xrightarrow{\nu} L \otimes B \]

arising from the pair \( \Theta, \Theta' : A \rightarrow L \otimes B \).

Specifically

\[ q_0, q_n, \ldots, q_n \rightarrow \nu q_0, \nu q_n, \ldots, \nu q_n \]

One has

\[ \nu \circ \Theta = \Theta', \quad \nu \circ \Theta = \Theta'. \]

Also

\[ \nu(Q_n) \subset J^n \otimes B, \quad \nu(Q_{\geq n}) \subset J^n \otimes B. \]

So where next?

\[ Q \xrightarrow{t^0} \bigoplus_{k \in \mathbb{Z}} t^k Q_{\geq k} \rightarrow \bigoplus_{k \in \mathbb{Z}} t^k J^k \otimes B \]

\[ Q \xrightarrow{t^0} Q^+ \xrightarrow{v^t} L^t \otimes B \]

linear map \( 1 \)

\[ R Q \xrightarrow{R^t Q^+} R_Q L^t (L^t \otimes B) \]

At the moment I am trying to maneuver things, but the problem is still the assertions.
I already decided that the good approach is to set up the maps on the supercomplex level, then claim they have appropriate property with respect to the filtration. Thus you want to identify your map $X(\text{RA}) \to J^{2m+1} \otimes X(\text{RB})$ with a specific composition of

$$
\begin{align*}
X(\text{RA}) \xrightarrow{\ell_x} X(\text{RQ}) \xrightarrow{x_2} & X_{> 0} = X_{> 1} \\
\xrightarrow{s_1} & X_{> 2} = X_{> 3} \\
\vdots \\
\xrightarrow{s_{2m-1}} & X_{> 2m-1} = X_{> 2m+1} \\
\end{align*}
$$

followed by a map $\ell : X_{> 2m+1} \to J^{2m+1} \otimes X(\text{RB})$.

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Objects: The map

$$
\xrightarrow{\ell} X(\text{RQ}) \xrightarrow{p_m(L_0)} X(\text{RQ}) \xrightarrow{\otimes} X(\text{RQ})_{> 2m+1}
$$

The last map $\ell : X(\text{RQ})_{> k} \to J^{k} \otimes X(\text{RB})$.

Relations: 1) $\ell \circ p_m(L_0) X_{> 1} : X(\text{RA}) \to J^{2m+1} \otimes X(\text{RB})$ coincides with my map.

2) $p_m(L_0) X_{> k}$ carries $FP X(\text{RA})$ into $FP^{-2m} X(\text{RQ})_{> 2m+1}$ for all $p$, so one has a map of towers $X_A \to X_{> 2m+1}$.

3) This class of this map in $HC^{2m}(X_A, X_{> 2m+1}) = HC^{2m}(A, Q_{> 2m+1})$. 
is essentially invariant for the universal quasi-homomorphism. Explain essentially overall factor of 2.

It still seems that the last map is the awkward point, owing to the fact that the filtration + grading are both involved. I would like to have a list of definitions and assertions whose proofs can be filled in by the reader.

Let us go over the last map carefully. We begin with $\theta, \theta': A \to L \otimes B$ cong. mod $J \otimes B$. Get homom.$$
Q \xrightarrow{w} L \otimes B$$

properties:
- $w(0) = 0$, $w(\theta') = \theta'$
- $w(a_0 a_1 \cdots a_n) = p_{0} q_{0}, \cdots q_{n}$
- $w(Q_n) \subseteq w(Q_{\geq n}) \subset J^n \otimes B$

Get linear map now 1.$$
Q \xrightarrow{f} S \otimes B \subset L^+ \otimes B$$

$f(0 a_1, \cdots a_n) = t^n p_{0} q_{0}, \cdots q_{n}$
Get homomorphism of graded algebras
\[ Q^t \xrightarrow{w^t} L^t \otimes B \]

comp. w. \( T \longrightarrow L^t \)
\[ f = \text{composition: } A \xrightarrow{t^D} Q^t \xrightarrow{w^t} L^t \otimes B \]
\[ f = w^t t^D \]

\( f \) gives rise to
\[ \text{homom. } f_x : RQ \longrightarrow R(S \otimes B) \longrightarrow S \otimes RB \]
\[ \text{map of } \]
\[ f_x : x(RQ) \longrightarrow x(S \otimes RB) \longrightarrow S \otimes x(RB) \]
\( w^t \) gives rise to
\[ R_{t^D} Q^t \xrightarrow{w^t} R_{L^t} (L^t \otimes B) \]
\[ X_{t^D} (R_{t^D} Q^t) \xrightarrow{w^t} X_{L^t} (L^t \otimes RB) \]

\[ \text{Let's try introducing notation.} \]
\[ f = p + t^D : A \longrightarrow S \otimes B \]
\[ g = w^t t^D : Q \longrightarrow S \otimes B \subset L^t \otimes B \]
\[ f_x : RA \longrightarrow S \otimes RB \quad \alpha : L^t \otimes B \]
\[ g_x : RQ \longrightarrow S \otimes RB \quad \eta : L^t \otimes B \]
\( f_{**} : X(RA) \rightarrow s_{\gamma} \otimes X(RB) \)
\( g_{**} : X(RQ) \rightarrow \) 

\( l : A \rightarrow Q \)
\( l_{**} : RA \rightarrow RQ \)
\( l_{**} : X(RA) \rightarrow X(RQ) \).

Then \( f = g \circ \Rightarrow f_\gamma = g_\gamma \epsilon_\gamma \Rightarrow f_{**} = g_{**} \epsilon_{**} \)

\[ t^D : Q \rightarrow Q^t \]
\[ t^D : RQ \rightarrow RQ^t \]
\[ t^{L_0} : X(RQ) \rightarrow X(RQ)^t \]
\[ w^t : Q^t \rightarrow L^t \otimes B \]
\[ w^t : (RQ)^t \rightarrow L^t \otimes RB \]
\[ w_{**}^t : X(RQ)^t \rightarrow L^t \otimes X(RB) \].

Before I can write this down I need I think.

\( R_T(Q^t) \cong (RQ)^t \)
\( X_T(R_T(Q^t)) \cong (X(RQ))^t \)
\( R_{L_t}(L^t \otimes B) = L^t \otimes RB \)
\( X_{L_t}(R_{L_t}(L^t \otimes B)) = X_{L_t}(L^t \otimes RB) = L^t \otimes X(RB) \)

and some of these need amplification by formulas.
Let us consider then the concrete statements I need and the proofs.

What do I need in order to link my construction with Nistor's?

Nistor objects.

\[ A^b = \text{mixed complex } (\Omega A, b, B) \]

\[ Q^b_{> k} = (\Omega Q_{> k}, b, B) \]

"last" map

\[ l^k : \Omega Q_{> k} \rightarrow J^k \otimes QB \]

\[ l^k : Q^b_{> k} \rightarrow J^k \otimes B^b \]

Here constructs \[ S_k : Q^b_{> k} \rightarrow Q^b_{> k+1} \]

\[ [S_k][c_k] = S \in H^2(C_b^{> k+1}, Q^b_{> k+1}) \]

\[ [c_k][S_k] = S \in H^2(Q^b_{> k}, Q^b_{> k}) \]

and notes that \([S_k]\) is the unique up to a class killed by \(S\).

(This is not what I want to use but so what)

He defines the biv. char of the map, quasi to be

\[ \chi^{2k} = [S_k] \cdots [S_1][c_{-1}] : H^2(C_b^{> k+1}, Q^b_{> k+1}) \]

13.15 I have to find minimum things to say. Let's try to organize the assertions.

The goal is to link my bivariant Chern character \( \otimes \chi^{2m}(b, b) \in H^2(C_b^{> k+1}, J^k \otimes \otimes B^b) \)
with Nistor's. To show they are essentially the same, better notation
\( \text{ch}^{2m}(\Theta, \Theta', \tau) \in \text{HC}^{2m}(A, B) \)
do you mention that one has a quasi-homom. consisting of two limos.

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Start with \( \Theta, \Theta' : A \rightarrow L \otimes B \) congruent
modulo \( J \otimes B \) and a \( J \)-adic trace \( \Phi \) on
\( J^{2m+1} \).

Aim to construct \( \text{ch}^{2m}(\Theta, \Theta', \tau) \in \text{HC}^{2m}(A, B) \).

Nistor construction (essentially)
\[ Q = QA \]
\[ Q_{>k}^b = B(\Omega Q_{>k}) \]
\[ l_k : gQ_{>k+1}^b \rightarrow gQ_{>k}^b \]
\[ S_k : gQ_{>k}^b \rightarrow gQ_{>k+1}^b [2] \]
\[ S_k l_k \sim S : gQ_{>k+1}^b \rightarrow gQ_{>k+1}^b [2] \]
\[ l_k S_k \sim S : gQ_{>k}^b \rightarrow gQ_{>k}^b [2] \]
version of Nistor's construction

\[ Q = QA, \quad \text{graded as a vector space} \]
\[ Q_n = \mathbb{R}^n. \]

\[ Q = QA \]

grading as vector space
assoc. filtration
2/1 grading

\[ D, \quad Y = (-1)^D \]

induced gradings on \( RQ, \quad X(RQ) \)

\[ D, \quad l_D = \mathbb{Z}(1, 0) \]

comm. \( \phi : RQ \rightarrow \Omega^2(RQ), \quad h_D = c^0(1, 0). \)

\[ l_{D_0} = [3, h_0], \quad [l_D, h_D] = 0 \]

Should I write down things.

OK what comes next???

I am trying to explain Joachim's construction of Nistor's invariant Chern character for the universal quasi-isomorphism.
Replace $S_{10}$ by \( \frac{1}{-k} \cdot \left( \frac{1}{k^2} \right) \) in the averageicket
\[ S_{10} = \left( \begin{array}{c} k^2 \\ -k \end{array} \right). \]

This leads to a class unique up to a class filler.

Next, we are trying to do this argument.

A = QA equipped with filtration

\[ \text{Points: } \{ x(14) \} \rightarrow S \rightarrow \Omega(10) \]

To give an origin of Nistor's construction.

\[ \text{Next - to} \]

\[ \text{link my construction to Nistor's construction.} \]

\[ \text{To recall Nistor's construction.} \]

\[ \text{\# T using F} \]

\[ \text{\# F} \]
new approach
first our construction
then link with Nistor's

new approach is to start by recalling with
Nistor's construction a suitable version

(QQ)_{2k} mixed subcomplex of QQ.

\[ \mathcal{L}_k \in \mathcal{H}^0((QQ)_{2k+1}, (QQ)_{2k}) \]

\[ \mathcal{L}_k \in \mathcal{H}^2(\mathcal{L}_k, (QQ)_{2k+1}) \]

\[ S_{\mathcal{L}_k} = S \in \mathcal{H}^2(QQ_{2k+1}, QQ_{2k}) \]

\[ \mathcal{L}_k \text{ unique up to a class killed by } S. \]

\[ \gamma = \frac{1}{2}(1 - \delta). \]

\[ \mathcal{L}_k = (QQ)_{2k+1} \text{ kernel.} \]

rep. \( S_k \) by \( \frac{1}{2}(S_k + \gamma S_k S) \) on subspace \( S_k \), \( S \) commute.
get not:

\[ \mathcal{L}_k \in \mathcal{H}^0(\mathcal{L}(QQ)_{2k+1}, \mathcal{L}(QQ)_{2k}) \]

\[ S_{\mathcal{L}_k} \in \mathcal{H}^2(QQ_{2k+1}, QQ_{2k}) \]

analogous identities.

\[ \mathcal{H}^0(Q, \mathcal{L}) \in \mathcal{H}^0(Q(Q), QQ_{2k+1}) \]

\[ \text{class of } \mathcal{L} \xrightarrow{\mathcal{S}_k} \mathcal{L} \quad \gamma \quad (QQ)_{2k+1} \]

\[ \mathcal{C}^{2m}(Q, \mathcal{L}) = S_{2m-1} \cdots S_3 S_1 \cdot \mathcal{H}^0(Q, \mathcal{L}) \]
Next present a version of this. I want to get things clear enough to understand myself. You have to decide what needs explaining.

Start with $F^p \Omega \geq k$.

