1990-1

Quillen’s own index for January 22 - April 11, 1990.

January 22: Notes on BRS cohomology.

January 25: An analogy between the \((b, S, 1 - \kappa, B)\) operators and the \((d, i_X, L_X, P_i_X)\) operators for manifolds with a circle action.

February 2: An analogue of the \(S\) operation. Transgression.

February 3: Lundell’s construction deforming \(S^2 \wedge U_n \to U_2 \to U_{2n} \to U_{2n+1}\).

February 4: Two methods for defining classes \(c_{2n-1} \in H^{2n-1}(P), 2n > \text{dim } B\), where \(P \to B\) is a principal \(U_n\)-bundle over \(B\).

February 6: Review of Bott map.

February 10: On \(U_N/\Delta_n S^1 \times U_{N-n}\). Chern-Simons forms on a \(U_n\)-bundle. Variation maps.

February 12, 15, 16: Notes about Feigin-Tsygan on Lie algebra cohomology and Riemann-Roch.

February 20: Lecture on Lie algebra cohomology.

February 21-28: Leray spectral sequence for the principal bundle \(G \to P \to B\). Spectral sequence arising from the bigraded differential algebra \(\Omega(P) \otimes g^* \otimes S g^* \otimes \phi\). Bott’s spectral sequence where \(E_2 = H^{\text{diff}}(G, S g^*) \Rightarrow H^1(BG)\). Review of Leray and Bott spectral sequences.

1990-2

March 11: More on Karoubi’s \(\kappa\) operator.

March 17: Formulas for a circle action on a manifolds and a discrete analogue in cyclic theory.

March 18: More on Karoubi’s \(\kappa\) operator and an \(S\) operator.

March 20: Formulas connected with the periodic complex \(\cdots \to Q \xrightarrow{\delta} (\Omega^1 Q) \xrightarrow{\delta} Q \xrightarrow{\delta} \cdots\). Bismut’s construction for an \(S^1\)-manifold. Explicit calculation of the space of invariant cochains.

March 21: Questions and ideas related to the March 20 work.

March 23: On the exact sequence \(0 \to sC_n^\rightarrow A \to \Omega^1 A \to 0\). On \(\rho_A \to k\) as a connection and explicit formulas for \(S\).

March 26: Karoubi’s \(\tilde{\kappa}\) on \(\Omega^1 A\).

March 27: More formulas related to \(\Omega^n = A \otimes A^n\).

March 28: Analysis of the Goodwillie theorems about derivations.

March 29, 30: More on derivations.

April 2: A theorem on exact sequences \(0 \to X \xrightarrow{i} E \rightarrow Y \to 0\).

April 3, 4, 6: More on Goodwillie-Rinehart.

April 7: On the map \(b' : A \times \Omega^\infty A \to A \otimes \Omega^\infty A \otimes A\). A contracting homotopy for the Hochschild complex in degrees \(> 1\).

1090-3 April 11: Feit conference. Outline of Serre’s lectures on Galois groups and cohomology.

April 12, 13: Derivations \(i_{1,2}^{(p)}\) and \(L_D\) induces by a derivation on \(\Omega A\). \(L_D^2 = [B, [b, H]]\).

April 14: Rinehart’s formulas.
April 16: On a representations of DG Lie algebra with basis $L, i$.  
April 25: Facts about mixed complexes.  
May 8: $I$-adic filtrations.  
May 21, 27: Defining $L_D, i^*_D$ on $\Omega A$.  
May 8-June 5: Homotopy for $X(A)$.  
June 8: Continuity of the homotopy with respect to the $I$-adic filtration.  
June 10: Some ideas.  
June 15, 21: New idea using the mixed complex $(\Omega, b, B)$.  

