Contents 1984

1984-1
January 1,2: Super-connections and super-traces.
January 4,5: Super-groups and super-Lie algebras.
January 6,7: Super-manifolds.
January 8: Odd $K$-elements.
January 9: Claims for the form $\text{tr}_E(e^{D^2+[D,L]+L^2})$ where $L$ is invertible.

1984-2
January 10: A new proof that the Chern character cohomology class is independent of the choice of connection.
January 11: Checking formulas of January 2.
January 13: Some Banach algebra formulas.
January 18: Notes on a wide ranging conversation with Connes.
January 24: Description of various notational conventions.
January 25: Comment on $K$-theory of Clifford algebra bundles.
January 27: Discussion of local Index formula for a family of Dirac operators with coefficients in a vector bundle.
January 28, 29: Bott’s theorem on Chern numbers. Witten’s cohomology identified with localized equivariant cohomology.
January 31: Notes on a conversation with Scott Petrach on Duistermatt-Heckman type proof of the Index theorem. Atiyah’s summary of a talk by Berry.
February 1: Conversation with Connes. Cyclic theory for a ring with no unit.
February 2: Account of conversation with Katz, Gabber and Deligne.
February 3: Conversation with Connes on $H^*(\Gamma) \to H^*_A(\mathbb{C}[\Gamma])$, wave packet transform and Novikov conjecture.
February 4: Connes method for extending cocycles from $A$ to $\text{End}_A(E)$ where $E$ is a projective module over $A$.

1984-3
February 5: Morita invariance of cyclic cohomology.
February 6,7: Connes construction to kill curvature.
February 8: Connes proof of Morita invariance.
February 9: Connes approach by tensoring an elliptic operator with a vector bundle. Relating cyclic cocycles and heat kernels.
February 10: Characterizing Dirac operators.

February 11: Constructing a cyclic cocycle belonging to a Dirac operator.

February 12: Constructing cyclic cocycles representing the Chern character of a Dirac operator.

February 17: Kasparov $K$-theory of graded $C^*$-modules.

1984-4

February 18: Check the Index theorem (using a heat kernel method) over a torus for the Dirac operator, $-i\not\!D = -i\gamma^\mu D_\mu + \epsilon L$ acting on $S \otimes E$ where $L$ is a selfadjoint operator.

February 19: Formula for the wave packet formula in Euclidean space. Bott’s idea related to his work on the Lefschetz fixed point theorem.

February 20: On a rigorous proof of the classical limit formula $\text{Tr}(e^{-\beta H}) \sim \int \left( \frac{dpdq}{2\pi\hbar} \right)^n e^{-\beta H}$ where $H = \frac{p^2}{2} + V(q)$ is quantized as the operator $-\hbar^2 \nabla^2 + V(x)$. On a proof of the Index theorem using the idea that the quantum partition function has a classical limit which is an integral over the cotangent bundle.

February 21: On Connes groupoid and algebra of operators depending on $h$.

February 22: The Index theorem for the Dirac operator interpreted as a supersymmetry operator. Partition function, Clifford algebras and Weyl algebra.


February 25: Fundamental class in $K$-theory.

February 26: Attempt to derive a proof for the Index theorem for the Dirac operator using an isometric embedding.

February 27: Thom isomorphism in $K$-theory for complex vector bundles.

February 28: Further discussion of the Index theorem for Dirac operators and a generalisation to an assertion about maps.

1984-5

February 29: Computation of the character of the Thom class $i!$ in the $K$-theory of a vector bundles equipped with a metric, connection and Spin$^c$ structure.

March 1: On the Berezin determinant.

March 3: Correspondances between approaches to the Index theorem in algebraic geometry and the Riemann-Roch theorem for complex manifolds.

March 4: Further discussion of the Index theorem for the Dirac operator.

March 5: Concept of a Dirac operator. Conversation with Graeme Segal.

March 6: Discussion of Riemannian geometry calculations connected with a family of Dirac operators.