Bifiltration of $\Omega = \Omega^2$ can transport via $X \sim \Omega$ to obtain $F^p X \geq k$.

Claim the canonical map $X \sim \Omega$ induces $F^p X \geq k \sim F^p \Omega \geq k$ for all $p, k$.

Cor. $\chi_{X,k} = \left( X_{\geq k}/F^p X_{\geq k} \right) \sim \Theta(\Omega_{\geq k})$.

So bivariant classes between $\Omega_{\geq k}$ can be constructed from maps of towers.

Now bring in $D_1, D_2, h_D$.

0836 First thing I want

$l_k \in HC^0(X_{\geq k+1}, X_{\geq k})$

given by inclusion $X_{\geq k+1} \subset X_{\geq k}$ (which carries $F^p X_{\geq k+1}$ into $F^p X_{\geq k}$)

$s_k \in HC^2(X_{\geq k}, X_{\geq k+1})$ given by

$1 - k^{-1} L_D : X_{\geq k+1} \rightarrow X_{\geq k}$

which carries $F^p X_{\geq k+1} \rightarrow F^{p-2} X_{\geq k}$ by ...
Outline: So let's use this pen a bit. What I seem to have evolved is the idea of starting with a version of Cartier's construction, namely: linear invariant Chern character for the universal quasi-hermon. This means describing classes
\[ \lambda_k \in H^0(\Omega^k, \Omega^k) \]

Let's try out the notation
\[ \lambda_k \in H^0(\Omega^b_k, \Omega^b_{k+1}) \]
\[ S_k \in H^2(\Omega^b_k, \Omega^b_{k+1}) \]

\[ \text{ch}^{2m}(\lambda, \lambda) = S_{2m-1} \cdots S_3 \cdot S_1 \cdot \text{ch}^0(\lambda, \lambda) \]
\[ \text{ch}(\lambda, \lambda) = \text{class of} \quad \lambda \in H^{2m}(A^b, \Omega^b_{2m+1}) \]

\[ \Omega^b_k \Rightarrow Q^b_k \Rightarrow \Omega^b_{k+1} \]

To describe these classes

Now discuss, describe X-version:

Have \[ \chi(\Omega^b) = \frac{\Omega^b}{\Omega} \]

Define \[ \text{FPX} \Rightarrow \Omega \]

Then you have the lemma about the behavior. Your problem is the fact you haven't written out the details. You haven't got the details straight in your own mind.
The point is to use \[ t \] the pieces together.

Suppose you define \( \operatorname{FP}_h \) to \( \cong \) \( \operatorname{FP}^h \) under the isom. \( X \cong \mathbb{Q} \). (Check that or note the \( \operatorname{FP}^0 \) stable under \( \psi \), \( \varepsilon \) etc.) do \( \operatorname{FP}_h \) is a subcomplex of \( X \). the SDR of \( X \) onto \( \operatorname{PX} \), \( \Sigma \) onto \( \mathbb{Q} \), and isom \( \operatorname{PX} \cong \mathbb{Q} \) have to induce the SDRs etc. where \( \Sigma \) replaced by \( \operatorname{FP}^0 \).

The problem with this approach is that it can't handle \( D \). \( L_0, b \) are defined on \( X(RQ) \) because \( RQ \) depends on \( Q \) as vector space. These words are all clear, but the point is

I need to make them convincing.

Given \( Q \) with grading \( Q = \bigoplus_{i=0}^{\infty} Q_i \) and filtration \( Q_h = \bigoplus_{i=0}^{h} Q_i \) compact with alg str. \( Q_{h+1} \cdot Q_{h+1} \subset Q_{2h+1} \), \( h < 0 \). Form graded algebra \( Q^h = \bigoplus_{i=0}^{\infty} t^i Q_{i+h} \subset T \otimes Q \). \( Q \) graded T algebra. Have \( T \otimes Q^h \rightarrow T \otimes Q \) alg. isom. \( \otimes \) graded T-module.

\[ T \otimes Q \rightarrow Q^h \tag{1} \]

\[ t D + \sum \rightarrow t D \]
\( \mathbb{Q} \xrightarrow{t} \mathbb{Q}^t \)

\( T \otimes \mathbb{Q} \xrightarrow{\sim} \mathbb{Q}^t \)

\( f \otimes x \xrightarrow{?} f \otimes t_x \)

\( t \mathbb{Q}^t + D \xrightarrow{\sim} t \mathbb{Q}^t \)

\[ R_T( T \otimes Q ) \xrightarrow{\sim} R_T( \mathbb{Q}^t ) \]

\[ T \otimes R_{Q^t} \]

Let's go into the derivation \( L_D \)

\[ F_P \]

\[ X( R_T( Q^t ) ) \]

\[ \mathbb{I}_T( Q^t ) \]

\[ X_t( \mathbb{R}( Q^t ) ) \]

\[ ( F_P X )^t \]

\[ \mathbb{R}( Q^t ) \]

Establish ahead of time that

\[ T \otimes X( R_{Q^t} ) \xrightarrow{\sim} X_t( R_T( Q^t ) ) \]

\[ f \otimes ? \xrightarrow{?} f \otimes t_{D_0( g )} \]

What about \( \otimes_{D_0} \) on \( X_t( R_T( Q^t ) ) \)?

You have \( D \) on \( Q \) extended to \( Q^t \) so add to commute with \( T \)-module structure, where have \( L_D \) on \( X_t( R_T( Q^t ) ) \). Now have

\[ X( \mathbb{R}( Q ) ) \xrightarrow{?} X_t( R_T( Q^t ) ) \]
\[
Q \xrightarrow{t^p} Q^t \\
D \xrightarrow{tD_t}
\]

\[
X_t(I_t(q^t)) \\
X(RQ)^t \subset T' \otimes X(RQ)
\]

\[
L_D \\
L_D
\]

You have in \(X(RQ)^t\) both \(tD_t\) and \(L_D\).

What happens is that on \(R^i, RQ^t\), we have \(tD_t - D\) vanishes in the image of \(t\) and in the image \([L(tD_t - D), t^{-1}]\).

\[
(D - tD_t)(RQ)^t \longrightarrow t^{-1}(RQ)^t
\]

So \(L_D - tD_t : F^p I_t(q^t) X_t(RQ^t) \longrightarrow t^{-1} F^{p-2} I_t(q^t) X_t(RQ^t)\)

\[
(F^p X)^t \\
(F^{p-2} X)^t
\]

means that \(F^p X \geq k\) \(\xrightarrow{L_D = k} F^{p-2} X\)

as claimed.
What about $y(-1)^D$ on $Q^t$? we $y = \text{extended in obvious ways to commute with } t$ and also $(-1)^{t^2}$ which changes $t$ to $-t$. We have

$$(-1)^D - (-1)^{t^2} : R_T^t(Q^t) \times \frac{(RQ)^t}{(RQ)^t} \longrightarrow t^{-1}(RQ)^t$$

OK on image of $t^D : RQ \longrightarrow (RQ)^t$

OK on $t^{-1}$

$$(-1)^D(t^{-1}) - (-1)^{t^2}(t^{-1}) = t^{-1} - (-t^{-1}) = 2t^{-1}.$$

So we have two auto. $(-1)^D$ and $(-1)^{t^2}$ of $X(T(R(Q^t))) = (F^p) X^{t}(R^t)$ preserving

$$F^p_{1-t^2} X^t(R^t) = (F^p X)^t$$

For any element $p(x) \in R_T^t(Q^t)$ have

Wait. Think of $x^t$ with the filter $(F^p X)^t$. Two auto of $R^t$ preserving $I^t$ but $(F^p X)^t$ is $F^p_{1-t^2} X(R^t) \circ \text{have }$

streaming elts.

$(I^t)^{n+1} \vdash (I^t)^n R^t$

$h(I^t)^n dR^t$ go through such generators to calculate $\lambda(-1)^{t^2} : (F^p X)^t \longrightarrow t^{-1}(F^p X)^t$
Recall the concrete approach.

\[ F^p X \geq_k \ \text{spanned by} \]
\[
(F^p X)^t = F^p(I^t) X (R^t)
\]

For \( p = 2n \) we have

\[
(I^t)^{n+1} + [I^t]^n R^t \implies h^p(G^t)^n d(R^t)
\]

\[ h_D = h^p(I, D) \]

I think what I want to do is to illustrate the \( X(RQ) \) approach from the \( SQ \) one. To do this, we have \( X(RQ) \)

On one hand, we have \( F^p \Omega \geq_k \)

\[
(F^p \Omega)^t = F^p(\Omega^t)
\]

\[
= [\Omega^p]^t \odot [^t] \oplus [\Omega^t]^n \oplus
\]

\[ F^p \Omega \geq_k \ \text{spanned by} \]

\[ [x_0 dx_1 \ldots dx_j, x_{p+1}] \]

\[ x_0 dx_1 \ldots dx_n \quad n > p \]

\[ \sum_{i} \text{ord}(x_i) \geq j \]

Translates into

\[ F^p X \geq_k \ \text{spanned by} \]

\[ (I^n)^{n+1} + [I^n R] \geq_k \]

\[ p(x_0) \omega(x_1, x_2) \ldots \omega(x_{p+1} x_2) \quad j \geq n+1 \]
If you don't have the proof in your mind you can't outline it. Recall so far we have defined Nito's character for union quasi-hom.

\[ h_k \in H^0_c \left( Q^n_{\geq k+1}, Q^n_{\geq k} \right) \]
\[ e_k \in H^2_c \left( Q^b_{\geq k}, Q^b_{\geq k+1} \right) \]
\[ S_k \in H^1_c \left( Q^b_{\geq k}, \mathbb{F}^b_{\geq k+1} \right) \]

\[ S_k(\eta) = S \quad \text{if } S_k = S \]

rep. \( S_k \) by \( \frac{1}{2} (S_k + \delta S_k) \) and so on.

So we get

\[ X(RA) \xrightarrow{\psi} X(RQ) \xrightarrow{P_m^p(c)} \mathbb{F} X(RQ) \xrightarrow{P_{m\alpha}^p} \mathbb{F} \mathbb{F} \mathbb{F}^{-2m+1} X_{\geq 2m+1} \]

which gives Nito's proof

\[ \text{Ch}_{2m-1}(\eta, \gamma) \in H^{2m}_c(\eta X_{\leq 2m+1}) = H^{2m}_c(A^b, \mathbb{F} \mathbb{F} \mathbb{F}^{-2m+1}) \]

Next I need last map, point is we have

\[ Q \longrightarrow \mathbb{L} \otimes B \]

\[ Q_{\geq k} \longrightarrow J_{\geq k} \otimes B \]
8/12 - 0520

go over the whole proof.

\( \theta, \theta' : A \to \mathbb{C} \times \mathbb{C} \mod \mathbb{C} \times \mathbb{C} \)

\( p = \tfrac{1}{2} (\theta + \theta') , \quad q = \tfrac{1}{2} (\theta - \theta') : A \to \mathbb{C} \times \mathbb{C} \)

\( p + q : A \to \mathbb{C} \times \mathbb{C} \quad \text{linear resp.} \ 1. \)

\( \nu = (p + q) : RA \to S \otimes RB , \quad IA \to K \otimes RB + S \otimes IB \)

\( X(\nu) : X(RA) \to X(S \otimes RB) \to \bigoplus_{i \geq 0} q(i) \otimes FP^{-2i} \)

link my construction to Nistor's.

\( Q = QA \quad \text{filtration} \quad Q \geq \kappa \quad \text{after} \ 1 \ \text{order} \ 2 \)

\( \text{induced fields} \quad LQ \geq \kappa \quad \text{mixed subvers.} \)

\( \nu_k \in HC^0 (\Omega^2 \Omega^2_{k+1} , LQ_{\geq \kappa}) \)

\( \exists \ S_k \in HC^2 (\Omega^2 \Omega^2_{\leq \kappa} , LQ^2 \geq \kappa) \)

unique up to \( S \)

\( S_k \restriction_k = S \quad \text{or} \quad S_k \subseteq S \).

can suppose \( [S_k, F] = 0 \) whenever needed.