1990-4  
July 3: On the super-symmetric time evolution operator $e^{\tau X + tX^2}$ where $\tau$ is the Grassmann variable and $t$ an ordinary variable.  
July 5-9: Cyclic homology of $A$ where $\Omega^1 A$ is projective, $A = B/I$ with $I$ nilpotent.  
July 10: Review of earlier work on tensor products.  
July 12-14: Coalgebras in the category of $A$-bimodules. Bimodule version of the bar construction.  
July 19, 20: On the $\mathbb{Z}/2$ complex  

$$R/I^{m+1} + [R, I^m] \xrightarrow{b} (\Omega^1 R/I^m \Omega^1 R)_i.$$  
July 27: $\mathbb{Z}/2$ graded complexes $X(Q)$ and $X(Q^s)$ associated to $Q = QA$ considered either as an algebra or a superalgebra.  
August 9, 11: On the superalgebra $A \ast k[F]$.  

1990-5  
August 30- September 6: Differential algebra calculations for subalgebras $S$ and $Q$ such that $S \otimes Q \equiv E$. Relative theory for a map $S \to A$ of algebras with relative constructions $R(A; S), Q(A; S), A \ast S A, \Omega(A; S)$,  
September 12: Proof that $\Omega^1(R; A) \simeq R \otimes_A M \otimes_A R$ where $R = T_A(M)$ and $M$ is an $A$-bimodule.  
September 19, 20: Fredholm modules over $A$ and calculations with $EA = A \ast \mathbb{C}[F] = (QA) \times \mathbb{Z}/2$.  
September 28, 29: Rough notes on $R = S \otimes Q$.  
October 9: On $\Omega^1 R$.  
October 11: On $R^e = R \otimes R^e$.  
October 12: Derivations and $R \otimes R^e$.  
November 1: Higher homotopies for traces. Summary of ideas for future reference: Kunneth theorem; deformation theory of $P\Omega(A)$; maps on periodic cyclic theory and asymptotic maps; using $X(A)$ to establish periodic cyclic homology; $(P\Omega, b, \Omega)$ gives cyclic homology and the stabilization mystery behind $K$-theory.  
November 3: Calculation with the $I$-adic filtration on $R \otimes S$ where $I$ is the ideal generated by $[R, S]$.  

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November 4: Polynomial families of lifting homomorphisms $A \rightarrow R$ where $A = R/I$ and $I^{m+1} = 0$.

November 8: Square zero extensions.

November 10: Traces and homology. List of ideas to develop later: Index theory on a torus; Morita type maps; homotopy.

November 11,12: Natural homomorphism: $K^1_{alg}(A) \rightarrow \text{Ker}\{\Omega^1 A \rightarrow \Omega^0 A, \kappa\}$ given by $g \mapsto \text{tr}(g^{-1}dg)$.

November 15: Fedosov’s proof of the Index theorem and Connes tangent groupoid.

1990-6

November 16: On $X(R) = \lim_{\leftarrow} X(R/I^n)$.

November 17: First order derivation of homomorphisms.

November 18: On the projection $\Omega^1 R \otimes R \rightarrow \Omega^2$ where $\Omega^1 R$ is a projection.

November 20, 21: Adic topological algebra.

November 21: Deformations and Block’s theorem.

November 28: On $\hat{R} = \lim_{\leftarrow} X(R/I^n)$. The Hochschild complex $A \otimes_{A}^{\mathbb{A}}\text{mathbb{I}}$ in a derived category framework.

November 29,30: Exploiting results from adic filtrations.

December 2: Reduced cyclic homology.

December 3: Why $\Pi C_n$ and $H_n^{DR}$ are not Morita invariant.

December 9: Notation for the opposite algebra $R^o$ and the enveloping algebra $R^e$. Summary of identities for Karoubi’s $\kappa$ operation.

December 12: On transformations of finite order.

December 18: Towards understanding homotopy and restricted homotopy.

December 20: Polynomial families of homomorphisms.

December 22: The $B$ operator on the Hochschild complex associated to $A = R/I$ where $\Omega^1 R$ is projective.

December 25,26,28: More on the $B$ operator on the Hochschild complex.

December 31: To show that the truncated complex $X^\alpha(R; I)$ is invariant under restricted homotopy.