March 7,8: Return to the calculation of $\text{ch}(i!1)$ where $i : M \to E$ is the zero section of a complex vector bundle.

March 8: On a formula for $\det w e^{-Jw^{-1}J}$.

March 9: Classical mechanics, quantum mechanics and classical limit formalism.

March 10: Review of the Thom class formula.
March 11: Signature operator and Gaussian-Thom form.
March 12: Discussion on the Dirac operator and $d + d^*$. 

1984-6

March 13: Splitting the operator $d + d^*$ on a Kahler manifold. Notes for a lecture on the Chern character form $\text{tr}_s \left( e^{(D+L)^2} \right)$.
March 14: The Thom form on the tangent bundle.
March 15: Conversation with Atiyah on a local Index theorem for connections with non-zero torsion. Convolution algebra of differential forms.
March 16, 17: Discussion of the limiting heat kernel as an $n$-form on the tangent bundle.
March 20: Index theorem on a torus for the Dirac operator with coefficients in a vector bundle equipped with a connection.
March 21: Limiting heat kernels.
March 23: Review of recent work.
March 24: Classical limit for the heat kernel for the Dirac operator with values in a vector bundle with a connection.
March 25: Discussion of Friedan-Winney paper.
March 26: Fermion Lagrangian quantization.
March 27: When does a quadratic form $S(q, q')$ define a symplectic transformation $(q', p') \mapsto (q, p)$?
Hyperbolic quadratic forms and orthogonal transformations.

1984-7

March 28: Path integral approach to the heat operator $e^{-\beta H}$ and the limiting heat operator.
March 29: The super-group $\mathbb{R}^{1,1}$, its convolution algebra, left and right infinitesimal translations: $\partial_\theta \mp \theta \partial_t$.
March 30: Critical points for the fermion Lagrangian.
April 1: Further discussion of the limiting heat kernel.
April 2, 3, 4: Motion in a uniform magnetic field.
April 6: Dirac operator, general metric, coframe $w^m$ calculation.

1984-8

April 7: Review calculation of April 6. Getzler’s filtration. Dirac operator for a constant magnetic field.
April 8: Notes on a conversation with Atiyah in Witten’s office.
April 9: Problems with the idea of a classical limit for the super-heat kernel. Parallel transport using a path integral.
April 10: Graded subalgebras of $A \otimes k[h]$ where $A$ is an algebra over $k$.
April 11: Further thoughts on the local Index theorem.
April 12: The Grassmannian graph construction.
April 15: An extension of the algebra of asymptotic differential algebras.
April 18: Process of going from the cyclic cohomology of $\Omega^0(M)$ to that of $\Omega^0(M, \text{End}(E))$ and restriction $\Omega^0(M) \hookrightarrow \Omega^0(M, \text{End}E)$ with applications to cyclic cohomology.

April 20: Harmonic oscillator heat kernel.

April 21: Implementation of the isomorphism taking the cyclic cohomology of $\Omega^0(M)$ to that of $\Omega^0(M, \text{End}E)$ by the cochain $\int dt \text{tr} \left( e^{D^2 + t[D, \theta] + (t^2 - t)\theta^2} \right)$.

April 22: Review of November and December’s work on transgression. More discussion on a local Index theorem.

1984-9

April 23: Relationship between the Chern character associated with a super-connection and that coming from the Grassmannian graph construction.

April 29: Summary of recent ideas.

April 30: Constructing cocycles from odd derivations and related universal algebra.

May 2: Twisted polynomial rings and a new approach to the heat kernel of the harmonic oscillator.

May 3: Existence of the heat operator $e^{tD^2}$ in the Weyl algebra of operators of the form $\int d\sigma f(v)e^{Dv}$.

May 4, 5: Deriving the trace of $e^{tD^2}$ using the path integral. Derivation of the Schrödinger kernel of the harmonic oscillator using the explicit metaplectic formula.

May 6: Weights in a Von Neumann algebra, KMS condition.