\( \nu_k \in HC^0 (\Omega^2 \Omega^2_{k+1} , LQ^2 \geq \kappa) \)

\( S_k \in HC^0 (\Omega^2 \Omega^2_{\leq \kappa} , LQ^2 \geq \kappa) \)

same props.

\( \gamma^\Omega Q_{\geq \kappa+1} = \gamma^\Omega Q_{\geq \kappa} \quad \text{for} \ k \ \text{even} \)
\[
\begin{align*}
\mathrm{Ch}^{2m}(\epsilon, \epsilon') &= \sum S_{2m-1} S_3 S_1 \cdot \mathrm{Ch}^0(\epsilon, \epsilon') \\
&\in \mathrm{HC}^{2m}(\Omega A, J_# \otimes \Omega B) \\
\mathrm{Ch}^0(\epsilon, \epsilon') &= \chi^{(2m)} = \frac{1}{2} (\epsilon - \epsilon') \\
\end{align*}
\]

next \( \Theta \Theta' \) induce

\[
\Omega A \xrightarrow{l_k} \Omega Q \xrightarrow{q} \chi \Omega Q = (\chi \Omega Q)_{\leq 0}.
\]

true map \( V_k \)

\[
\Omega Q \xrightarrow{l_k} J_k \otimes \Omega B
\]

Claim this agrees with my

\[
\mathrm{Ch}^{2m}(\Theta, \Theta') = [\mu_m \times u_k] \\
\in \mathrm{HC}^{2m}(\chi A, J_{#}^{2m+1} \otimes \Omega B)
\]

I want to go over the steps of the proof many times today until it becomes incredibly clear in my mind.

First step is to pass from the mixed complexes \( \Omega A \otimes \) to the correct towers.

\( \Omega A \otimes \) spanned by \( \chi \otimes \epsilon_k, \ldots \epsilon_n \), \( \sum \) odd \( \epsilon_k \geq k \).

compatible with \( \Omega A \) alg structure / grading.
b) hence compatible with $b, K$, etc.

Recall linear form $X(RQ)^* \cong \Omega Q$

and the description of the structure on $X(RQ)$ in terms $\Omega Q$ and its operations.

this structure

$$b(x, y) = \sum_{i=0}^{n-1} k^{2i} b(x, y) + \sum_{i=0}^{n-1} k^{2i} d(x, y)$$

0735 go over the steps.

$$0, 0 \text{ yield hom. } Q \to \mathbb{L} \otimes B$$

$$\implies \mathbb{L} \otimes k \to \mathbb{J}^k \otimes B \quad \forall k.$$ 

get $\Omega Q \to \Omega_{\mathbb{L}}(\mathbb{L} \otimes B) = \mathbb{L} \otimes \Omega B$

DG alg homom. $\implies$

$$\Omega Q \otimes k \to \mathbb{J}^k \otimes \Omega B \quad \forall k.$$ 

Claim: get map $\mathbb{J}^k \otimes \Omega B$.

$$\Omega Q \otimes k \to \mathbb{J}^k \otimes \Omega B \quad \forall k$$

compatible $\mathbb{L} \otimes b, K$, etc. This means we have

$$X(RQ) \otimes k \to \mathbb{J}^k \otimes X(RB)$$

comp with differentials, $F^p_\otimes \mathbb{J}^k \otimes F^p_{LB}$. 

c) Go over facts, see what is obvious, + what requires proof.

\[ \theta, \theta' \text{ induce } \varphi : Q \to L \otimes B \]
\[ \Rightarrow \quad Q_\geq k \to \Omega^k (L \otimes B) \quad \forall k \]

get hom \[ \Omega^k Q \to \Omega^k (L \otimes B) = L \otimes \Sigma^k B \]
\[ \Rightarrow \quad \Omega^k (Q_\geq k) \to \Omega^k (L \otimes B)_\geq k = J^k \otimes \Sigma^k B \quad \forall k \]

get map \[ \Omega^k (Q_\geq k) \to J^k \# \otimes \Sigma^k B \quad \forall k \]

compat with \[ \alpha, \beta, \kappa, \rho, \ldots \]

get hom \[ R^k Q \to L \otimes RB \]
\[ \Rightarrow \quad R^k (Q_\geq k) \to J^k \otimes RB \]

get map \[ X(Q_\geq k) \to J^k \# \otimes X(RB) \]

compat with \[ \delta, \gamma, \kappa \]
\[ \Rightarrow \quad F^p X_\geq k \to J^k \# \otimes F^p \mathbb{I} \]

This is all straightforward. The main point not essentially tautological is why

\[ \Omega^k (Q_\geq k) \to J^k \# \otimes \Omega B \]

commutes with \( b \).

Now the \( t \) version.

\[ \text{hom. } \quad Q^t \to L^t \otimes B \]
\[ L^t(Q^t) \to \Omega L^t (L^t \otimes B) \]
\[ \Phi(Q^t) \to L^t \otimes \Omega B \]
So review the definition of \( X \rightarrow \mathbb{J} \# \otimes X(\mathbb{R}B) \).

possibilities:

\[
\begin{align*}
X \rightarrow \mathbb{J} \# \otimes X(\mathbb{R}B) & \sim \mathbb{J} \# \otimes \mathbb{B} \\
\text{trace} & \\
\end{align*}
\]

Why does this commute?

\[
\begin{align*}
X(\mathbb{R}Q) \rightarrow \mathbb{J} \# & \sim \mathbb{J} \# \otimes \mathbb{B} \\
\end{align*}
\]

\[
\begin{align*}
X(\mathbb{R}(L \otimes B)) & \rightarrow \mathbb{J} \# \\
\end{align*}
\]

doesn't work without difficulty.

So instead what I propose to do is to use

\[
\begin{align*}
\dot{\Theta}^{t \mathbb{R}Q} & \rightarrow \dot{\Theta}^{t \mathbb{J} \# \otimes \mathbb{B}} \\
\end{align*}
\]
Then apply
\[ X_T(R_T(Q^t)) \longrightarrow X_L(R_L(L \otimes B)) \]
\[ \quad \text{is} \quad \text{is} \]
\[ Q_T(Q^t) \longrightarrow Q_L(L \otimes B) \otimes L \]
\[ \text{to} \quad \text{to} \]
\[ (Q_Q)^t \longrightarrow L^t \otimes \Omega B \]

Repeat the logic:
\( \theta, \theta' \) induce < hom. \[ Q \longrightarrow L \otimes B \]
such that \[ Q \geq h \longrightarrow J^t \otimes B \] for all \( h \).
Thus we have a homomorphism:
\[ Q^t \longrightarrow L^t \otimes B \]
of graded \( T \)-algs. Comm. dge.

\[ X_T(R_T(Q^t)) \longrightarrow X_L(R_L(L \otimes B)) \]
\[ \quad \text{is} \quad \text{is} \]
\[ Q_T(Q^t) \longrightarrow Q_L(L \otimes B) \otimes L \]
\[ \text{to} \quad \text{to} \]
\[ (Q_Q)^t \longrightarrow L^t \otimes \Omega B \]

\[ R_T(Q^t) \longrightarrow R_L(L \otimes B) = L^t \otimes RB \]
\[ I_T(Q^t) \longrightarrow I_L(L \otimes B) = L^t \otimes IB \]

\[ (X(R_Q)^t)^U \]
\[ (F^P_X)^t \]

\[ X_T(R_T(Q^t)) \longrightarrow X_L(L \otimes RB) = L^t \otimes X(R_B) \]
\[ \text{is} \quad \text{is} \]
\[ Q_T(Q^t) \longrightarrow Q_L(L \otimes B) \otimes L \]
\[ \text{to} \quad \text{to} \]
\[ (Q_Q)^t \longrightarrow L^t \otimes \Omega B \]

\[ L^t \otimes IB = L^t \otimes F^P IB \]
Also comm. diag.

\[ X_t(R_t(Q^t)) \to X_t(L_t^t(\mathcal{B} \otimes B)) \]

\[ \Omega_t(Q^t) \to \Omega_t(L_t^t \mathcal{B}) \]

leading to

\[ X_t \to L_t^t \otimes X(R^t B) \]

\[ \Omega(Q^t) \to L_t^t \otimes \Omega B \]

1545 2\frac{1}{2} hours.

Review what we did learn, namely

\( \theta, \theta' \) induce

\[ Q \to L \otimes B \]

\[ Q_{\theta^t} \to L_{\theta^t} \otimes B \]

where

\[ \text{hom of filtered alg.} \]

whence

\[ \Omega_t(Q^t) \to \Omega_t(L_t^t \mathcal{B}) \to \Omega_t(L_t^t \mathcal{B} \otimes \mathcal{B}) \]

\[ (\Omega Q)^t \to L_t^t \otimes \Omega B \]

also

\[ X_t(R_t(Q^t)) \to X_t(R_t^t(L_t^t \mathcal{B})) \]

\[ X_t \to L_t^t \otimes X(R^t B) \]

have comm. diag.

\[ X_t(R_t(Q^t)) \to X_t(R_t^t(L_t^t \mathcal{B})) \]

\[ \Omega_t(Q^t) \to \Omega_t(L_t^t \mathcal{B}) \otimes \mathcal{B} = L_t^t \otimes \Omega B \]
Canonical K^* equiv.

\[ L^t_B \otimes X(RB) \]

Find last map compatible with filtrations, i.e.

\[ F^p X^t \rightarrow L^t_B \otimes F^p X(RB) \]

and that it is canonically K^* equiv. equivalent to trace map:

\[ (\Omega Q)^t \rightarrow L^t_B \otimes \Omega B \]

respecting \( F^p \)-filtrations.

Next, now that the last map is under control, we need to focus on the key point, which is how \( D \) enters. Here we take off the facts that use the fact \( RA, X(RA) \) depends only on \( Q \) as vector space with \( I \).

Let's try to find an order:

\[ \begin{array}{c}
A^t \xrightarrow{p^t B} S \otimes B \subseteq L^t \otimes B \\
\end{array} \]

\[ \begin{array}{c}
Q \xrightarrow{?} S \otimes B \\
\end{array} \]

So far, we have established

\[ (F^p X^t) = \sum \frac{t^k}{k!} F^p X \]

This comes from the definition

\[ X^t \sim (\Omega Q)^t \]
(I_k)^{2} = k \sum \tau_{k} = 2k \tau_{k} + \ldots + \tau_{k}^{2} \tau_{k} = \frac{k^{2} \tau_{k}^{2}}{k^{2}} = \tau_{k}^{2}

It follows that $I_{0}(k)$ is an important subset of $I_{1}(k)$.

From this one knows that $I_{1}(k)$ is actually generated by $I_{0}(k)$.

Then the whole thing collapses, can assume $k = 0 = \beta_{k}$.

Then this all goes away.

What is $I_{1}(k)$? Can define it as

$F_{0}(\tau(\alpha), R(\tau(\alpha)))$

for $\tau(\alpha) = \alpha_{k}$.

What does this mean in concrete terms?

What is $\alpha_{k}$? Can define it as

$U_{(\alpha)} = F_{0}(\tau(\alpha), R(\alpha))$

for $\alpha_{k}$.
I) Return to how $D$ enters. You control $FPX \leq k$ by means of

$$(FPX)^t = Fp \text{I}^{(Q_t)} X_T(R_T(Q_t^t))$$

Okay

Now you have $D$ on $Q$, $RQ$
Set $L_D$ on $X(RQ)$. Extend $D$ to $Q^t$

$Q^t \subset T \otimes Q$

How $D$ enters. Now consider $D$.

Try this way. Start with $Q = \mathbb{Q}$,
$Q_{\mathbb{Q}^t} = \mathbb{Q} \oplus Q_{n}$, define $D$ on $Q$ by $D = 1$ on $Q_n$.
Extend $D$ as derivation on $RQ$, $L_D$ on $X(RQ)$.

grade $Q$ as $v_s$ with $1$ induces grade on alg $RQ$ and on src $X(RQ)$. Identify degree operators.

$Q \stackrel{t}{\longrightarrow} Q^t$

extends to isom of $T$-modules

$T \otimes Q \longrightarrow Q^t$

induces

$R_T(T \otimes Q) \cong R_T(Q^t)$

$T \otimes RQ \otimes (RQ)^t$

This is the extension of $t^D : RQ \longrightarrow (RQ^t)$
to a $T$ module map.

