

## Contents 1990

### 1990-1

Quillen's own index for January 22 - April 11, 1990.

January 22: Notes on BRS cohomology.

January 25: An analogy between the  $(b, S, 1 - \kappa, B)$  operators and the  $(d, i_X, L_X, Pi_X)$  operators for manifolds with a circle action.

February 2: An analogue of the  $S$  operation. Transgression.

February 3: Lundell's construction deforming  $S^2 \wedge U_n \rightarrow U_{2n}$  to a map  $S^2 \wedge U_n \rightarrow U_{n+1}$ .

February 4: Two methods for defining classes  $cs_{2n-1} \in H^{2n-1}(P)$ ,  $2n > \dim B$ , where  $P \rightarrow B$  is a principal  $U_n$ -bundle over  $B$ .

February 6: Review of Bott map.

February 10: On  $U_N/\Delta_n S^1 \times U_{N-n}$ . Chern-Simons forms on a  $U_n$ -bundle. Variation maps.

February 12,15,16: Notes about Feigin-Tsygan on Lie algebra cohomology and Riemann-Roch.

February 20: Lecture on Lie algebra cohomology.

February 21-28: Leray spectral sequence for the principal bundle  $G \rightarrow P \xrightarrow{\pi} B$ . Spectral sequence arising from the bigraded differential algebra  $\Omega(P) \otimes \mathfrak{g}_\chi^* \otimes S\mathfrak{g}_\phi^*$ . Bott's spectral sequence where  $E_2 = H_{\text{diff}}^*(G, S\mathfrak{g}^*) \Rightarrow H^1(BG)$ . Review of Leray and Bott spectral sequences.

### 1990-2

March 11: More on Karoubi's  $\kappa$  operator.

March 17: Formulas for a circle action on a manifolds and a discrete analogue in cyclic theory.

March 18: More on Karoubi's  $\kappa$  operator and an  $S$  operator.

March 20: Formulas connected with the periodic complex  $\cdots \rightarrow \overline{Q} \xrightarrow{d} (\Omega^1 Q)_\natural \xrightarrow{\beta} \overline{Q} \rightarrow \cdots$ . Bismut's construction for an  $S^1$ -manifold. Explicit calculation of the space of invariant cochains.

March 21: Questions and ideas related to the March 20 work.

March 23: On the exact sequence  $0 \rightarrow s\mathcal{C}^{n+1} \rightarrow \mathcal{C}^n \xrightarrow{\tilde{s}} s\mathcal{C}^n \rightarrow 0$ . On  $\rho_A \rightarrow k$  as a connection and explicit formulas for  $S$ .

March 26: Karoubi's  $\tilde{\kappa}$  on  $\Omega^1 A$ .

March 27: More formulas related to  $\Omega^n = A \otimes \overline{A}^n$ .

March 28: Analysis of the Goodwillie theorems about derivations.

March 29,30: More on derivations.

April 2: A theorem on exact sequences  $0 \rightarrow X \xrightarrow{i} E \xrightarrow{p} Y \rightarrow 0$ .

April 3,4,6: More on Goodwillie-Rinehart.

April 7: On the map  $b' : A \times \overline{A}^{\otimes n} \otimes A \rightarrow A \otimes \overline{A}^{\otimes(n-1)} \otimes A$ . A contracting homotopy for the Hochschild complex in degrees  $> 1$ .

### 1090-3 April 11: Feit conference. Outline of Serre's lectures on Galois groups and cohomology.

April 12,13: Derivations  $i_D^*$  and  $L_D$  induces by a derivation on  $\Omega A$ .  $I_D^2 = [B, [b, H]]$ .

April 14: Rinehart's formulas.

April 16: On a representations of DG Lie algebra with basis  $L, i$ .

April 25: Facts about mixed complexes.

May 8:  $I$ -adic filtrations.

May 21, 27: Defining  $L_D, i_D^*$  on  $\Omega A$ .

May 8-June 5: Homotopy for  $X(A)$ .

June 8: Continuity of the homotopy with respect to the  $I$ -adic filtration.

June 10: Some ideas.

June 15,21: New idea using the mixed complex  $(\Omega, b, B)$ .

#### 1990-4

July 3: On the super-symmetric time evolution operator  $e^{\tau X + t X^2}$  where  $\tau$  is the Grassmann variable and  $t$  an ordinary variable.