May 7: Computing $\text{Tr}(e^{-H}e^{wD})$ where $H = -D^2$ and $vD = v^\mu D_\mu$ using the KMS condition.

May 8: Calculations associated with showing the existence of the heat operator.

May 9: Fundamental solution of the heat operator.

May 10: Construction of the heat operator $e^{-tH}$ where $H = p^2 + V(q)$.

May 11: Hadamard’s method of constructing the heat kernel for the Laplacian of a curved manifold.

1984-10

May 12: Volterra’s method for constructing $e^{-tH}$. Discussion of a method of proof of the existence of the heat operator for the harmonic oscillator by deformation of the classical case where $\hbar = 0$.

May 13: The heat operator for the covariant derivation Laplacian with coefficients in a vector bundle.

May 14, 15: Working with pseudo-differential operators in $\mathbb{R}^n$.

May 16: Review of past few day’s work. Attempt to construct $e^{-tH}$ as a kernel on the tangent groupoid of a manifold.

May 17, 18, 19: Attempt to put together a complete proof of the existence of the heat operator depending on Plank’s constant, together with an asymptotic evaluation of the trace as $\hbar \to 0$.

May 20: Review parametrix method and Fredholm theory.

May 21: Review local Index theorem for families.

May 22: Proof of Index theorem for $M = \mathbb{R}^n/\Gamma$.

May 24: On Kasparov $K$-theory.
May 25: Character form and examples coming from projections. Analogue for the plane of the Hilbert operator. Fourier transform of \( \frac{1}{|z|^k}. \)

May 26: Operators in Kasparov’s \( K \)-theory. Cocycle computation for constant coefficient operators in \( \mathbb{R}^n. \)

May 28: Discussion of the general picture for families of Dirac operators.

May 29: Laplace transfer of \( \text{tr}_s \left( e^{(L+2dL)} \right) \) where \( L \) is a family of odd-degree skew adjoint operators endomorphisms of a super-vector space. Grassmannian graph construction.

May 30, 31: Grassmannian graph approach to the local Index theorem. Representation of the \( k \)th Chern character form as \( \frac{1}{2} \text{tr}_s \left( \frac{\sqrt{s}}{\sqrt{\lambda}} dL \right)^{2k}. \) Links between the heat kernel and resolvant approaches to the Chern character.

June 2: Transgression problem from the point of view of the Grassmannian graph construction.

1984-12

June 10: Transgressing the character form \( \text{ch}_k = \frac{1}{2} \frac{1}{k!} \text{tr}_s \left( \frac{\sqrt{s}}{\sqrt{\lambda}} dL \right)^{2k} \) to the group acting on the Grassmannian graph.

June 11: Abstract discussion of character forms.

June 24: Review of the relationship between character forms defined by the super-connection formalism and that by the Grassmannian graph method, i.e., the link between the heat operator approach and the parametrix approach.

June 25: Review of Witten’s approach to the determinant line bundle over a space of connections.

June 26: Baum-Connes problem: to define a good notion of equivariant cohomology.

June 28: Review of transgression situation.


June 30: Understanding of the relationship between the super-connection and the Grassmannian graph formalisms. Discussion of the super-character: \( \text{tr}_s e^{(L+|D|+D^2)} \)

July 1, 2: Work on an attempt at writing paper for Gelfand on the determinant line bundle.

July 7: Description of two interesting problems.

July 9: Link between Connes \( S \)-operator and Bott periodicity. Discussion of the bilogarithm and Bott periodicity. Local Index theorem for families - odd degree case.

July 10, 11: Work on local Index theorem for Dirac operators using super-connections. The \( \eta \)-invariant for the Dirac operator over a circle.

1984-13

July 12: Further discussion of the transgression process. Comment on multiplicative \( K \)-theory.

July 13: Constructing regulator maps \( K_{2n-1} \mathbb{C} \to \mathbb{C}^\times \) by Karoubi’s method using relative \( K \)-theory and Deligne cohomology. Discussion of parallel transport from the space of connections on the trivial bundle over \( S^1 \) to the unitary group.