$T \otimes (RQ) \cong X(T \otimes RQ) \cong X_T((RQ)^t) = X(RQ)^t$
Consider $X(RQ)$

Suppose you understand $Q^t = \bigoplus_{i=1}^n Q^t_i \subset T \otimes \mathcal{K}$

graded K-algebra

last map (trace map)

$QB^t$ reduces $Q \longrightarrow L \otimes B$ homo of $Q^t \rightarrow T \otimes \mathcal{K}$ filt. alg.

i.e. hom. $Q^t \longrightarrow L^t \otimes B$ for $T$-alg

get $X(RQ)^t = X_T(R_T Q_t^t) \rightarrow X_{t^c}(R_{t^c}(L^t \otimes B)) = L^t_{t^c} \otimes X(RB)$

$(RQ)^t = R_T(Q^t)^t \rightarrow \Sigma_{i=1}^n (t^c \otimes \mathcal{K}) \otimes L^t_{t^c} = L^t_{t^c} \otimes \Sigma \mathcal{K}$
Also the bottom map, compact, with Hodge filtration i.e.
\[ \text{FP}(\Omega Q)^k \rightarrow L_t \otimes \text{FP} \Omega B \]
whence
\[ \text{FP}X_{\geq k} \rightarrow J_k \otimes \text{FP IB} \]

So now I understand the last map and trace map. But now I have to understand the role being in D.

Bring in D. Here use \( X(RA) \) depends on \( Q \) as vector up with 1. Grading on \( Q \) induces gradings on \( RA, X(RA) \). Degree of \( D, L \).

First claim is consistency of the induced grading + filtration. Because
\[ R_{\geq k} = \bigoplus Q_n^{\geq k} \]

it follows that
\[ RA_{\geq k} = \bigoplus RA_n^{\geq k} \]

Why is this true? Matter of defn. Here is \( X_{\geq k} = X(RA)_{\geq k} \) defn? Concretely what happens is this:
\[ X_{\geq k} \]

i. \( X_{\geq k} \) spanned by \[ f(x_0) \cos^m(x_1, x_2 m) \sum_{m=0}^\infty \int_{dP(x_2 m)} x_i \geq k \]
\[ X_n \text{ spanned by } \\{ p(x) \cdots p(x_n) \} \quad \Sigma l x_i = \frac{1}{2} \ n \]

question is why \[ X_{\geq k} = \bigoplus_{n \geq k} X_n \]

Enough to consider \( R_{\geq k} \) and \( R'_{\geq k} = \bigoplus_{n \geq k} R_n \)

\[ \omega(x, x) = p(x, x) - p(x_i) p(x) \]

\[ \therefore R_{\geq k} \subset R'_{\geq k} \]

But \( p(q_n) R_{\geq k} \subset R_{\geq k+1} \)

\[ \Rightarrow R'_k R_{\geq k} \subset R_{\geq k+1} \quad \text{etc.} \]

Let's try a more abstract proof.

\[ T \otimes Q \to Q^+ \]

\[ E^k \otimes E^k \to E^{k+1} \otimes E^{k+1} \]

isom. of graded \( T \) modules resp. 1.

\[ \text{indeces } R_T(T \otimes Q) \sim R_T(Q^+) \to R_T'(T' \otimes Q) \]

\[ T \otimes RQ \to (RQ)^+ \subset T' \otimes RQ \]

\[ T \otimes Q \to Q^+ \subset T' \otimes Q \]

\[ R_T(T \otimes Q) \sim R_T(Q^+) \to R_T'(T' \otimes Q) \]

\[ T \otimes RQ \to T' \otimes RQ \]
\[ x \xrightarrow{1_0} T \otimes Q \xrightarrow{\sim} Q^t \subset T' \otimes Q \]
\[ RQ \xrightarrow{\sim} R_T(T \otimes Q) \xrightarrow{\sim} R_T(Q^t) \xrightarrow{s^1} R_T(T' \otimes Q) \]
\[ \text{logic? at the moment } Q \text{ is only a graded vector space with } 1 \in Q_0, \text{ this makes you to prove and } Q_{2k} \overset{\text{def}}{=} \bigoplus_{1 \leq k \leq m} Q_{2k}. \]
\[ \text{equiv. to } T \otimes Q \xrightarrow{\sim} Q^t \subset T' \otimes Q \]
\[ 1 \otimes x \xrightarrow{1} t^{D_x} \]

Then \[ RQ \xrightarrow{\sim} R_T(T \otimes Q) \xrightarrow{\sim} R_T(Q^t) \xrightarrow{s^1} R_T(T' \otimes Q) \]
\[ \text{The first thing that happens is the grading. This is unclear because you mix the grading + filtration.} \]
\[ \text{you have } Q \xrightarrow{t^D} T' \otimes Q \text{ this map induces} \]
\[ RQ \xrightarrow{\sim} R_T(T \otimes Q) \]
\[ \text{a homom. } 2^\infty \xrightarrow{g(x)} g(t^{D_x}), \text{ so we get a grading on } RQ \text{ is } \text{self}. \text{compat. with alg structure.} \]
Repeat: You say that $RQ$ depends only on $Q$ as vector space with $\mathcal{A}$, and this means the grading on $Q$ induces an alg. grading on $RQ$. Why?

$Q \xrightarrow{t^D} T' \otimes Q$ hom. map

induces $RQ \rightarrow R_{t^D}(T' \otimes Q)$

$x \rightarrow T' \otimes RQ$

so we get a homomorphism $RQ \rightarrow T' \otimes RQ$

and the image is a graded subalgebra.

This homom. is inj (set $t=1$)

with the specialization map $T' \otimes RQ \rightarrow RQ$. Thus $RQ$ has the structure of a graded alg.

$\Rightarrow X = t^D$.

Now that the grading is straight we consider the filtration:

$T \otimes Q \rightarrow T' \otimes Q$

induces $R_{t^D}(T' \otimes Q) \rightarrow R_{t^D}(T' \otimes Q)$

$T \otimes RQ \rightarrow T' \otimes RQ$ graded $T$-module map $x \mapsto t^D x$ graded $T$-alg map
grading works as follows
\[ \mathcal{Q} \rightarrow T \otimes \mathcal{Q} \]
induces hem.

\[ R \mathcal{Q} \rightarrow R_1(T \otimes \mathcal{Q}) = T \otimes R \mathcal{Q} \]
\[ \rho(x) \rightarrow t^1 \rho(x) \times \text{hem.} \]

This is a lifting of \( R \mathcal{Q} \) to a graded subalgy of
\( T \otimes R \mathcal{Q} \), lifting means wrt specialization
\[ T \otimes R \mathcal{Q} \rightarrow R \mathcal{Q} \]
to \( \mathcal{Q} \rightarrow 1 \).

Similarly get map of sets.
\[ x(R \mathcal{Q}) \rightarrow x_1(R_1(T \otimes \mathcal{Q})) = T \otimes x(R \mathcal{Q}) \]
This is a lifting of \( x(R \mathcal{Q}) \) to a graded
super subcomplex of \( T \otimes x(R \mathcal{Q}) \),
But what about the filtrations
assoc. to these gradings. By defn we have
\[ T \otimes \mathcal{Q} \rightarrow \mathcal{Q}^t \subset T \otimes \mathcal{Q} \]
\[ T \otimes R \mathcal{Q} \rightarrow (R \mathcal{Q})^t \subset T \otimes R \mathcal{Q} \]
\[ T \otimes x(R \mathcal{Q}) \rightarrow x(R \mathcal{Q})^t \subset T \otimes x(R \mathcal{Q}) \]
provided \( Q_{>k}, (R \mathcal{Q})_{>k}, x(R \mathcal{Q})_{>k} \) defined in

terms of the grading, e.g. \( (R \mathcal{Q})_{>k} = \bigoplus (R \mathcal{Q})_n \ V_k \).
But then it follows that we have
\[ \text{some notation} \]
1) It follows that we have
\[ Q^t \subseteq T \otimes Q \]
inducing isomorphisms:
\[ R_T(Q^t) \cong (RQ)^t \quad x_T(R_T(Q^t)) \cong x(RQ)^t \]
Why, because:
\[ T \otimes Q \cong Q^t \subseteq T \otimes Q \]
\[ R_T(T \otimes Q) \cong R_T(Q^t) \to R_T(T \otimes Q) \]
\[ \cong \]
\[ T \otimes RQ \cong (RQ)^t \subseteq T \otimes RQ \]

So where are we now? Past stages involve
- Recall of Nistor \[ N.8 \]
- End maps \[ N.11 \]
- Grading \[ N.12 \]

So now what next. I have I think understood why
\[ T \otimes x(RQ) \cong x_T(R_T(Q^t)) \cong x(RQ)^t \subseteq T \otimes x(RQ) \]
So now I can examine the behavior of \( L_D, h_D \)
\( L_D, h_D \) operators on \( x_T(R_T(Q^t)) \)
The point is that we have \( L_D, h_D \)
acting on \( x(RQ) \)
So what do we do next.

Identify my map with

\[ X(\text{RA}) \xrightarrow{t^D} X(\text{RQ}) \xrightarrow{P_{m(L_0)}} \delta X_{2m+1} \xrightarrow{l} J_{2m+1} \otimes X(\text{RB}) \]

I have to list the identifications.

\[
\begin{align*}
X(\text{RA}) & \xrightarrow{t^*} X(\text{RQ}) \\
& \xrightarrow{\pi_-} S_q \otimes X(\text{RB}) \\
& \xrightarrow{1 - L_0} \pi_{-S_{q,3}} \otimes X(\text{RB}) \\
& \xrightarrow{1 - t_0^D} \pi_{-S_{q,3,3}} \otimes X(\text{RB}) \\
& \xrightarrow{l} J_{2m+1} \otimes X(\text{RB})
\end{align*}
\]

Let's be explicit about the maps.

\[
A \xrightarrow{t^D} Q \xrightarrow{t^*} Q^t \xrightarrow{L_{t} \otimes B}
\]

Let's be explicit about the maps.

\[
\begin{align*}
X(\text{RA}) & \xrightarrow{t^D} X(\text{RQ}) \\
& \xrightarrow{t^*} X(\text{RQ})^t \\
& \xrightarrow{L_{t} \otimes B X(\text{RB})}
\end{align*}
\]
T) Now what??
I think you must go back to the map. It is defined by

\[ Q^t \longrightarrow L^t \otimes B \]

\[ \text{getting} \quad X_t(R_t(Q^t)) \longrightarrow X_t(R_t(L^t \otimes B)) \]

\[ X(RQ)^t \longrightarrow L^t \otimes X(RB) \]

and restricting to degree k to get

\[ X(RQ)_{2k} \longrightarrow J_k \otimes X(RB). \]

Other point is that we have a \( T \)-module map

\[ \text{tr} X(RQ) \stackrel{t^D}{\longrightarrow} X(RQ)^t \longrightarrow L^t \otimes X(RB) \]

which means that all we need is to take

\[ X(RQ) \longrightarrow X(RQ)^t \longrightarrow L^t \otimes X(RB) \]

What I am trying to say is that

\[ X(RQ)^t \longrightarrow L^t \otimes X(RB) \]

is a \( T \)-module homomorphism. The former is free over \( T \). generated by \( t^D X(RQ) = \bigoplus t^t X(RQ)_n \)

Thus

\[ X(RQ)_{2k} = \bigoplus_{n \geq h} X_n \]

\[ t^k X(RQ)_{2k} = \bigoplus_{n \geq h} t^k (t^n X_n) \]

\[ t^n X_n \longrightarrow t^n J_n \otimes X(RB) \]

\[ t^k X_n \longrightarrow t^k J_k \otimes X(RB) \]
Let's set it up. Chain

\[ A \rightarrow \varnothing \rightarrow \mathcal{S} \otimes \mathcal{B} \]

\[ A \rightarrow \mathcal{S} \otimes \mathcal{B} \rightarrow \text{the composition} \]

\[ A \rightarrow \mathcal{Q} \rightarrow \mathcal{Q}^t \rightarrow \mathcal{S} \otimes \mathcal{B} \rightarrow \mathcal{S} \otimes \mathcal{S} \]

which gives commutativity

\[ X(RA) \rightarrow X(RQ) \rightarrow \mathcal{S} \otimes X(RB) \]

\[ X(RA) \rightarrow X(RQ) \rightarrow \bigoplus_{k=0} X(RQ)^{2k} \rightarrow \mathcal{S} \otimes X(RB) \]

so why am I confused at this point?