July 5-9: Cyclic homology of  $A$  where  $\Omega^1 A$  is projective,  $A = B/I$  with  $I$  nilpotent.

July 10: Review of earlier work on tensor products.

July 12-14: Coalgebras in the category of  $A$ -bimodules. Bimodule version of the bar construction.

July 16-18: Review of square zero extensions of algebras. Hochschild homology and Connes exact sequence in the case of a presentation  $A = R/I$  with  $R$  smooth.

July 19, 20: On the  $\mathbb{Z}/2$  complex

$$R/I^{m+1} + [R, I^m] \begin{matrix} \xleftarrow{b} \\ \xrightarrow{d} \end{matrix} (\Omega^1 R/I^m \Omega^1 R)_{\natural}.$$

July 27:  $\mathbb{Z}/2$  graded complexes  $X(Q)$  and  $X(Q^s)$  associated to  $Q = QA$  considered either as an algebra or a superalgebra.

August 9,11: On the superalgebra  $A * k[F]$ .

#### 1990-5

August 30- September 6: Differential algebra calculations for subalgebras  $S$  and  $Q$  such that  $S \otimes Q \cong E$ . Relative theory for a map  $S \rightarrow A$  of algebras with relative constructions  $R(A; S)$ ,  $Q(A; S)$ ,  $A *_S A$ ,  $\Omega(A; S)$ ,

September 12: Proof that  $\Omega^1(R; A) \simeq R \otimes_A M \otimes_A R$  where  $R = T_A(M)$  and  $M$  is an  $A$ -bimodule.

September 19, 20: Fredholm modules over  $A$  and calculations with  $EA = A * \mathbb{C}[F] = (QA) \times \mathbb{Z}/2$ .

September 28,29: Rough notes on  $R = S \otimes Q$ .

October 9: On  $\Omega^1 R$ .

October 11: On  $R^e = R \otimes R^o$ .

October 12: Derivations and  $R \otimes R^o$ .

November 1: Higher homotopies for traces. Summary of ideas for future reference: Kunnet theorem; deformation theory of  $P\Omega(A)$ ; maps on periodic cyclic theory and asymptotic maps; using  $X(A)$  to establish periodic cyclic homology;  $(P\Omega, b, \Omega)$  gives cyclic homology and the stabilization mystery behind  $K$ -theory.

November 3: Calculation with the  $I$ -adic filtration on  $R \otimes S$  where  $I$  is the ideal generated by  $[R, S]$ .

November 4: Polynomial families of lifting homomorphisms  $A \rightarrow R$  where  $A = R/I$  and  $I^{m+1} = 0$ .

November 8: Square zero extensions.

November 10: Traces and homology. List of ideas to develop later: Index theory on a torus; Morita type maps; homotopy.

November 11,12: Natural homomorphism:  $K_1^{alg}(A) \rightarrow \text{Ker}\{\Omega^1 A_{\natural} \rightarrow \Omega^0 A_{\natural,\kappa}\}$  given by  $g \mapsto \text{tr}(g^{-1}dg)$ .

November 15: Fedosov's proof of the Index theorem and Connes tangent groupoid.

## 1990-6

November 16: On  $X(R) = \varprojlim X(R/I^n)$ .

November 17: First order derivation of homomorphisms.

November 18: On the projection  $\Omega^1 R \otimes R \rightarrow \Omega^2$  where  $\Omega^1 R$  is a projection.

November 20, 21: Adic topological algebra.

November 21: Deformations and Block's theorem.

November 28: On  $\hat{R} = \varprojlim X(R/I^n)$ . The Hochschild complex  $A \otimes_A^{mathbb{I}} I$  in a derived category framework.

November 29,30: Exploiting results from adic filtrations.

December 2: Reduced cyclic homology.

December 3: Why  $\overline{HC}_n$  and  $H_n^{DR}$  are not Morita invariant.

December 9: Notation for the opposite algebra  $R^o$  and the enveloping algebra  $R^e$ . Summary of identities for Karoubi's  $\kappa$  operation.

December 12: On transformations of finite order.

December 18: Towards understanding homotopy and restricted homotopy.

December 20: Polynomial families of homomorphisms.

December 22: The  $B$  operator on the Hochschild complex associated to  $A = R/I$  where  $\Omega^1 R$  is projective.

December 25,26,28: More on the  $B$  operator on the Hochschild complex.

December 31: To show that the truncated complex  $X^n(R; I)$  is invariant under restricted homotopy.