July 14: Construction of left invariant character forms on the loop group.

July 15: Dirac operators and Weitzenbock formula using the principal frame bundle.

July 17, 18: Link between super-connections and the \( \zeta \)-formula for \( \text{ch}_1 \) of the determinant line bundle. Odd version of Grassmannian graph.
July 17, 18: Transgression for the $e_1$ class. Bott periodicity from the differential point of view.
July 19: Continuous cohomology and periodicity. Dilogarithm.
July 20: Connes description of the multiplicative map on $K^{	ext{alg}}_1(A)$ associated to a trace on $A$. Novikov conjecture and Mischenko’s construction.

1984-14
July 21: On Bismut’s proof of the Index theorem for the case when $M = \mathbb{R}^n/\Gamma$.
July 24: Ito’s equation.
July 25: More on Bismut’s paper and Ito’s ideas. Review of what we know on the Index theorem and path integrals.
July 26, 27: Physicist’s proof of the Index theorem using path integrals and fermion integrals. Comparison with Bismut’s approach. Review of the super-group $R^{1,1}$ and Freidan-Windey. Bismut’s factorization of parallel transport in $S \otimes E$.
July 28: Review construction of $\langle x_0 | e^{-\frac{h}{2} \partial^2} | x_0 \rangle$. Random walk on $\mathbb{C}^*$: $dz_t = z_t i dw_t$, where $w_t$ is Brownian motion.
July 29: Understanding the fermion integral: $\int \mathcal{D}\psi(t) e^{i(\psi\dot{\psi} - \psi A \psi)} dt$. Idea’s behind Bismut’s paper. Possible approach to computing the heat kernel in a Weyl algebra.
July 30, 31: Fermion integrals and quantization. Some super-connection identities.

1984-15
August 1: Bismut’s form on $\mathcal{L}M$, the loop space of $M$.
August 2: Interpretation of equivariant cohomology of $\Omega M$ and $\Omega BU(k)$. The $\Omega BU(1)$ case - Bismut’s form is not an equivariant form but a Witten form.
August 3: Differential geometry of the normal bundle. Comment on Bismut’s map $H(\Omega^*(\Omega M)^{S^1}, d - ui_X) \to H_{DR}(M)$. Comments on ‘The Index Theorem and Equivariant Cohomology of the Loop Group’, by J-M Bismut.
August 4: Continuing with attempt at a local Index theorem. Review of Godbillon-Vey class and characteristic class for a codimension $n$ foliation with trivial normal bundle.
August 5: Calculations for a 1-parameter family of connections.
August 6: $\partial_t \mathcal{D}$ for the Dirac operator associated to a 1-parameter family of metrics.
August 7: Transverse connection on $T_{X/Y}$.
August 8: Curvature calculations.

1984-16
August 18: Berligne-Vergne proof of the Index theorem.
August 19: The Dirac operator on a Riemann surface.
August 20, 22: Bismut’s Witten form on $\mathcal{L}M$.
August 23: Constructions using $X/Y$, and $S^1$ bundle over base $Y$.
August 24: Bismut’s form is defined for an $S^1$ action. Analogies between cyclic theory and equivariant cohomology of the free loop space. Links between $\mathcal{L} M$ and $S^{-1} \text{Ext}_A(k^3, A^2)$.
August 29: Summary of recent ideas (generalize Bismut to Kac-Moody).
August 30: Gaussian processes.
August 31: Comment on constructing the heat operator.

September 1: Can the heat operator be constructed via
\[ T \left( e^{\int dt \psi_t \psi} \right) = e^{-t \nabla^2 + \theta D?} \]

September 2: Applying Connes-Gezler theory.

September 5: The Clifford algebra as a deformation of the exterior algebra and an attempt at an analogous treatment of the Weyl algebra.

September 6,7: Twisted polynomial algebra. The convolution algebra belonging to the Weyl algebra and formula for \( e^{-t \rho^2} \).