Idea for

[x version of Nistor construction]

grading on \( \mathcal{Q} \) = grading on \( RA, X(RQ) \) = filtration on \( RA, X(RQ) \).
\(8/14-0536\)

\(X\) version of what Hst0r does

\(Q = QA = \mathbb{Z}A\) with \(o\), \(Q = \oplus Q_n\), \(Q_n = \mathbb{Z}A\)

\(RQ\), \(X(RQ)\) inherit gradings

\(X(RQ)_n\) spanned by \(p(x_1) \cdots p(x_n)\)

\(q_p(x_1) \cdots p(x_j) p(x_{j+1})\)

\[\sum |x_i| = n.\]

Associated filtration \(X(RQ)_\geq k = \oplus X(RQ)_n\)

spanned by above els with \(\sum |x_i| \geq k\)

\(D\) on \(RQ\), \(L_D\) on \(X(RQ)\), \(h_D\)

Next I can I define \(I(Q) \geq k\)?

space define \(I(Q) = \mathbb{K}er (RQ \rightarrow Q)\)

\((I(Q))_\geq k = I(Q) \cap (RQ)_\geq k\)

define then

\[(I(Q))_\geq k = \sum_{k_1 + \cdots + k_n = k} (I(Q))_{\geq k_1} \cdots (I(Q))_{\geq k_n}\]

\[(F^m \mathbb{Q} X(RQ))_\geq k = (I(Q))_{\geq k} + \sum_{k_1+k_2=k} (I(Q))_{\geq k_1} (RQ)_\geq k_2\]

\[\leq \sum_{k_1+k_2=k} \mathbb{Q} ((I(Q))_{\geq k_1} \cup (RQ)_\geq k_2)\]

defines a bifiltration \(FPX_{\geq k}\) of \(X = X(RQ)\)

Can state lemma

\(L_D, h_D: FPX_{\geq k} \rightarrow FP^{-2}X_{\geq k}\)

\(L_D - k: FPX_{\geq k} \rightarrow FP^{-2}X_{\geq k+1}\)

\(r - (-1)^k: FP^{-2}X_{\geq k} \rightarrow FP^{-2}X_{\geq k+1}\).
Other lemma is under the rain.
\[ X(Q) \sim \mathcal{Q} \]
\[ F^\mathcal{P} X_{2k} \sim F^\mathcal{P} (Q)_{2k} \]
and the canonical bijection equin. \[ X(Q) \sim \mathcal{Q} \]
induces a bijeq \[ F^\mathcal{P} X_{2k} \sim F^\mathcal{P} (Q)_{2k} \].

Next the trace map \[ \varphi \rlap{\hspace{1em}} \larrow \sigma \varphi \rightrightarrows \varphi \rlap{\hspace{1em}} \leftarrow \sigma \varphi \]
\[ Q \rightarrow L \otimes B \]
\[ Q_{2k} \rightarrow J_{2k} \otimes B \]
\[ \text{Reorder instead like } p + t \text{ you form} \]
\[ Q \rightarrow S \otimes B \]
\[ \text{lin. resp. 1} \]
\[ \text{resp. grading} \]

Third lemma says \[ F^\mathcal{P} X_{2k} \rightarrow J_{2k} \otimes F^\mathcal{P} B \].

8/15-0532 Review program

1) my construction \[ X(QA) \rightarrow J^{2m+1} \otimes X(QB) \]
\[ F^\mathcal{P} \rightarrow \mathcal{P} \]
\[ \text{resp. grading} \]

2) version of Nijita construction \[ \mathcal{S}_{k} \in \text{HC}^2(\sigma_-(Q)_{2k+1}, \sigma_+(Q)_{2k+1}) \]
\[ \exists \mathcal{S}_{k} \in \text{HC}^2(\sigma_-(Q)_{2k+1}, \sigma_+(Q)_{2k+1}) \]
unique up to \( \text{ker } \mathcal{S} \)
\[ \mathcal{S}_{k} \in \mathcal{P} \]
\[ \mathcal{S}_{k} = \mathcal{S} \] defines

\[ \mathcal{L} \rightarrow \mathcal{Q} \rightarrow \mathcal{L}_2 \rightarrow \mathcal{Q}_2 \rightarrow J_{2m+1} \sim \mathcal{Q}(Q)_{2k+1} \]
1) By Chen's theorem for universal quasi-homomorphisms:
\[ \chi^2_m(c, c') \in \text{HC}^{2m}(\mathcal{RA}, \mathcal{K'}Q) \geq 2m+1 \]
trace map
\[ \Omega \geq k \rightarrow \text{J}^k \# Q \]

class \[ l \in \text{HC}^{0}(\Omega \geq k, \text{J}^k \# Q) \]

put
\[ \chi^2_m(c, c') = \bigtriangleup \chi^2_{m+1}(c, c') \]
\[ \in \text{HC}^{2m}(\mathcal{RA}, \text{J}^{2m+1} \# Q) \]

3) X-version of Nistor construction:

X-version of Nistor construction:
Consider \( k = X(RQ) \)
grading \( Q = \oplus \mathcal{Q} \)
Inherited gradings on \( RQ, X(RQ) \)
\( D, \mathcal{L}_D, \mathcal{h}_D \).
Also \( \gamma \)
Assoc. filtration \( (RQ)_{\geq k}, X(RQ)_{\geq k} = X_{\geq k} \)
\( (IQ)_{\geq k} = IQ \cap (RQ)_{\geq k} \)
\( (IQ')_{\geq k} \)
\[ h(IQ'' d(RQ))_{\geq k} = \]

Lemma 1. \( L_D, \mathcal{h}_D : PP^{2k} X_{2k} \rightarrow PP^{2k} X_{2k+1} \)
\[ \mathcal{L}_D - k : PP^{2k+1} X_{2k+1} \rightarrow PP^{2k+2} X_{2k+1} \]
\[ \gamma = (-1)^k : \ldots \rightarrow PP^{2k+1} X_{2k+1} \]

\( R^t = \Theta t_{R^t} < T \otimes R \)
\( I^t = \Theta t(\mathcal{I} \cap R^t) \)
\[ = (T' QI) \cap R^t < T \otimes I \]
y) What's confusing is that $I$ is an arbitrary ideal. We need to know:

$X_T(R^t)$ is torsion-free over $T$

because then the local map

$X_T(R^t) \rightarrow X_{T'}(T \otimes R)$

is injective, identifying

$X_T(R^t) \cong X_T$

But then we have an ideal $I^t \subset R^t$ and

$(FPX)^t = FP^t X(R^t)$

Then all we need is a relative version of $b_D : F^p_T \subset F^{p+2}_T$.
We do need $\phi : R^t \rightarrow S^2_T(R^t)$

torsion-free

i.e. want $\phi : R \rightarrow S^2 R$ to be compatible with filtration.

$L_D - k : \bar{I}_{2k} \rightarrow R_{2k+1}$
because $I_{2k}/\bar{I}_{2k+1} \subset R_{2k}/R_{2k+1}$

killed by $L_D$
or better: $(L_D - k)(I_{2k}) \subset (L_D - k)(R_{2k}) \subset R_{2k+1}$

Note: you don't need $I_{2k} = I \cap R_{2k}$
2) only that $R_i \geq c = I \geq k + e$ etc so that $I_t$ is an ideal in $R^t$.

Point is that you went over all this before

3) $X$ version of Nistor's construction. Consider $X = X(RQ)$.
grading on $Q$ as v.s. induces gradings on $RQ, X
D, L, h, D^*$
assoc. filt. $X \geq k$
$I \geq k = IQ \cap RQ_{\geq k}$
$(I^n)_{\geq k} = I^n_{\geq k}$
$L((I^n)^{\geq k}) = L(I^n)^{\geq k}$
$FPX_{\geq k} = FPX_{\geq k}$

Lemma 1: behavior $D, h, D^*$
$X \simeq \mathbb{Q}$
canonical v.s. isomorphism
$X \simeq \mathbb{Q}$
canonical homotopy equivalent

Lemma 2: canonical homotopy again $X \simeq \mathbb{Q}$ induces $FPX \simeq \mathbb{Q}$
$FPX_{\geq k} \simeq FPQ_{\geq k}$

Final step - trace map $\eta: A \to L \otimes B$

Induce $Q \to L \otimes B$
$Q \simeq k \otimes B$ hom. of filtered algs.

Let's try to understand what is needed. I have supposedly identified Nistor's $\partial^2 (\xi, \eta)$
with $X(RA) \to X(RQ) \xrightarrow{P_m(L^2)} X(RQ) \to 2m + 1$.
and its filtration behavior. i.e. \( F^p \xrightarrow{\alpha} \Xi F^{p-2m+1} \)

(so that we get \( X_A \xrightarrow{\alpha} X_{2m+1} \))

Anyway, I now need the \( X \)-version of the trace map. There are still some problems here. So let's review. I have to get agreement with my construction:

\[
\begin{align*}
X(RA) & \longrightarrow X(S \otimes RB) \xrightarrow{\alpha} S_0 \otimes X(RB) \longrightarrow J^{2m+1} \otimes X(RB) \\
X(RQ) & \longrightarrow X(S \otimes RB) \xrightarrow{\alpha} S_0 \otimes X(RB)
\end{align*}
\]

So far, we have used

\[
\begin{align*}
A & \xrightarrow{p+\beta} S \otimes B \\
\alpha & \xrightarrow{\text{linear map}} \text{compat with gradings}
\end{align*}
\]

leads to

\[
\begin{align*}
X(RA) & \xrightarrow{(p+\beta)\#} \\
X(RQ) & \xrightarrow{\alpha} X(S \otimes RB) \xrightarrow{\alpha} S_0 \otimes X(RB) \\
X(RQ) & \xrightarrow{\text{two steps}} \xrightarrow{\alpha} X(RB)
\end{align*}
\]

\[
J^{2m+1} \otimes X(RB)
\]
critical point to understand is

\[ x^* : X(RQ) \rightarrow S_p \otimes X(RB) \]

\[ U \]

\[ X(RQ) \rightarrow S_p \otimes X(RB) \]

\[ \triangleright \]

\[ j^k \otimes X(RB) \]

commutes.

But first I have to define trace map.

\[ Q \rightarrow L \otimes B \text{ fill.} \]

\[ Q_{\otimes k} \rightarrow J^k \otimes B \]

\[ Q^t \rightarrow L^t \otimes B \text{ hom. of } \eta \text{ T-alg.} \]

\[ R_T Q^t \rightarrow R_T (L^t \otimes B) \]

\[ R Q^t \rightarrow L^t \otimes RB \]

\[ X_T (RQ^t) \rightarrow X_{L^t} (L^t \otimes RB) \]

\[ L^t \otimes X(RB) \]

\[ (\Omega Q)^t \rightarrow L^t \otimes \Omega B \]

So we know that \( Q^t \rightarrow L^t \otimes B \)

induces the trace map after applying the relative \( X \cdot R \) in the two cases.
so now I have $X$ version of the trace map of Nistor.

so what should be possible is to describe following picture

$$X(RA) \rightarrow X(RQ) \rightarrow X(RQ) \rightarrow \cdots \rightarrow \bigwedge_{2n+1} \otimes X(RB)$$

\[ \text{trace map} \]

and its filtration behavior.

So I define the trace map as induced by the homomorphism:

$$Q^t \rightarrow L^t \otimes B$$

$$RQ^t = R_t(Q^t) \rightarrow R_t(L^t \otimes B) = L^t \otimes RB$$

$$\begin{align*}
F^P X(RQ^t) & \rightarrow X_t(L^t \otimes RB) \\
L^t \otimes X(RQ^t) & \rightarrow L^t \otimes F^P X(RB)
\end{align*}$$

1409 Start again to clean up things.

$\sharp$ $X$-version of trace map.

hom. of filtered algs. $Q \rightarrow L^t \otimes B$, $Q^t \rightarrow L^t \otimes B$

hom. of graded alg. $Q^t \rightarrow L^t \otimes B$

compatible with evident hom. $T \rightarrow L^t$.

So get $X_t(R_t(Q^t)) \rightarrow X_t(R_t(Q^t))$. 
such that

\[ F^p_{I_Q^t} X_T(R_T(Q^t)) \rightarrow F^p_{L^t \otimes IB} X(L^t \otimes RB) \]

i.e.

\[ X(RQ)^t \rightarrow L^t \otimes X(RB) \]

\[ (FPX)^t \rightarrow L^t \otimes F^p_{IB} \]

Like the following

Hom of f-alg.

\[ Q \rightarrow L \otimes B \quad Q \rightarrow J^k \otimes B \quad \text{such that} \]

\[ Q^t \rightarrow L^t \otimes B \quad \text{comp. with} \quad T \rightarrow L^t \]

gives Hom of r-alg.

\[ Q \rightarrow L \otimes B \]

yields

\[ R_T(Q^t) \rightarrow R_{L^t}(L^t \otimes B) \]

\[ (RQ)^t \rightarrow L^t \otimes RB \]

Such that

\[ (IQ)^t \rightarrow L^t \otimes IB \]

Then yields

\[ \xrightarrow{I_Q^t} \]

\[ X_{\frac{1}{2}}(RQ)^t \rightarrow X_{L^t}(L^t \otimes RB) \]

\[ X(RQ)^t \rightarrow L^t \otimes X(RB) \]

Such that

\[ F^p_{I_Q^t} X(RQ)^t \rightarrow L^t \otimes F^p_{IB} X(RB) \]

Thus get \( Q^k \)

\[ X(RQ)^k \rightarrow J^k \otimes X(RB) \]

s.t.

\[ (FPX)^k \rightarrow J^k \otimes F^p_{IB} \]
But in fact we have the extra information that $X(RQ)^t \rightarrow L^*_y \otimes X(RB)$ is a $T$-module map and that

$$T \times X(RQ) \xrightarrow{t^D} X(RQ)^t$$

In other words $T \times Q \xrightarrow{t^D} Q^t$

$$T \times RQ \xrightarrow{t^D} RQ^t$$

$$T \times X(RQ) \xrightarrow{t^D} X(RQ)^t$$

So how can I describe this sensibly?

I know that I can also describe the trace map

$$X(RQ)^t \rightarrow L^*_y \otimes X(RB)$$

as the unique $T$-module map extending the maps as follows

$$\mathbb{Q} \xrightarrow{t^D} \mathbb{Q}^t \rightarrow L^*_y \otimes B$$

linear map $1$ induces

$$RQ \xrightarrow{t^D} RQ^t \rightarrow L^*_y \otimes RB$$

induces

$$X(RQ) \xrightarrow{t^D} X(RQ)^t \rightarrow L^*_y \otimes RB$$

better

$$\mathbb{Q} \rightarrow L^*_y \otimes B$$

$$RQ \rightarrow R_{L^*_y}(L^*_y \otimes B) = L^*_y \otimes RB$$

$$X(RQ) \rightarrow X_{L^*_y}(L^*_y \otimes RB) = L^*_y \otimes X(RB).$$
In concrete terms we have the sort of map like \( p + t \):

\[ Q \rightarrow Q^t \rightarrow L^t \otimes B \]

which yields

\[ X(RQ) \rightarrow X(L^t \otimes RB) \rightarrow L^t_0 \otimes X(RB) \]

which yields

\[ T \otimes X(RQ) \rightarrow L^t_0 \otimes X(RB) \]

\[ \xrightarrow{\mu} \]

\[ X(RQ)_t \]

\[ \xrightarrow{\phi} \]

\[ X(RQ)_n \]

\[ \xrightarrow{\#} \]

\[ J^k \otimes X(RB) \rightarrow J^{t+1} \otimes X(RB) \]

\[ \xrightarrow{\text{induced by } J^k \subset J^{t+1}} \]

\[ \text{for } n \geq k. \]

Let's go back & keep on trying to get to the bottom of things. Start again:

I have

\[ X(RA) \rightarrow X(RQ) \rightarrow X(L^t \otimes RB) \rightarrow L^t_0 \otimes X(RB) \]

Try to list the steps to follow:

my map

\[ A \rightarrow S \otimes B \]

induces

\[ X(RA) \rightarrow X_S (L^t_0 \otimes (S \otimes B)) = S^t_0 \otimes X(RB) \]

followed by \( \mu_n \).
Keep an eye on reviewing. My map can be described as follows.

One has \( p + t g : A \rightarrow S \otimes B \) lim. map 1.

This includes

\[
X(RA) \rightarrow X_s(R_s(S \otimes B)) = S_q \otimes X(RB)
\]

which we can then follow by

\[
\bigoplus_{m \geq 1} X(RB).
\]

I propose to factor \( p + t g \) into

\[
\begin{array}{ccc}
A & \xrightarrow{i} & Q \\
\text{hom.} & & \text{lim. map 1}
\end{array}
\]

and grading.

Which includes

\[
X(RA) \rightarrow X(RQ) \rightarrow X(R(Q^1, z_0)) \rightarrow S_q \otimes X(RB)
\]

\[
X(RQ) \xrightarrow{t_d} X(RQ) t_{\geq 0} \xrightarrow{w_x} S_q \otimes X(RB)
\]

OK, so there's still this part I don't get.

Let us consider namely the use of \( S \) versus \( L \).

Let's go on. My map

\[
X(RA) \rightarrow X_s(R_s(S \otimes B)) = S_q \otimes X(RB)
\]

factor \( p + t g : A \rightarrow S \otimes B \)

into

\[
\begin{array}{ccc}
A & \xrightarrow{i} & Q \\
\text{lim. map 1} & & \text{and grading}
\end{array}
\]
What is easy is the following

\[ X(\mathcal{RA}) \xrightarrow{w} X(\mathcal{RQ}) \xrightarrow{w_{2m+1}} X(\mathcal{QB}) \]

\[ \xrightarrow{P_m(CD) \gamma_2} \quad \xrightarrow{P_m(CD) \gamma_2} \]

\[ S_{2m+1} \otimes X(\mathcal{RB}) \]

\[ \xrightarrow{e_{2m+1}} \quad \xrightarrow{\text{ev}_1} \]

\[ J^{2m+1} \otimes X(\mathcal{RB}) \]

\[ \text{inv. resp. 1.} \quad \vdots \]

I think I want to keep after this because there might be a cleaner version.
So what actually happens?

Go back to

\[
\begin{align*}
Q \xrightarrow{t^D} Q^t & \overset{5}{\xrightarrow{\gamma}} L^t \otimes B \\
\text{linear rep.} & \text{homom.} \\
D & \xleftarrow{t^D} \mathcal{D}
\end{align*}
\]

\[
RQ \xrightarrow{t^D} (RQ)^t & \overset{5}{\xrightarrow{\gamma}} L^t \otimes RB
\]

\[
\begin{align*}
X(RQ) & \xrightarrow{t^D} (X(RQ))^t \overset{5}{\xrightarrow{\gamma}} L^t \otimes X(RB) \\
\text{linear rep.} & \text{and grading}
\end{align*}
\]

Concentrate on the fact that

\[
Q \xrightarrow{t^D} L^t \otimes B
\]

\[
\text{linear rep.} \\
\text{and grading}
\]

\[
\therefore \text{get } X(RQ) \xrightarrow{t^D} L^t \otimes X(RB)
\]

Here seems to be a point: Consider

\[
Q \xrightarrow{t^D} S \otimes B
\]

\[
\text{linear rep.} \text{ and grading}
\]
This induces:

\[ X(RQ) \rightarrow X(S \otimes RB) \rightarrow S_\mathfrak{p} \otimes X(RB) \]

compatible with grading:

\[ X(RQ) \rightarrow S_\mathfrak{p} \otimes X(RB) \]

Other point is that \( S_\mathfrak{p} \) is a \( T \)-module, so we have extension:

\[ T \otimes X(RQ) \rightarrow S_\mathfrak{p} \otimes X(RB) \]

Somehow think along these lines. You have:

\[ A \xrightarrow{p+q} S \otimes B \]

graded alg.

\[ \cdots RA \rightarrow S \otimes RB \]

graded

\[ d(RA) \]

Actually what is \( S(RA) \)?

algebra generated by RA.

So what do we have??

The graded
go over steps - what to say

\[ X(\text{RA}) \rightarrow X(\text{S} \otimes \text{RB}) \rightarrow S^p \otimes X(\text{RB}) \rightarrow T^{2m+1}_\text{S} \otimes X(\text{RB}) \]

\[ A \xrightarrow{p+g} S \otimes B \]

graded alg

\[ \text{RA} \rightarrow S \otimes \text{RB} \]

\[ \text{RA} \rightarrow \text{RQ} \rightarrow S \otimes \text{RB} \]

I am beginning to think that I can use \( S(\text{RA}) \) instead of \( \text{RQ} \). How does this

If \( A \) is a vector space with 1 then there is a corresponding \( N \)-graded vector space with 1 generated by \( A \) namely

\[ S_A = A \oplus A \oplus A \oplus ... = C[t] \otimes A / (t)^{n+1}_A \]

Then \( A \xrightarrow{p+g} S \otimes B \) you do factors in this way

\[ \text{RA} \rightarrow R(S_A) \rightarrow \text{RQ} \rightarrow S \otimes \text{RB} \]

\[ \text{S}(\text{RA}) \]

\[ \text{A} \xrightarrow{p+g} S \otimes B \]

\[ \text{A} \oplus \text{A} \rightarrow S'_A \]

vector space level

\[ \text{but I don't see } F \text{ connected with } R(\text{A} \oplus \text{A}) \]

\[ \text{okay} \]
get back into the spirit, find what to say

\[ A \xrightarrow{p \oplus q} S \oplus B \]

\[ X(RA) \rightarrow X_S(R_S(S \oplus B)) = S \oplus X(RB) \]

Recall what I liked yesterday about the trace map at the end.

Hom. of filtered alg.

\[ Q \rightarrow L \otimes B \]

Hom. of gr alg.

\[ a^t \rightarrow L^t \otimes B \]

\[ T \rightarrow L^t \]

\[ Q^t \rightarrow L^t \otimes B \]

Yields

\[ R_{\text{r}}(Q^t) \rightarrow R_{L^t}(L^t \otimes B) \]

\[ (RA)^t \rightarrow L^t \otimes RB \]

s.t.

\[ (IQ)^t \rightarrow L^t \otimes IB \]

Then get

\[ X_T((RA)^t) \rightarrow X_{L^t}(L^t \otimes RB) \]

\[ X(RA)^t \rightarrow L^t \otimes X(RB) \]

s.t.

\[ F_{\text{r}}^{\text{p}} X(RA)^t \rightarrow L^t \otimes F_{\text{r}}^{\text{p}} X(RB) \]

Thus we get

\[ X(RA)_k \rightarrow J^k \oplus X(RB) \quad \forall k \]

\[ \rightarrow P P X_{2k} \rightarrow J^k \oplus F_{\text{r}}^{\text{p}} X_{2k} \]

Note all done on filtered algebra level.

Next go back to factor of \( p \oplus q \)

\[ A \rightarrow Q \rightarrow \]
\[ A \xrightarrow{t} Q \xrightarrow{h} S \otimes B \]

\[ X(RA) \xrightarrow{t} X(RQ) \xrightarrow{2m+1} S \otimes X(RB) \]

\[ y \cdot X(RQ) \xrightarrow{2m+1} y \cdot X(RB) \]

\[ J^{2m+1} \otimes X(RB) \]

to get straight

\[ Q \xrightarrow{t} S \otimes B \leq L^{t} \otimes B \]

linear map

\[ Q \xrightarrow{t} Q \xrightarrow{w} L^{t} \otimes B \]

Let's go over the steps.

my map + filtration behavior yields

\[ ch^{2m}(\theta, \theta') \in HC^{2m}(X_{A}, J^{2m+1} \otimes \mathbb{Q}) \]

The version of Nistor's construction

\[ ch^{2m}(\theta, \theta') \in HC^{2m}(\pi A, X(RQ), 2m+1) \]

\[ \times (\theta, \theta') \in HC^{\bullet}(\pi A, X(RQ), 2m+1), J^{2m+1} \otimes \mathbb{Q} \]

\[ ch^{2m}(\theta, \theta') = \times (\theta, \theta') \cdot ch^{2m}(\theta, \theta') \in HC^{2m}(\pi A, J^{2m+1} \otimes \mathbb{Q}) \]
exactly what is at stake.