September 9: Review of standard tensor calculus formulas.

September 17: Singer’s suggestion that \( \text{tr} \left( e^{\int (-\dot{x}^\alpha A_\alpha(x) + \frac{1}{2} \psi^\alpha \psi F_{\mu \nu}(x)) dt} \right) \) is the trace of super-parallel transport.

September 18: Formulas in Getzler’s proof of the Index theorem.

1984-17

October 15, 16: Construction of the heat operator by constructing its kernel invariantly using the filtration idea.

October 18: Formal construction of \( e^{-tH} \) at \( t = 0 \) using the nilpotency of \( t, (t^2 = 0) \). Relations between Berligne-Vergne forms and Chern forms.

October 19, 20: Attempt to understand how to construct the Thom form for a complex vector bundle \( E \) using \( \mathbb{F}(E) = S/S^1 \) and \( \xi^n + c_1(E) + \cdots + c_n(E) = 0 \). Review Gaussian-Thom form coming from the super-connection formalism.

October 21: Chern classes of representations.

October 22, 23: The formula \( U = \det e^{-d\frac{1}{2}dz|z|^2} \) for the Thom form. Pfaffian and related formulas.

October 26: Constructing heat operators \( e^{-tH} \) where \( H \) is a Laplacean type operator. Review of Atiyah’s lecture on mixed volumes and isoperimetric inequalities.

October 27, 28: Moment map calculations. Relations between convex bodies, toral varieties and the moment map as defined by Atiyah.

October 29: Strrook’s remarks on Stratonovich’s versus Ito’s stochastic differential equations.

October 31, November 2,3: Constructing \( e^{t \Delta} \) where \( \Delta = \rho^\mu \partial_i \partial_j + \text{lower order terms} \).

November 4: The algebra of smooth kernels in the tangent groupoid.

November 5: Equivalent descriptions of a singularity.

November 6,7: Thom form and Bott’s residue calculations.

November 8: Feynman’s formula: \( \frac{1}{\pi} = \int_0^1 \frac{dt}{((1-t)a + t b)^2} \).

November 9: Singular transgression form.

November 10: Proof that \( c_1(\theta(1) \otimes \pi^* E) = 0 \) in \( H^*(PE) \) using differential forms.

November 11: Physicist’s integration process for differential functions on the loop space \( \mathcal{L}M \).

November 12: Constructing heat operators.

November 13: Reflection positivity.

1984-18

November 14: Review of geometric approach to the heat operator.
November 15: Homogeneous distribution on $\mathbb{R}^n$. Kernels on the tangent groupoid.

November 17: Gaussian measure. Current operators.

November 18: Return to construction of $e^{t\Theta + \theta D}$. Bismut’s separation of parallel transport in $S \otimes E$ using Brownian motion.

November 19: Summary of links between the fermion integral, Bismut’s use of Brownian motion in $\text{Lie}(\text{Spin}(n))$ and Vergne’s Laplacian in the group direction.

November 20: Roe’s thesis.

November 22: Comments on the map $\Omega(\mathcal{L}M)^{S^1} \to \Omega(M)$.

November 26: Notes on the basic 2-form on $\mathcal{L}M$ coming from the Riemannian structure on $M$.

November 27-30: Integrating differential functions on $\mathcal{L}M$.

December 1: Fermion integration formulas.

December 2: On the skew form $w(f, g) = \int_0^1 fg' dt$, a generalization and the fermion integral.

December 5,6: Weiner process on the line and related problems.

December 7: Notes on Bismut’s construction. Bosonic and fermionic algebras.

December 8-11: Gaussian measure.

December 12: Equivariant cohomology $H_{S^1}(\mathcal{L}BU(1))$.

December 19-26: On $\Lambda(V)$ and why $\det(w)e^{-\nabla + J}$ is denominator free. Pfaffian algebras and details for a paper with Matthei.

1984-19
Lecture notes on infinite determinants.

1984-20
Notes on lectures by Connes and Hurder.