Next point is to relate these two.

We need $X$ version of Nistor's construction.

\[ Q = \bigoplus_{n} Q_n, \quad Q_n = 2^n Q \]

\[
\pi_{\mathcal{D}} = \bigoplus_{n} \pi_{\mathcal{D}, n}
\]

filtration compatible with $\mathfrak{g}$.

\[ Q_{<k} = \bigoplus_{n} Q_{<k, n} \]

grading on $Q$ induces grading on $RQ$, $X(RQ)$.

$D$, $L_D$, $\theta_D$ (comm. $\phi$).

Associated filtrations.

Outlines

my construction $\quad \text{ch}^{2m}(0,0') \in H^c_{2m}(X_{\mathcal{A}}, \mathbb{Z}) \neq 0$

Nistor's $\quad \text{ch}^{2m}(c, c') \in H^{2m}(\mathcal{R}A, \mathbb{Z})$

$X$-version of Nistor's construction.

Up to now have considered $RQ$, $RQ_{<k}$

Now you want to look at $RQ$ and $X(RQ)$.

To define $X_{\geq k}$, $R^k X_{\geq k}$

\[ b(px, \ldots, px_m, d(px_{m+i})) \quad \sum |x_i| = n. \]
0/7 - 05:47
my construction

\[ A \xrightarrow{p \circ h} S \otimes B \]

\[ X(RA) \xrightarrow{F^p} X_0(R_0(S \otimes B)) = S_B \otimes X(RB) \]

\[ J_{#} \otimes X(RB) \]

\[ J_{#} \otimes X(RB) \]

\[ F_{#}^{p \circ h_1} \otimes X_B \]

\[ X_{#} \]

Next a version of Nistor's construction

Let \( Q = QA \) be the free product \( A * A \) in the category of unital algebras. Let \( Q \) be the canonical automorphism of order 2 interchanging the two copies of \( A \), let \( QA \) be the canonical algebra of the \( QA \) equipped with the graded product

\[ x \circ y = xy - (-1)^{\deg x \deg y} xy \]
0539

Our version of Nistor's construction

\[ Q = QA, \ g_A, \ X \ \text{rep}. \ A \times A \text{ in the cat. of unital algebras} \]
the kernel of \( A \times A \to A \)
the canonical autom of order 2

\[ Q = QA \text{ equipped with Fed prod } \]

\[ \lambda a = a + da, \ \lambda a = a - da \]

\[ \Omega = \cdot \]

get grading of \( Q \) on vector space

\[ Q = \bigoplus Q_n \quad \text{s.t. } \lambda Q_n = (-1)^n Q_n \quad \text{and such that} \]

\[ Q_n = \Omega^A, \quad Q_{\geq 0} = \bigoplus_{n \geq 0} Q_n \]

8/18 Review again.

\[ X \text{ version of Nistor's construction} \]

\[ Q = \bigoplus Q_n, \quad 1 \in Q_0 \quad \text{grading of } Q \text{ as vector space} \]

induces gradings \( RQ = \bigoplus RQ_n, \quad x(RQ) = \bigoplus x(RQ_n) \)

\[ x(\sum \lambda x_i) = \sum \lambda x_i \quad \text{where } \sum |x_i| = n \]

get filtrations \( (RQ)_h \)

\[ (RQ)_h \text{ spanned by } x \quad \text{where } \sum \text{ord}(x_i) > h \]

Start again 06/17

\[ X \text{ version of Nistor's construction} \]

Recall \( x(RQ) \neq RQ \) and \( FP_{IQ} x(RQ) = FP_{IQ} RQ \)
also \( x(RQ) \sim RQ \) \( FP_{IQ} x(RQ) \sim FP_{IQ} RQ \)

claim this extends to filtered algebras

\[ Q_{\geq 0} \cdot Q_{\geq 0} = Q_{\geq 0}, \quad 1 \in Q_{\geq 0} \]
\( \overline{\lambda}_{T}(Q^t) \) is the directed summum of
\( Q^t \otimes_T \ldots \otimes_T Q^t \)
which embeds in \( \overline{\lambda}_{T}(T' \otimes Q) = T' \otimes \overline{\lambda}_{Q} \)
\( \overline{\lambda}_{T}(Q^t) \twoheadrightarrow (TQ)^t \subset T' \otimes \overline{\lambda}_{Q} \)

Relative versions over \( T, T' \),
\( \chi_{T}(R_T Q^t) \Rightarrow \Omega_{T}(Q^t) \)
\( \chi_{T}(R_{T'}(T' \otimes Q)) \Rightarrow \Omega_{T'}(T' \otimes Q) \)
\( T' \otimes \chi(QR) \subset T' \otimes \overline{\lambda}_{Q} \)
gives
\( \chi(QR)^t = (TQ)^t \)
\( \chi(QR)^{t,k} = (TQ)^{t,k} \)

Similarly,
\( I_T(Q^t) = \text{Ker} (R_T Q^t \rightarrow Q^t) \)
\( I_T(Q^t) = \text{Ker} (TQ \rightarrow Q^t) \)
\( \bigoplus_{t,k} \text{Ker} (TQ \rightarrow Q_{t,k}) \)
\( (IQ)^t = \bigoplus_{t,k} (IQ \rightarrow Q_{t,k}) \)

\( I_T(Q^t)^m = (IQ)^m = (\sum t^k (IQ)^{t,k})^m \)
\( = \sum_{k_1, \ldots, k_m} t^{k_1} \ldots (IQ)^{t,k_m} \)

So that
\( F_P^{*} \chi_{T}(R_T Q^t) = F_P^{*} \Omega_{T}(Q^t) = \bigoplus_{t,k} F_P^{*} (IQ)^{t,k} \)

\( F_P^{*} \chi(QR)^t = \)
Review earlier stuff.

A filtered alg, $\mathbb{Q} \geq 0$

$Q^{i} = \bigoplus t^{i} Q_{\geq i}$

$T = \mathbb{C}[t^{i}] \subseteq T'$

Identify decreasing filtrations on $\bigoplus V$

with a graded $T$-submodule of $T \otimes V$.

A graded $T$-submodule of $T \otimes V$ has the form $\bigoplus \bigoplus k \leq V_{2k}$ where $(V_{2k})$ is a decreasing filtration of $V$.

If $(V_{2k})_{k \in \mathbb{Z}}$ is a decreasing filtration of a vector space $V$ equipped with an $\mathbb{R}$-basis, let $V_{2k}$ be a decreasing filtration of $V$.

Put $V^{\mathbb{Q}} = \bigoplus \bigoplus V_{2k}$. This is a graded $T$-submodule of $T \otimes V$.

Equivalence between decreasing filtrations on $V$ and graded $T$-submodules $\mathbb{R}^{+}$.

**Theorem.** A filtered alg, $\mathbb{Q} \geq 0$

$Q^{i} = \bigoplus t^{i} Q_{\geq i}$

$Q^{\mathbb{Q}}(Q^{i}) \to Q^{i}(T \otimes \mathbb{Q}) = T \otimes Q^{\mathbb{Q}}$

Direct summand of $\bigoplus (Q^{i} \otimes Q^{i} \otimes Q^{2}) \to T \otimes Q^{\mathbb{Q}}$

$Q^{\mathbb{Q}}(Q^{i}) \to T \otimes Q^{\mathbb{Q}}$
Similarly, for canonical indicies, $X(\mathbf{RQ}) = X(\mathbf{FR})$.

And similarly for $h(\mathbf{RQ}) = h(\mathbf{FR})$.

Provided $\mathbf{RQ} = \mathbf{FR}$.

Also,

$$X(\mathbf{RQ}) = X(\mathbf{FR})$$

Provided $\mathbf{RQ} = \mathbf{FR}$.
So far I have reviewed the filtered algebra version of $F^p x(RQ) = P^p Q$ and $w$.

Next need grading:

$q = \bigoplus_{n} q^n, \quad l \in q_0$

$q^a_k = \bigoplus_{n \geq k} q^n$

$D$ degree op., $D=n$ on $q_n$

Use $RQ$, hence $x(RQ)$ depends only on $q$ as vector space with $1$.

$RQ$, $x(RQ)$ inherits grading.

$x(q^n)$ spanned by

$p_{x_1} \cdots p_{x_m} \quad q (p_{x_1} \cdots p_{x_m} d(p_{x_{m+1}}))$

$D$ on $RQ$ is unique derivation extending $D$ on $q$

deg $p_{x^n} x(RQ)$ is $L_D = L(1, D)$

canonical $\phi$ and $b_D = h^\phi (1, D)$.

$\bigotimes Q$ means $\bigotimes Q \xrightarrow{\sim} Q^+$

\[
\begin{align*}
    t^{-m} \circ x & \quad \mapsto \quad t^{-m}(t^D x) \\
\end{align*}
\]

sum. of graded $T$-modules.

\[
\begin{align*}
    R_f (T \otimes Q) & \xrightarrow{\sim} R_f (Q^+) \\
    \bigotimes RQ & \quad \sim \quad \bigotimes x(RQ) \xrightarrow{\sim} x(RQ)^+. \\
\end{align*}
\]
Key results concern $L_0$, $h_0$, $\gamma$ relative to $FPX$.

1) $h_0 : FPX \to F^{p-2}X$, also $L_0$

2) $L_0 - tQ \to FPX \to F^{p-2}X$,

3) $\gamma = (-1)^k : FPX \to F^{p-2}X$.

Translate into:

1) $h_D : (FPX)^t \to (FP^{-2}X)^t$

2) $L_D - tQ : (FPX)^t \to (FP^{-2}X)^t$

3) $\gamma = (-1)^t : (FPX)^t \to (FP^{-2}X)^t$

where $(FPX)^t = \sum t^k (FP_{IQ} x(R))_{2k} \subset (X(RQ))^t$.

But because of

$$X_T(R_T(Q^t)) \sim X(RQ)^t$$

$$FP_{T(Q^t)} X_T(R_T(Q^t)) \sim (FPX)^t$$

1) is a relative form of the calculation

$$h_D : FPX \to F^{p-2}X$$

2) results from the definition of $FP_{I(Q^t)}$ which extends $D-tQ$ to $R(Q^t)$, which maps $Q^t$ into $t^{-1}Q^t$.\[\]
\[
\begin{array}{ccccccc}
X \rightarrow \mathbb{F}^p X \rightarrow \mathbb{F}^p X \rightarrow O \\
X \rightarrow X \rightarrow O \\
X \rightarrow X \rightarrow O
\end{array}
\]

\[
\begin{array}{c}
X \rightarrow X \rightarrow O \\
X \rightarrow X \rightarrow O
\end{array}
\]

\[
\begin{array}{c}
X \rightarrow X \rightarrow O \\
X \rightarrow X \rightarrow O
\end{array}
\]

\[
X \rightarrow X \rightarrow O
\]

**Proof:** \( \mathcal{X} X_{\geq k+1} = \mathcal{X} X_{\geq k} \)

and \( \mathcal{X} F^p X_{\geq k+1} = \mathcal{X} F^p X_{\geq k} \)

So the point which I forgot is that \( y = (-1)^k \) on \( F^p X_{\geq k} \) means \( y F^p X_{\geq k+1} = y F^p X_{\geq k} \) for \( k \) even and all \( p \).
\[ q^t = \bigoplus t^k q^t \land k \subset T' \times Q \]

Injective
\[ q^t \otimes q^t \subset q^t \otimes (T' \times Q) \subset (T' \times Q) \otimes (T' \times Q) \]
\[ = T' \otimes Q \otimes^2 \]

8/20 - 0599

D section

Grading
\[ Q = \bigoplus Q^i \]

RQ, \( x(RQ) \) defined only

Reduced grading \( RQ^i, x(RQ)_i \)

Degree of \( D \) on \( RQ \), \( L_D \) on \( x(RQ) \)

First result is consistency of filtration + grading, clear from element description:

\[ px_{i_1} \ldots px_{i_m} \sum |x_{i_1}| = n \text{ or } \sum \text{order}(x_i) > k. \]

Alternative viewpoint

\[ x(RQ) \overset{t \cdot b}{\longrightarrow} T' \otimes_x x(RQ) \]

\[ \overset{1 \otimes f}{\longrightarrow} \]

\[ T \otimes x(RQ) \longrightarrow x(RQ) \]

\[ x(RQ) \overset{1 \otimes t}{\longrightarrow} T \otimes x(RQ) \overset{\longrightarrow}{\longrightarrow} x(RQ)^t \subset T' \otimes x(RQ) \]

Lift out \( x_T(R_T(Q))^t \)

\[ T \otimes x(RQ) \overset{\longrightarrow}{\longrightarrow} x_T(R_T(Q))^t \overset{\longrightarrow}{\longrightarrow} x(RQ)^t \subset T' \otimes x(RQ) \]
Point is that the fact that

Point to make?

On one hand you have \( X_{>k} \) understood, described, controlled, by \( X^t \) which sits

\[ X_T \rightarrow X^t < T \otimes X \]

On the other hand you have

\( Q \rightarrow Q^t \) linear map hot, kettles

such that \( T \otimes Q \rightarrow Q^t \)

hence \( T \otimes X \rightarrow X_T \)

Do it again:

On one hand you have the

filtration \( Q_{>k} \) described by \( Q^t < T \otimes Q \)

On the other hand you have the grading

described by \( D \), (also by the subspace \( tDQ < \otimes T \otimes Q \)).

Repeat. You have

\[ \text{filtration } Q_{>k} \leftarrow Q^t < T \otimes Q \]

\[ \text{grading } Q_n \leftarrow D \]

how do you expect the assertion that the

filtration \( Q_{>k} \) arises from the grading

\[ T \otimes tDQ \otimes \rightarrow Q^t \]

in more detail: \( T \) makes \( Q \) into
In more detail, the map \( t^D : Q \rightarrow T' \otimes Q \)
induces an isomorphism of graded \( T \)-modules,
\( T' \otimes Q \rightarrow Q^t \).

This means that \( t^D \) actually maps \( Q^t \)
the image of \( t^D \) is contained in \( Q^t \)
and that the extension to a \( T \)-module map
is an isomorphism.

Repeat: have
- filtration \( Q \leq \bigoplus \)
- grading \( Q \leq D \)
- grading defines a map \( t^D : Q \rightarrow T' \otimes Q \)
which extends to a graded \( T \)-module map
\( T \otimes Q \rightarrow T' \otimes Q \)

and the claim is that
\[ Q^t = \bigoplus_{k \geq 0} Q^k \leq t^D Q \bigoplus \]

Proof: \( Q^t = \bigoplus \)
\[ = \bigoplus \bigoplus_{n \geq k} t^n Q^n \bigoplus\]
\[ = \bigoplus_{n \geq k} t^{n-k} Q^n \bigoplus\]
\[ = C \big[ \big[ t \big] \big] \otimes \bigoplus_{n \geq k} t^n Q^n \bigoplus\]
Assume $T \otimes Q \cong Q^t$

look at degree $k$ and you get

$$
\oplus \bigoplus t^p \otimes Q_{k+p} \cong t^k Q_{\geq k}
$$

Repeat: have

filtration $Q_{\geq k}$ described by the graded $T$-module $Q^t \cong T \otimes Q$

grading $Q_n$ by the operator $D$

the grading defines a map of graded vector spaces

$$t^D : Q \to T \otimes Q$$

which extends to a graded $T$-module map

$$T \otimes Q \to \bigoplus T' \otimes Q$$

and we have

$$Q_{\geq k} = \bigoplus Q_n \iff T \otimes Q \cong Q^t$$

Now that $T \otimes A$ depends only on $A$

as $S$-bimodule with $1 \mapsto$

\[
\begin{align*}
X_T R_T (T \otimes Q) & \cong X R_T (Q^t) \otimes (R_T (T' \otimes Q)) \\
T \otimes X R_Q & \cong X (R_Q)^t \otimes T' \otimes X R_Q
\end{align*}
\]

conclusion is that $t^D : Q \to Q^t$ induces

$$T \otimes X (R_Q) \cong X_T (R_T (Q^t)) \cong X (R_Q)^t$$
Not clear how much of this I have to say.

So what do we have at the moment?

Just the statement that $t^D: Q \to Q^t$ induces $T \otimes X(RQ) \sim X_t(R_t(Q^t)) \simeq X(RQ)^t$.

Behavior of $D$, $D^t$, $h_D$.

extend $h_D$ to $X(RQ)^t$.

\[ B/21 - 0507 \quad \text{What is the problem.} \]

\[ X(RQ)_k = \bigoplus_{n\geq k} X(RQ)_n \quad \text{for} \]

$X_t(R_t(Q^t)) \sim X(RQ)^t = \bigoplus t^k X(RQ)_{\geq k} \subset T \otimes X(RQ)$

there's a glitch with the definition of $D$, $h_D$, $h_D$ on $X(RQ)^t$. This is need the following

a relative form.

The point is to generalize the objects $Q$.

$Q$ has grading as vector space such that the associated $\mathbb{Z}/2$ grading and associated filtration are comp. with alg. structure.
The problem is that I still can't organize the proof in my mind. I want now to sit down and outline the whole thing. The main topics

Nistor's construction (our version)
X-version of Nistor's construction

\[ X \text{ analogue of } (\mathcal{D}^X_{\mathcal{F}}, b_R) \]

bifiltration \((P^X_{\mathcal{F}})\)

behavior of \(L_0, b_0, \) \& \(\psi\) \text{ or } \(P^X_{\mathcal{F}}\)

Link between our + Nistor's constructions

I can review pieces I understand but I would like to get a hold of the whole picture.

Let's take the graded approach first.

I would like to start to emphasize the structure on \(Q\) graded as vector space.

2/4 graded as algebra

emphasize also canonical iden
\[ X(\mathcal{Q}) = \mathcal{Q} \quad \mathcal{F}^0_{\mathcal{D}} X(\mathcal{Q}) \approx \mathcal{F}^0_{\mathcal{D}}\mathcal{Q} \]

and canonical deg.
\[ X(\mathcal{Q}) \approx \mathcal{Q} \quad \mathcal{F}^0_{\mathcal{D}} X(\mathcal{Q}) \approx \mathcal{F}^0_{\mathcal{D}}\mathcal{Q} \]
So far we emphasize structure on $Q$ vs. with $I$

canonical ident. $\mathbf{X}(\mathbf{Q}) \cong \mathbf{Q} \otimes \mathbf{FP}$

first point is the filtration on $Q$ leads to filtrations on $(\mathbf{Q})_2$, $\mathbf{X}(\mathbf{Q})_2$, $\mathbf{FP}_2 \mathbf{X}(\mathbf{Q})_2$

consequence is that we get a tower

$$\mathbf{X}_2 = (\mathbf{X}_2 / \mathbf{FP}_2 \mathbf{X}_2) \cong \mathbf{Q} \otimes \mathbf{Q}_2$$

grading. $\mathbf{R}_Q, \mathbf{X}(\mathbf{Q})_2$ depend only on $Q$ as vector space with $I$.

filtration on $Q$ induces a filtration on $\mathbf{R}_Q, \mathbf{X}(\mathbf{Q})_2$. Why precisely?

$Q^t \subset T \otimes Q$

$$R_t(Q^t) \rightarrow R_t \otimes (T \otimes \mathbf{R}) = T \otimes \mathbf{R}$$

compatible. The image is a graded $T$-submodule of $T \otimes \mathbf{R}$. hence of the form $(\mathbf{Q})^t$ for some filtration.

fight with Erica over bike
structure on $Q$

grading as v.s. with 1

assoc. filt. | loop alg. structure

2/2: grading

get induced $T$ and filtrations on each of the objects $RQ$, $F^P RQ$, $RQ$, $IQ$, $x(RQ)$, $F^P IQ$, $x(RQ)$
compatible with the structure on these objects + relations between them discussed before.

Explain $Q^t < T^\otimes Q$.

$T$ subalgebra

$\Omega_T(Q^t) \rightarrow \Omega_{T'}(T^\otimes Q) = T^\otimes RQ$

image is evidently $(RQ)^t$ for the filtrator.

Take

$F^P \Omega_T(Q^t) \rightarrow T^\otimes F^P RQ$

$\boxtimes_t \Omega^P_t(Q^t)$

$[\Omega^P_t(Q^t), Q^t] \rightarrow T^\otimes [\Omega^P_t Q, Q]$

in degree $\bullet$ coefficient

all this is the easy part.
Point is that $RQ$ depends on $Q$ as vector space with $1$, and similarly $X(RQ)$.

How do you get induced grading on $RQ$?

$$t^D : Q \to T \otimes Q \quad \text{lin. rep. } 1.$$  

$$RQ \to R^\alpha_t (T \otimes Q) = T \otimes RQ$$  

Obviously we expand on what we understand without getting the whole picture.

$Q$ graded as vector space with $1$

$$Q = \bigoplus Q_n \quad 1 \in Q_0.$$  

$$D = n \circ Q_n$$

$$t^D : Q \to T \otimes Q$$

$$D \leftarrow t^D_t$$

$$t^D \quad \text{lin. rep. } 1.$$  

$$RQ \to R^\alpha_t (T \otimes Q) = T \otimes RQ$$

lifting $\mathbb{G}$ of $RQ$ rel. spec. at $t = 1$.

To graded subspaces... get grading on $RQ$

get $D \circ RQ \leftarrow t^D_t$ for above map, which is then $t^D$

Similarly get grading and $t^D$ is degree.
What to prove next?
Need something reasonable.

Concentrate. Note that

\[ a \rightarrow T \otimes a \rightarrow (T \otimes a)^t = T \otimes a \]

\[
\begin{align*}
RQ & \rightarrow R_{T}(T \otimes a) \rightarrow R(T(Q^t)) \rightarrow R_{T}(T \otimes a) \\
& \rightarrow T \otimes RQ \rightarrow (RQ)^t \subset T \otimes RQ
\end{align*}
\]

1. \( a \) is graded, \( \Rightarrow \) \( RQ \) graded
2. \( a \) is filtered \( \Rightarrow \) \( RQ \) filtered

to see 1) take \( a \rightarrow T \otimes a \)

generate \( RQ \rightarrow T \otimes RQ \)

and \( D \) on \( RQ \) \( \Rightarrow \) \( \) get \( (RQ)^t \) \( \subset \) \( \) homogeneous

to see 2) take \( a \rightarrow T \otimes a \)

generate \( R(T(Q^t)) \rightarrow T \otimes RQ \)

image is \( (RQ)^t \)

3. When \( q_{a1} = \otimes q_{\eta} \) have \( T \otimes a \rightarrow (T \otimes a)^t \subset T \otimes a \)

\[ T \otimes RQ \subset R(T(Q^t)) \rightarrow T \otimes RQ \]

composition injective

Conclusion
try again

3) When \((Q^\otimes R^t)\) assoc. grading have

\[ Q \subseteq T \otimes Q \rightarrow Q^t \subseteq T \otimes Q \]

\[ \text{No you're confusing things.} \]

\[ \text{You want to say that the grading} \]

\[ RQ \subseteq T \otimes RQ \]

determines the filt., i.e.

\[ T \otimes RQ \rightarrow (RQ)^t \]

grading of \(Q\) gives \(RQ \rightarrow T \otimes RQ\)

homog. image \(\oplus t \otimes (RQ)^t\) gives grading \(RQ\).

filt. of \(Q\) gives \(R_T(Q^t) \rightarrow T \otimes RQ\)

image is \((RQ)^t\). Now grading \(\rightarrow\) filt on \(Q\)

says \(T \otimes Q \rightarrow Q^t \Rightarrow T \otimes RQ \rightarrow R_T(Q^t) \Rightarrow T \otimes RQ \rightarrow (RQ)^t\) whereas grading \(\rightarrow\) filt on \(RQ\).